

ON THE SYMMETRIES OF THE MANEV PROBLEM AND ITS REAL HAMILTONIAN FORM

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Abstract. The Manev model and its real form dynamics are known to possess Ermanno–Bernoulli type invariants similar to the Laplace–Runge–Lenz vector of the Kepler model. Using these additional invariants, we demonstrate here that both Manev model and its real Hamiltonian form possess the same $\mathfrak{so}(3)$ or $\mathfrak{so}(2, 1)$ symmetry algebras (in addition to the angular momentum algebra) on angular momentum level sets. Thus Kepler and Manev models are shown to have identical symmetry algebras and hence sharing more features than previously expected.

1. Introduction

Since Kepler and Newton the elliptical trajectories became the new archetype of the (bounded) planetary motion and the circular orbit nowadays is viewed upon rather as a degenerate ellipse than as an embodiment of perfection. The advent of Einstein's theory did not produce a new archetype of heavenly motions, apart from the exceptional case of a collapse into the black holes. Nevertheless, among the variety of relativistic effects the perihelion shift of the inner planets and the light deflection in a gravity field are definitely the best recognizable effects in the Solar system. Maybe it is time to accept a new archetype of heavenly motions: *precessing ellipse* (or more generally, precessing conics). If precessing conics give us