

POSITION UNCERTAINTY MEASURES ON THE SPHERE

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Abstract. Position uncertainty (delocalization) measures for a particle on the sphere are proposed and illustrated on several examples of states. The new measures are constructed using suitably the standard multiplication angle operator variances. They are shown to depend solely on the state of the particle and to obey uncertainty relations of the Schrödinger-Robertson type. A set of Hermitian operators with continuous spectrum is pointed out the variances of which are complementary to the longitudinal angle uncertainty measure.

1. Introduction

Recently an interest is shown in the literature to the problem of a quantum particle on the circle [3, 4, 5, 7, 8, 9, 13] and on the sphere [4, 6, 10]. In [5, 6, 8, 10] overcomplete families of states (coherent states) for these systems are constructed. One of the difficulties for these systems is the position (and momentum) uncertainty measures for the particle (or equivalently, the wave function spread measure). This is a consequence of the issue related the choice of the operator for the azimuthal angle φ . For a particle on the sphere there is a second problem, related to the non-hermiticity of the operator $-i\partial/\partial\vartheta$, where ϑ is the longitudinal angle. The problem of correct definitions of uncertainty measures is (and should be) closely related to the construction and justification of the uncertainty relations (UR's), and of coherent and squeezed states as well.

From the Dirac correspondence rule between Poisson bracket $\{f, g\}$ of two classical quantities f and g and the commutator of the corresponding operators \hat{f} and \hat{g} ,

$$\{f, g\} \longrightarrow i[\hat{f}, \hat{g}] \quad (1)$$

it follows that $[\hat{p}_\varphi, \hat{\varphi}] = -i$, where $\hat{\varphi}$ is the azimuthal angle operator, and \hat{p}_φ is the angular momentum operator. Formally this commutation relation is satisfied