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OLD AND NEW STRUCTURES ON THE TANGENT BUNDLE

MARIAN IOAN MUNTEANU

"Al. I. Cuza" University of Iaşi, Faculty of Mathematics, Blvd. Carol I, n. 11
700506 Iaşi, Romania

Abstract. In this paper we study a Riemanian metric on the tangent bundle $T(M)$ of a Riemannian manifold M which generalizes Sasakian metric and Cheeger–Gromoll metric along a compatible almost complex structure which together with the metric confers to $T(M)$ a structure of locally conformal almost Kählerian manifold. This is the natural generalization of the well known almost Kählerian structure on $T(M)$. We found conditions under which $T(M)$ is almost Kählerian, locally conformal Kählerian or Kählerian or when $T(M)$ has constant sectional curvature or constant scalar curvature.

1. A Brief History

A Riemannian metric g on a smooth manifold M gives rise to several Riemannian metrics on the tangent bundle $T(M)$ of M. Maybe the best known example is the Sasakian metric g_S introduced in [18]. Although the Sasakian metric is *naturally* defined, it is very *rigid* in the following sense. For example, Kowalski [11] has shown that the tangent bundle $T(M)$ with the Sasakian metric is never locally symmetric unless the metric g on the base manifold is flat. Then, Musso and Tricerri [13] have proved a more general result, namely, that the Sasakian metric has constant scalar curvature if and only if (M, g) is locally Euclidean. In the same paper, they have given in explicit form a positive definite Riemannian metric introduced by Cheeger and Gromoll [9] and called this metric *the Cheeger–Gromoll metric*. In [19] Sekizawa computed the Levi-Civita connection, the curvature tensor, the sectional curvatures and the scalar curvature of this metric. These results are completed in 2002 by Gudmundson and Kappos [10]. They have also shown that the scalar curvature of the Cheeger–Gromoll metric is never constant if the metric on the base manifold has constant sectional curvature. Furthermore, Abbassi and Sarih have proved that $T(M)$ with the Cheeger–Gromoll metric is never a space of constant sectional curvature (cf. [2]). A more general metric is given by