Thirteenth International Conference on
Geometry, Integrability and Quantization
June 3–8, 2011, Varna, Bulgaria
Ivaïlo M. Mladenov, Andrei Ludu
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2012, pp 215–232

VORTEX PATTERNS BEYOND HYPERGEOMETRIC*

ANDREI LUDU

Department of Mathematics, Embry-Riddle Aeronautical University
Daytona Beach, 32114 USA

Abstract. We prove that loop vortices are created by a point-like magnetic dipole in an infinite superconductor space. The geometry of the vortex system is obtained through analytic solutions of the linearized Ginzburg-Landau equation described in terms of Heun functions, generalizing the traditional hypergeometric behavior of such magnetic singularity.

1. Introduction

Three-dimensional superconducting structures allow interesting physical properties such as giant vortices or multi-vortex states, or even (for larger size) a combination of the two states. The traditional approach relies on the Ginzburg-Landau (GL) free-energy minimization procedure, performed on a basis of linear combinations of solutions of the corresponding linearized GL equation [9, 21, 28]. While finite mesoscopic superconductor samples (size comparable to coherence length ξ, and penetration depth λ) of different geometries, subjected to external uniform magnetic field, have been theoretically investigated [4, 9, 17, 21, 27, 28, 31, 32], there are no exact theoretical models proving that vortex states are possible in unbounded space. In this case the presence of an external applied magnetic field is un-physical (requests infinite energy) and one needs to generate the magnetic field from a localized source. A simple example of this type, which is investigated in this paper, is a microscopic magnetic dipole placed at the origin of an infinite three-dimensional superconducting space. Such types of magnetic field were studied in [25] and it was numerically demonstrated the existence of interconnected vortex loops for spherical or prismatic boundaries. Attempts to solve similar equations in the presence of an electric dipole, have been made [22]. The Coulomb problem for a class of general Natanzon confluent potentials was exactly solved in

*Reprinted from J. Geom. Symmetry Phys. **26** (2012) 85–103.