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GAUSS–MANIN SYSTEMS OF POLYNOMIALS OF TWO VARIABLES CAN BE MADE FUCHSIAN*

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Abstract. We prove modulo a conjecture due to A. Bolibrukh that every monodromy group in which the operators of local monodromy in their Jordan normal forms have Jordan blocks of size only ≤ 2 can be realized by a fuchsian system of linear differential equations on Riemann's sphere without additional apparent singularities. This implies that the Gauss–Manin system of a polynomial of two variables can always be made fuchsian if a suitable basis in the cohomologies is chosen.

1. Introduction

1.1. Regular and Fuchsian Systems

In the present paper we consider *regular* (resp. *fuchsian*) systems, i. e. linear systems of ordinary differential equations depending meromorphically on complex time (which runs over Riemann's sphere), with *moderate growth rate* of the solutions in neighborhoods of the poles (resp. with logarithmic poles). Fuchsian systems are always regular. By definition, the growth rate is *moderate* at a given pole if any solution to the system when restricted to a sector with vertex at the pole and of arbitrary opening grows no faster than some real power of the distance to the pole. Restricting to a sector is necessary because the poles, in general, are ramification points for the solutions.

When a linear change (meromorphically depending on the time) of the dependent variables is performed, then the system changes and the only object which remains invariant under such changes is its *monodromy group*. A *monodromy*

*Dedicated to Professor O. A. Laudal.