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THE STRUCTURE OF FORMAL SOLUTIONS TO NAVIER'S EQUILIBRIUM EQUATION

JOHN SARLI[†] and JAVIER TORNER[‡]

[†] *Department of Mathematics, California State University, San Bernardino, U.S.A.*

[‡] *Department of Physics, California State University, San Bernardino, U.S.A.*

Abstract. The local Lie structure of the orientation-reversing involutions on $\mathbb{R}^3$ is used to construct a family of orthogonally invariant operators that produce all formal solutions, up to biharmonic equivalence, of Navier's equation for elastic equilibrium. In this construction the value of Poisson's ratio associated with each solution is determined by the hyperbolic geometry of $sl_2(\mathbb{R})$. Empirically feasible values of the ratio are associated with 'spacelike' operators whereas values outside of this range are associated with 'timelike' operators.

1. Introduction

In the theory of linear elasticity, Navier's equation says that the displacement $\mathbf{U} = (u_1, u_2, u_3)$ of a point in a body subjected to surface forces and after acceleration has vanished must satisfy the equilibrium equation

$$\nabla^2 \mathbf{U} = \frac{1}{2\nu - 1} \nabla\nabla \cdot \mathbf{U}\,,$$

where ν is Poisson's ratio, a dimensionless constant of the material that expresses the ratio of transverse compression to longitudinal extension under deformation. In Poisson's original formulation this constant was presumed to have the value of $\frac{1}{4}$ for all materials satisfying the generalized Hooke's law hypothesis, thus prompting Poisson to characterize linearized elastic response as "directionless". Later predictions by Cauchy and others (based on thermodynamic properties of the strain-energy function — an excellent discussion can be found in [1], Chapter 8) that ν could vary over the interval $(0, \frac{1}{2})$ were verified by photoelastic stress measurement ([5], 250 ff). Typically, hyperbolic