Fourth International Conference on
Geometry, Integrability and Quantization
June 6–15, 2002, Varna, Bulgaria
Ivaïlo M. Mladenov and Gregory L. Naber, Editors
Coral Press, Sofia 2003, pp 303–315

MASS AND CURVATURE

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Abstract. In this work we present a brief summary of the "geometry of mass". We show how the notion of mass of elementary particles is related to the geometrical concept of curvature. In particular, the bosonic mass matrices are related to the extrinsic curvature of specific sub-manifolds of the Higgs bundle and the underlying gauge bundle. In contrast, the mass matrix of the fermions is related to the intrinsic curvature of bundles that geometrically represent "free fermions" within the context of spontaneously broken Yang–Mills gauge theories.

1. Introduction

When seen from a geometrical viewpoint the electric charge of an electron can be considered as the coupling constant of a $U(1)$ Yang–Mills gauge theory. That means that the electrons charge parameterizes the most general **Killing form** on $\mathfrak{u}(1) \equiv \mathrm{Lie}(U(1))$. One of the still outstanding and deep secrets of nature is why all of the electric charges of free particles is given by an integer multiple of this coupling constant. In general, the electric charge of a particle may be defined as

$$\text{charge} = \int_{\mathcal{S}} *F_{\text{elm}} \tag{1}$$

where $F_{\text{elm}} \in \Omega^2(\mathcal{M})$ denotes the electromagnetic field generated by the charge and $\mathcal{S} \subset \mathcal{M}$ any closed two dimensional space like surface of an orientable spacetime $(\mathcal{M}, g_{\mathcal{M}})$.

In other words, electric charge is tied to the curvature of the total space of the underlying principal $U(1)$-bundle. However, by formula (1) electric charge is also tied to the metric on spacetime ($*$ denotes the Hodge map that is defined