

A GEOMETRIC MODEL FOR EXTENDED PARTICLES

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Abstract. Here we combine the ideas of a quantum functional theory intended to describe intrinsically extended particles with those of a geometro-stochastic one describing stochastically extended particles. The main ingredients of the former are a physical wave u replacing the point \mathbf{x} and a functional wave $X[u, t]$ replacing the probability wave function of the conventional quantum theory. The latter introduces a proper wave function accounting for the unavoidable errors in the measurement of continuous observable such as the position and momentum.

1. Introduction

In the nineteen fifties of previous century, Destouches [1, 2] developed his functional quantum theory as a generalization of de Broglie's theory. His basic idea was that elementary particles need not be pointlike. Being extended and non rigid is a better conception. Rather than conceiving the particle as a bulk of fluid, we have supposed that it is composed of pointlike quantum modes. This enabled the construction of our **Geometro-Differential Model** (G-D-M) for extended particles and its quantization by a method of induced representation [3, 4, 9, 10]. The geometric structure have been drawn from a recent **Geometro-Stochastic Theory** (G-S-T) which seems to be a candidate for the unification of quantum mechanics and general relativity devoid of many of the inconsistencies of both theories [6–8]. It deals with an extension of particles attributed to the impossibility of sharply measuring a position (or momentum) of a particle. It is a stochastic extension. The aim of the present work is to describe a scheme of the extended particles which incorporate both the intrinsic and stochastic extensions. To achieve this, we shall combine our G-D-M with the G-S-T.

2. Functional Quantum Theory

Inconsistencies of quantum mechanics led some researchers to the conclusion that their cause lies in the pointlike conception particles. So, let us begin with the quantum functional theory which is one of the oldest quantum theories advocating an extension for extended particles. Its main feature is that a physical system is not really distinguishable from the remaining part of the Universe. Representing the action of the latter on the system is an approximation. A better approximation can be arrived at, when the exterior can influence the intrinsic characteristics of the system. As a consequence, an elementary particle may be represented by a function (the physical wave) u describing these characteristics and, as such, it must be conceived as a non rigid extended body.

To understand the physical wave, recall that in classical mechanics a point particle is represented by a function $\mathbf{x}(t) \in \mathbb{R}^3$ of time giving its successive positions. In quantum mechanics, it is represented by a variable \mathbf{x} as the argument of the wave function. In the functional theory, this point is replaced by the physical wave u depending on a spatiotemporal variable ξ having no physical meaning. The position \mathbf{x} becomes a functional $F[u]$ generalizing de Broglie's singularity.

The abstract function u can be treated by associating a specific physical model such as a bulk of fluid. Then, it obeys an ordinary quantum mechanical wave equation with a non linear quantum potential Q [1].

Since the conventional point \mathbf{x} is replaced by a function u , the conventional wave function $\psi(\mathbf{x}, t)$ has to be replaced by a functional X

$$X[u, t] \tag{1}$$

depending on u . This functional yields probabilities and obeys a spectral decomposition

$$X[u, t] = \sum_i a_i(t) X_i[u]. \tag{2}$$

In the pointlike approximation, it should become a wave function $\Psi(\mathbf{x}, t)$

$$X[u, t] \rightarrow \Omega X[F[u], t] = \Psi(\mathbf{x}, t) \tag{3}$$

that yields previsions on the point \mathbf{x} and is the analogue of the conventional wave function $\psi(\mathbf{x}, t)$.

3. Geometro-Differential Model

First, we adopt the most important physical foundations of the functional theory. Namely, the particle is extended and non rigid. Its intrinsic characteristics can be influenced by the exterior. However, we change the notation u to Ψ and choose a different physical model to treat this physical wave. The functional is a bilocal

field with an external and an internal part. It may be of a product form but this is not the general case

$$X[\Psi](x, \xi) = \psi(x, \xi) = \psi(x)\Psi_x(\xi). \quad (4)$$

The points $x = (x^0, \mathbf{x})$ belong to an external Minkowski space $M \simeq \mathbb{R}^4$ and the points ξ belong to an internal Minkowski space-time M' .

The intuitive image of the extended particle is that the internal part corresponds to an internal quantum mode described by the physical wave and localized in the internal space-time of the particle. The particle as a whole is considered as an external mode localized in the external space-time. The quantization of both modes proceeds with the method of induced representation [5]. We shall present the latter in connection with a pointlike relativistic particle. For this purpose, we consider the Poincaré group P which transforms points of the Minkowski space M through translations a and Lorentz matrices l

$$x' = g(a, l) = lx + a. \quad (5)$$

The group law shows a semidirect form

$$g(a, l)g(a', l') = g(a + la', ll'). \quad (6)$$

The Lorentz transformations $l = rv_L$ are composed of ordinary rotations r and velocity boosts v_L and leave invariant the Minkowski scalar product $\eta_{ij}x^i x^j$. The quotient of this group with respect to the Lorentz subgroup yields the Minkowski space

$$M = P/L. \quad (7)$$

The reducible configuration induced representation $\widehat{U}(P)$ is obtained from the irreducible representation $D(L)$ of the Lorentz subgroup

$$\widehat{U}(P) = D(L) \uparrow P. \quad (8)$$

The action on an element $\widehat{\psi}$ of the Hilbert space \widehat{H}

$$[\widehat{U}(a, l)\widehat{\psi}](x) = D(l)\widehat{\psi}(l^{-1}(x - a)) \quad (9)$$

shows that $D(l)$ mixes the spin components of the wave function. A parallel construction can be carried out when the subgroup is a semi-direct product of translations and rotations

$$H = [T]R. \quad (10)$$

The momentum space is a homogeneous space with respect to the subgroup H

$$C = P/H. \quad (11)$$

A unitary and irreducible representation of H is

$$\Delta^{ms}(ar) = \exp(ima \cdot e_0)\Delta^s(r), \quad e_0 = (1, 0, 0, 0). \quad (12)$$

It contains the well known representation Δ^s of rotations of spin s and an exponential of the time component of the translation vector multiplied by the mass m .

The induced momentum representation

$$\tilde{U}^{ms}(P) = \Delta^{ms}(H) \uparrow P \quad (13)$$

$$[\tilde{U}^{ms}(a, l)\tilde{\varphi}^{ms}](v) = \exp(im a \cdot v) \Delta^s[(l, l^{-1}v)_R] \tilde{\varphi}^{ms}(l^{-1}v) \quad (14)$$

obtained in this way is irreducible and labeled with the mass m and spin s of the particle. The representation of rotations contains a Wigner rotation $(l, l^{-1}v)_R$ and mixes spin components of $\tilde{\varphi}^{ms} \in \tilde{H}^{ms}$.

The reducibility of the configurational representation entails that the corresponding localized states $\hat{\psi}(x)$ may be virtual. In contrast, the momentum states $\tilde{\varphi}_{ms}(v)$ can be interpreted as real or material states. Hence, the intertwining operator K_{ms} projecting the configurational representation onto the momentum representation can be interpreted as a process of materialization of the localized state. The range of the inverse intertwining operator I_{ms} is an irreducible component of the configurational representation and corresponds to a process of localization of the material state. The obtained states are both material and localized.

Now, we just give the integral forms of the intertwining operators corresponding to the processes of materialization and localization where the plus and minus signs refer to particles and antiparticles, respectively.

Localization (I_s^\pm is a constant matrix and $d\Omega_1^\pm(v) = d^3\mathbf{v}/v^0$):

$$I_{ms}^\pm \tilde{U}^{ms} = \hat{U} I_{ms}^\pm \quad (15)$$

$$\begin{aligned} \hat{\psi}^{ms, \pm}(x') &= (I_{ms}^\pm \tilde{\varphi}^{ms})(x') \\ &= \frac{m}{4\pi^{3/2}} \int_{C_1^\pm} d\Omega_1^\pm(v) \exp[\mp(imv \cdot x')] D(v_L) I_s^\pm \tilde{\varphi}^{ms}(v). \end{aligned} \quad (16)$$

Materialization (K_s^\pm is a constant matrix and $d\mu(x) = d^4x$):

$$K_{ms}^\pm \hat{U}(P) = \tilde{U}^{ms}(P) K_{ms}^\pm \quad (17)$$

$$\begin{aligned} \tilde{\varphi}^{ms, \pm}(v) &= (K_{ms}^\pm \hat{\psi})(v) \\ &= \frac{m}{4\pi^{3/2}} \int_M d\mu(x) \exp[\pm(imv \cdot x)] K_s^\pm D(v_L^{-1}) \hat{\psi}(x). \end{aligned} \quad (18)$$

The composition of a materialization followed by a localization yields a propagation of localized states

$$\Pi_{ms}^\pm = I_{ms}^\pm K_{ms}^\pm, \quad \hat{\psi}^{ms, \pm}(x') = (\Pi_{ms}^\pm \hat{\psi})(x') = \int_M d\mu(x) \Pi_{ms}^\pm(x', x) \hat{\psi}(x). \quad (19)$$

The integral form of the propagator is obtained from the integral forms of $I_{m,s}^\pm$ and $K_{m,s}^\pm$

$$\Pi_{m,s}^\pm(x', x) = \frac{m^2}{2(2\pi)^3} \int_{C_1^+} d\Omega_1^+(v) \exp[\mp imv \cdot (x' - x)] S(v_L). \quad (20)$$

The spin matrix $S(v_L) = D(v_L) I_s^\pm K_s^\pm D(v_L^{-1})$ equals one for the scalar particles and the propagator is identified with the Dyson function. The causal propagator is then obtained in the usual way by $\Pi_{m,s}^c(x) = \theta(x^0) \Pi_{m,s}^+(x) + \theta(-x^0) \Pi_{m,s}^-(x)$.

Returning to our extended particle, recall that it is composed of an external mode with mass m and spin s and an internal mode with mass μ and spin σ . This conception can be described by a fiber bundle. The external space-time M constitutes the base manifold and the internal M' space corresponds to the fiber. States of the internal mode belong to the fiber of a Hilbert bundle

$$E^D(M, \hat{H}, \hat{U}) \quad (21)$$

with the same base, the Hilbert space \hat{H} as a total space and the induced representation \hat{U} as the structural group. The functional wave is a bilocal field where the two modes are quantized with the inducing method

$$X[\Psi](x, \xi) = \hat{\psi}(x, \xi) = \hat{\psi}(x) \hat{\Psi}_x(\xi). \quad (22)$$

In order to apply this method, we can use four fiber bundles exhausting all combinations of configuration and momentum representations for both the external and internal modes

$$E(M, \hat{H}, \hat{U}), \quad E^{\mu\sigma}(M, \tilde{H}^{\mu\sigma}, \tilde{U}^{\mu\sigma}) \quad (23)$$

$$E_{m,s}(C, \hat{H}, \hat{U}), \quad E_{m,s}^{\mu\sigma}(C, \tilde{H}^{\mu\sigma}, \tilde{U}^{\mu\sigma}). \quad (24)$$

All possible intertwining operators give the same results. One way is to achieve a complete materialization of a completely localized state $E \rightarrow E_{m,s}^{\mu\sigma}$. Then, a complete localization $E_{m,s}^{\mu\sigma} \rightarrow E$ leads to the propagator

$$\Pi_{m,s}^{\mu\sigma}(x', \xi'; x, \xi) = \Pi_{m,s}^\pm(x', x) \otimes \Pi^{\pm\mu\sigma}(\xi', \xi). \quad (25)$$

The latter is a tensor product of the external and internal pointlike propagators (20). Interaction is represented by a connection Γ acting on the internal propagator for infinitesimal paths

$$\begin{aligned} & \Pi_{m,s}^{\mu\sigma}(x_n, \xi_n; x_{n-1}, \xi_{n-1}) \\ &= \Pi_{m,s}^\pm(x_n, x_{n-1}) \otimes \hat{U} \left(\exp \int_{x_{n-1}}^{x_n} \Gamma(x) \right) \Pi^{\pm\mu\sigma}(\xi_n, \xi_{n-1}). \end{aligned} \quad (26)$$

The total propagator is a path integral of infinitesimal propagators

$$\Pi_{ms}^{\mu\sigma}(x, \xi; x', \xi') = \lim_{N \rightarrow \infty} \int \prod_{n=1}^N \Pi_{ms}^{\mu\sigma}(x_n, \xi_n; x_{n-1}, \xi_{n-1}) \prod_{n=1}^{N-1} d\sigma(x_n) d\mu(\xi_n) \quad (27)$$

where $d\sigma(x_n)$ and $d\mu(\xi_n)$ are invariant measures of a space-like slice (foliation) of the external space-time and the internal space-time, respectively.

4. Geometro-Stochastic Theory

Our model of extended particles has many mathematical similarities with a geometro-stochastic theory but actually it is very different from the physical point of view. The G-S-T claims a consistent unification of the quantum theory and general relativity by analyzing and unifying their most fundamental principles. It is based on two components. The stochastic component is related to measurement theoretical issues dealing with the localization and extension of particles and accounts the inaccurate nature of any actual measurement of position and momentum, i.e., an irreducible indeterminacy. This implies that all particles have a stochastic extension accounting for that indeterminacy. Such test particles may play the role of quantum micro-detectors. On the other hand, the geometric component is based on an operational definition of space-time whereby its classical nature is questioned in the quantum realm since the classical test particles serving its definition are absent. If they are replaced by stochastic test particles, a notion of local quantum frame can be defined and used to construct a quantum space-time as a Hilbert bundle.

Let us begin with the stochastic theory whose interpretations are easier to understand in the non relativistic case. The main idea is that all measuring apparatuses are imperfect and plagued with an irreducible indeterminacy that should be considered in the formalism of a quantum theory. For instance, when a measurement of position yields a value $\mathbf{x} \in \mathbb{R}^3$ it is interpreted as the real position of the particle in conventional quantum mechanics. However, in stochastic quantum mechanics, the measurement may yield another value $\mathbf{q} \in \mathbb{R}^3$ with a confidence function (or probability density) $\chi_{\mathbf{q}}(\mathbf{x})$. The corresponding probability for \mathbf{q} to belong to a Borel set $\hat{\Delta}$ is a confidence measure

$$\mu_{\mathbf{q}}(\hat{\Delta}) = \int_{\hat{\Delta}} \hat{\chi}_{\mathbf{q}}(\mathbf{x}) d\mathbf{x}. \quad (28)$$

A stochastic value $(\mathbf{q}, \mu_{\mathbf{q}})$ is the association of an ordinary value \mathbf{q} to its confidence measure $\mu_{\mathbf{q}}$. The configuration and momentum confidence functions

$$\hat{\chi}_{\mathbf{q}}(\mathbf{x}) = (2\pi)^3 |\hat{\eta}(\mathbf{x} - \mathbf{q})|^2 \quad \text{and} \quad \tilde{\chi}_{\mathbf{p}}(\mathbf{k}) = (2\pi)^3 |\tilde{\eta}(\mathbf{k} - \mathbf{p})|^2 \quad (29)$$

are related to functions $\hat{\eta}$ and $\tilde{\eta}$ which can be interpreted as proper state of stochastically extended microdetectors. These functions uniquely determine irreducible phase space representations.

Transition to the relativistic case can be made by generalizing the Galilei group theoretic definitions to the Poincaré group and considering a space-like hypersurface $\Sigma = \sigma \times V_m^+$ of the relativistic phase space. The space-like hypersurface σ of the Minkowski space M has invariant measure $d\sigma(q)$ and V_m^+ is the mass hyperboloid with invariant measure $d\Omega(k) = d^3\mathbf{k}/2k^0$. Note that the induced representation method uses the unit mass hyperboloid C_1^+ with $d\Omega_1^+(v) = 2d\Omega(v = k/m)$. We have not unified the stochastic and induced representation conventions yet. One starts with the proper state vector $\tilde{\eta}$ describing a stochastically extended test particle marking the origin of a quantum Lorentz frame. Translating it with the amount q and boosting it to the momentum p yields the proper state vector

$$\tilde{\eta}_{q,p}(k) = [\tilde{U}(q,p)\tilde{\eta}](k) = \exp(ik \cdot q)\tilde{\eta}(p,k) \quad (30)$$

of another particle at stochastic position q , having stochastic momentum $p = mv$, and marking another point of the same quantum frame. Projection of generic state vectors $\psi_\eta \in H_\eta \subset L^2(\Sigma)$ on the proper state vectors

$$\psi_\eta(q,p) = \langle \tilde{\eta}_{q,p} | \tilde{\psi} \rangle = \int_{V_m^+} \tilde{\eta}_{q,p}^*(k) \tilde{\psi}(k) d\Omega(k) \quad (31)$$

defines an irreducible phase space representation. Probabilities are expectation values of positive operator valued (POV) measures defined with the proper state vectors

$$P((q,p) \in \Delta) = \langle \psi | E(\Delta) \psi \rangle = \int_\Delta d\Sigma(q,p) |\psi_\eta(q,p)|^2 \quad (32)$$

$$E(\Delta) = \int_\Delta |\eta_{q,p}\rangle d\Sigma(q,p) \langle \eta_{q,p}|. \quad (33)$$

Here Δ is a Borel set in Σ . The invariant measure of Σ is $d\Sigma(q,p) = d\sigma(q) d\Omega(k)$. The propagators are also defined by means of the proper state vectors

$$K(q,p; q', p') = \langle \tilde{\eta}_{q,p} | \tilde{\eta}_{q',p'} \rangle. \quad (34)$$

The second component of the geometro-stochastic theory is its fiber bundle geometric structure

$$E(M, H_\eta, U(P)) \quad (35)$$

lifting the classical nature of space-time to a quantum one. The stochastic quantum mechanics Hilbert space H_η becomes a fiber over the space-time M of mean locations x . The role of structural group is played by the stochastic phase space representation U_η . The main point is that the proper state vectors can be considered as a local quantum frame Φ_x

$$\Phi_x = \{\eta_{x;(q,p)}; (q,p) \in M \times M'\} \quad (36)$$

with no contradiction with the uncertainty principle. Then all interactions, including gravity, appear as connections. The above definitions have been oversimplified (a rigorous exposition can be found in [7] and [8]).

5. Intrinsically and Stochastically Extended Particles

We have presented two approaches for possible extension of the notion a particle. It may be considered as intrinsically extended as in our G-D-M based on the functional theory, or it can be considered as stochastically extended according to the G-S-T. We think that the G-D-M can be improved by incorporating the stochastic component in it. Recall that our extended particle is composed of two modes. The internal mode, being indirectly accessible to measurement, may be considered as pointlike. In contrast, the external mode is directly accessible to measurement and can be described stochastically. We assume that both modes are scalar particles ($s = \sigma = 0$) and anti-modes non considered for simplicity.

We can construct four state spaces. First, the space where the external and internal modes are both in the momentum representation

$$L^2(V_m^+ \times C_1^+) = \{\tilde{\Phi}_m^\mu(k; \zeta)\}, \quad \tilde{U}_m^\mu = \tilde{U}_m(a, \Lambda) \otimes \tilde{U}^\mu(a', \Lambda'). \quad (37)$$

These states are completely real with momenta k and ζ belonging to the external and internal mass hyperboloids V_m^+ and C_μ^+ . In the other cases,

$$H_\eta \otimes \tilde{H}^\mu = \{\tilde{\Psi}_\eta^\mu(q, p; \zeta)\}, \quad \tilde{U}_\eta^\mu = U_\eta(a, \Lambda) \otimes \tilde{U}^\mu(a', \Lambda') \quad (38)$$

$$H_\eta \otimes \hat{H} = \{\hat{\Psi}_\eta(q, p; \xi)\}, \quad \hat{U}_\eta = U_\eta(a, \Lambda) \otimes \hat{U}(a', \Lambda') \quad (39)$$

$$H_\eta \otimes \hat{H}^\mu = \{\hat{\Psi}_\eta^\mu(q, p; \xi)\}, \quad \hat{U}_\eta^\mu = U_\eta(a, \Lambda) \otimes \hat{U}^\mu(a', \Lambda') \quad (40)$$

the external mode is stochastic. The internal modes are real, localized, and both real and localized, respectively. The external representation is stochastic U_η , while in the internal space we have the momentum \tilde{U}^μ , the reducible \hat{U} , and irreducible configuration representations \hat{U}^μ , respectively.

The probabilities can consistently be defined in $H_\eta \otimes \tilde{H}^\mu$. Wave functions can be defined as projections on tensor products of stochastic proper state vector $\tilde{\eta}$ with the perfectly localized vectors $\tilde{\varphi}_\zeta$

$$\tilde{\Psi}_\eta^\mu(q, p; \zeta) = \langle \tilde{\eta}_{q,p;\zeta} | \tilde{\Psi} \rangle, \quad |\tilde{\eta}_{q,p;\zeta}\rangle = |\tilde{\eta}_{q,p}\rangle \otimes |\tilde{\varphi}_\zeta\rangle, \quad \langle \tilde{\varphi}_\zeta | \tilde{\varphi}_{\zeta'} \rangle = \zeta^0 \delta(\vec{\zeta} - \vec{\zeta}'). \quad (41)$$

The probability that a simultaneous measurement of the stochastic position and momentum yield a value (q, p) in Δ and that the internal momentum ζ be in $\tilde{\Delta}'$ is expressed in the usual way

$$P_\Psi(\Delta \times \tilde{\Delta}') = \int_{\Delta \times \tilde{\Delta}'} d\Sigma(q, p) d\Omega_1^+(\zeta) |\tilde{\Psi}_\eta^\mu(q, p; \zeta)|^2. \quad (42)$$

The passage from the momentum-momentum to the stochastic-momentum representation can be performed by the stochastic intertwining operator W_η

$$W_\eta : L^2(V_m^+ \times C_1^+) \rightarrow H_\eta \otimes \tilde{H}^\mu. \quad (43)$$

Its integral form is concerned with external variables only

$$\tilde{\Psi}_\eta^\mu(q, p; \zeta) = \int d\Omega(k) \tilde{\eta}_{q,p}^*(k) \tilde{\Phi}_\eta^\mu(k; \zeta). \quad (44)$$

Localization can be obtained by the action of an intertwining operator $I_\eta^{\mu,+} = W_\eta \otimes I^{\mu,+}$ where W_η intertwines the internal momentum and phase space representations. The second operator $I^{\mu,+}$ is the internal induced representation localization operator

$$I_\eta^{\mu,+} : \tilde{\Phi}_\eta^{\mu,+}(q, p; \xi) \rightarrow \hat{\Psi}_\eta^{\mu,+}(q, p; \xi) \quad (45)$$

$$\hat{\Psi}_\eta^{\mu,+}(q, p; \xi) = \frac{\mu}{2\pi^{3/2}} \int_{V_m^+ \times C_1^+} d\Omega(k) d\Omega_1^+(\zeta) \exp(-i\mu\zeta\xi) \tilde{\eta}_{q,p}^*(k) \tilde{\Phi}_\eta^{\mu,+}(k, \zeta).$$

Materialization K_η^μ is the product of the inverse external intertwining operator W_η^{-1} with internal materialization K^μ

$$K_\eta^{\mu,+} : \hat{\Psi}(k, \zeta) \rightarrow \tilde{\Phi}_\eta^{\mu,+}(k, \zeta) \quad (46)$$

$$\tilde{\Phi}_\eta^{\mu,+}(k, \zeta) = \frac{\mu}{2\pi^{3/2}} \int_{\Sigma, M'} d\Sigma(q, p) d\xi \tilde{\eta}_{q,p}(k) \exp[i\mu\xi\zeta] \Psi(q, p; \xi).$$

The propagation is obtained as in the pointlike case. Namely, by composing a materialization $K_\eta^{\mu,+}$ followed by a localization $I_\eta^{\mu,+}$. The total propagator is a product of the external stochastic propagator and an internal pointlike propagator

$$K_\eta^{\mu,+}(q, p, \xi; q', p', \xi') = K(q, p; q', p') \Pi^{\mu,+}(\xi, \xi'). \quad (47)$$

The quantum space-time structure of the G-S-T may be retained with appropriate changes in the fibers over the external space-time points

$$E^{\eta\mu}(M, H_\eta \otimes \tilde{H}^\mu, \tilde{U}^\mu). \quad (48)$$

The fiber H_η is now replaced by $H_\eta \otimes \tilde{H}^\mu$ since the phase space point (q, p) is added to the internal momentum ζ . Corresponding by the local quantum frames are labeled with these points

$$\Phi_x = \{\eta_{x;(q,p;\zeta)}; (q, p; \zeta) \in \Sigma \times C_1^+\} \quad (49)$$

and the structural group

$$\tilde{U}_\eta^\mu = U_\eta(a, \Lambda) \otimes \tilde{U}^\mu(a', \Lambda') \quad (50)$$

is the product of the external phase space and internal induced momentum representations.

6. Conclusion

Our work is based on the idea that an intrinsic extension of particles may be combined with their stochastic extension. This has been achieved by incorporating a stochastic component in our geometro-differential model. The stochastic properties affect the external mode only. The processes of localization, materialization and propagation have been successfully generalized to this case. Moreover, it seems that even the quantum geometric structure can be generalized. The latter point merits a refinement since the present exposition is rather very sketchy. Further investigation in connection with gauge fields is to be considered also.

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