LANGUAGE AND CONCEPT DEVELOPMENT IN GEOMETRY

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The object of this paper is to present and analyze some observations of the language children in 1st grade (6-7 years old) use when they talk about geometrical objects. The observations are made during a number of encounters with the pupils during one year. The work is part of an ongoing collaboration project between a university college and a primary school. The work can be described as empirical research centered around teaching units where the researcher partly has been a passive observer and partly an active participant in the process. The analysis is based on constructivistic theory and theory about levels of development.

The pupils in this study started in 1st grade in August, and the episodes that are described took place in the period from October to February. I want to investigate how these children develop their language and concepts about geometrical objects in various surroundings, using various artefacts and being subject to varying degree of interaction from me. More precisely I want to get information about what properties of the geometrical objects they seem to notice, how they describe these properties, and how they construct names for geometrical objects for which they have not already learnt a name. The main purpose of obtaining such information is to improve the possibilities for the teacher to meet the pupil on his/her level and with an appropriate language in order to initiate better learning. In the teaching units (episodes) that I will describe, I sometimes use the Norwegian words for mathematical concepts. In these cases the word is marked with (NO), and I also give the literal translation (lit.) from Norwegian to English, which sometimes differ from the usual English word.

THEORETICAL FRAMEWORK

One important theoretical basis for the study is social constructivism. Steffe and Tzur (1994) discuss the relation between social interaction and radical constructivism. Many authors interpret constructivistic learning as a process that takes place in solitude, where the learner constructs his/her knowledge in the absence of social interaction. As Jaworski (1994) points out part of the reason why constructivistic learning has been viewed as an individual and lonely process is due to Piaget whose view of the learner is more that of an individual than that of a social participant. Steffe and Tzur (1994, p. 9) view learning as "the capability of an individual to change his or her conceptual structures in response to perturbation." In the episodes that I will describe I will discuss how concepts and language may have developed in different ways due to the degree of interaction with me (degree of perturbation).

Steffe and Tzur (ibid., p. 24) further state that

> “Children cannot construct our knowledge, because our knowledge is essentially inaccessible to them. The best they can do is to modify their own knowledge as a result of interacting with us and with each other.”

Similarly the children’s knowledge is also inaccessible to us. In order to learn as much as possible about their mathematical knowledge I will argue that it is valuable to observe and interact with the children in as many different situations as possible. By doing this we can better adjust our actions in order to modify the children’s knowledge in the direction that we want it to develop. This is in accordance with Steffe and Tzur (ibid., p. 12) when they state the following:

> ”The mathematics of children is not independent of our mathematical concepts and operations because it is constructed partially through their interactions with our goals, intentions, language and actions.”

The process of constructing permanent objects is discussed by von Glasersfeld (1995) where he refers to a fundamental work by Piaget (1937). A crucial point in this discussion is the difference between *recognition* and *re-presentation.* Recognition has to do with our ability to recognize an object from some, possibly partial, presentation in our perceptual field. On the other hand, re-presentation has to with the ability to construct to oneself an image of an object without being exposed to a presentation of it. The difference between these concepts will play a crucial role in some of the examples that will be presented later. When a pupil is able to re-present a concept, I consider the concept to be developed to a higher level than if only the ability of recognition is present.

Another important theoretical basis for my study is the van Hiele theory (levels) of geometrical thinking. In the literature the number of levels varies from three to five (or even six). I shall be concerned only with the first two levels, here in the formulation of Schoenfeld (1986, p. 251).

> *First level:* Gestalt recognition of figures. Students recognize entities such as squares and triangles, but they recognize them as wholes; they do not identify properties or determining characteristics of those figures.
>
> *Second level:* Analysis of individual figures. Students are capable of determining objects by their properties.

Going from the first to the second level, the students move from a visual recognition of the objects to a more descriptive and analytic recognition. The next levels are of a more abstract/relational nature. Nickson (2000) discusses a reconceptualization of the van Hiele level 1, presented by Clements *et.al.* (1997):

> “Their results suggest that on this level children are not merely interpreting shapes visually, but they are thinking in a more syncretic way and bringing together visual responses with some recognition of the components and properties of shape.” (Nickson 2000, p. 62).

I will later refer to this as level 1+.

METHOD

Methodologically the study resembles what Wittmann (1998) describes as "empirical research centered around teaching units". Data have been collected in different ways depending on my role in the various teaching units. In cases where I have been an active participant the content of the important episodes has been written down right after they took place. When I have played the role of a passive observer, I have taken notes of conversations during the process and pictures of the scenes. All pupils involved came from the same class, but I am not working with a fixed group of pupils throughout the study.

DESCRIPTION AND ANALYSIS OF THE EPISODES

The first episodes take place in a cathedral. I accompanied the pupils during an excursion to the cathedral, and we looked for geometrical shapes and discussed what names to give to them. Here I was taking an active part in the dialogues. In the cathedral geometrical objects can be met in somewhat different surroundings than in everyday life, and we also have the opportunity to see objects that are not so often seen elsewhere. An example of this is that in the cathedral the children became very aware of the concept of the octagon, various eightfold symmetries and other occurrences of the number eight. The high altar of the cathedral is surrounded by walls, and the floor inside of these walls is shaped like a large octagon (Figure 1).

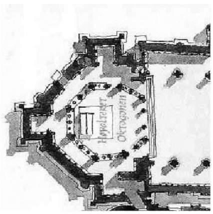


Figure 1

There are actually only seven walls, because there is an opening facing the choir of the church (to the right in Figure 1). The pupils and I looked at the shape of the floor and I asked if they could see what kind of geometrical figure this floor was shaped like. It should be mentioned that before the excursion to the cathedral the children had worked with geometrical shapes, and they were familiar with certain triangles and quadrilaterals. They even knew names like square and rectangle. No suggestions for the shape of the floor came up, so I introduced an interaction by suggesting that we should try to find out how many edges and corners the surrounding walls had. I told the children to walk along the walls and count up every time they passed a corner. When we reached the start of 'the missing wall' they had counted to seven. One pupil said: "If there had been a wall from here to where we started, it would have been eight, so it must be an åttekant" (NO: Åttekant = octagon, lit.: eight-edge). I interpret this in the following way: Based on the concept that this pupil has of triangles (NO: Trekant, lit.: three-edge) and quadrilaterals (NO: Firkant, lit.: four-edge) as figures with three, respectively four edges, she is able to generalize to the concept 'åttekant' because there were eight edges (NO: kanter).

Shortly after this little episode the pupils discovered that some of the columns in the church were also shaped like an octagon. Although the perimeter of the columns was much shorter than the perimeter of the octagon around the altar, and the columns actually were solid octagonal cylinders, the pupils could apply the concept by focusing on the property of having eight edges. What they actually did also with the columns was to walk around them, counting the edges. Some of the columns have a different shape, not so easy to describe with simple mathematical terms, and they also investigated these columns and discussed what name to give to the shape based on the properties they observed.

My understanding of the pupils' level of understanding here agrees with the van Hiele level 1+ , as they seem to focus on *some* properties of the octagon (eight edges) and not on a holistic recognition of the figure. There is, however, no evidence in this observation that they have identified other properties of the octagon, other than the eight edges.

The cathedral's main appearance is that of a gothic cathedral, but the oldest parts of it have roman arches, both as openings and as decorations on the walls. The children observed some of these arches and their first interest was to count them. On one of the walls there were 11. Afterwards I asked the children what names they would give to these arches. What did the arches look like? They answered that they looked like circles, but they were not complete (NO: hele, lit.: whole). So I asked how much of a circle they were. They said one half and suggested that we should call them half circles (NO: halvsirkel). The class teacher could confirm that they had worked with the concept 'circle' before, but not 'half circle'. In this case I propose that the children themselves constructed a new word based on their previous experience with the concept 'half' and the concept 'circle'.

Figure 2

The next episode takes place in the classroom where I did some paper folding activities together with the children. I wanted them to make an eight-pointed star. The activity involved folding eight pieces of paper, and then gluing these pieces together to a star with eight arms. Each piece has the shape shown in Figure 2. I asked what name they would give to this object. At first I did not get any suggestion, and I did not give any clues. Although they were familiar with quadrilaterals like squares and rectangles, they did not seem to recognize this object as a quadrilateral.

After a while one pupil suggested that we should call it a 'to-trekant' (lit.: two-three-edge). I interpret this suggestion to mean that the pupil sees two triangles (NO: to trekanter) put together, and hence gives to it a name that for him is natural, namely a 'to-trekant'.

I did not pursue this at the time because we were too busy making the star, but I see that if we had been able to agree that the object also could be called a quadrilateral, we would have had the opportunity to discover the property that *a quadrilateral can be divided into two triangles*. The point to make here is that the children's naming can be a good starting point for discovering further properties of the objects.

The episodes with the 'half circles' and the 'two triangle' are two episodes where a child is naming an object from looking at the object, and observing some properties of the object that he/she is familiar with. In one episode, with the half circle, this led to the concept which is generally used for that type of object, in the other episode it did not. It is possible that the outcome of the first episode was influenced by my leading questions. If given more time without my influence, the children might have come up with other suggestions. In the second episode the name (to-trekant = two-triangle) came up without any leading questions, although there was interaction on my part just by drawing attention to the figure. These two episodes seem to fit with the ideas of Steffe and Tzur (1994) that the children's concepts and language is developed and modified as a result of interaction, and in different ways depending of the nature of the interaction.

In the next two episodes the pupils work with given tasks without much interference from others. A student teacher is leading the work, and I am a passive observer. I take notes of the dialogues and pictures of the objects that are produced. The children communicate with the student teacher and with each other. My main interest here is to analyze the level of development of the concepts that are being handled.

The first episode consists of two tasks. The first task is to sort and categorize geometrical objects. The pupils were shown various geometrical objects made of colored plastic, and with a very distinct shape such as regular triangles, other simple polygons and circles. They were asked to name the objects, and this they could do without hesitating. Most often they would use mathematical terms like rectangle and circle. One of the objects they were shown was a regular hexagon, and they answered that 'this is a sekskant' (NO: sekskant = hexagon, lit: six-edge). The next task was to construct polygons with sticks of equal length given the name of the polygon. When they were asked to make a hexagon out of the sticks, two of the suggestions that came up can be seen to the left in Figure 3.

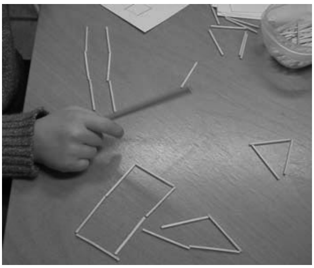


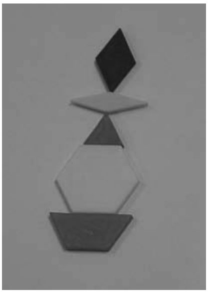
Figure 3

In both attempts they have grasped the idea that a hexagon should consist of six sticks, but also in both cases the idea of six vertices is missing. In the proposed hexagon on top left the edges are not even connected. These are the same pupils that give the name 'sekskant' to the regularly shaped plastic piece that they are shown, but when asked to make a 'sekskant' they do not copy the plastic piece, but construct an object that has some of the properties that the plastic piece has. Also here I see a situation where *some* properties of the objects are used to construct the figures. The two tasks differ, from the viewpoint of the observing teacher, in the sense that the second task contains more possibilities to obtain information about the level of development. The experiment with the plastic hexagon indicates that only a gestalt recognition is taking place, whereas the experiment with the sticks suggests that the property of being composed of six sticks is identified, and at the same time the idea of six vertices is missing.

In view of the language of von Glasersfeld (1995) my interpretation of this incident is that the children were able to *recognize* the hexagon, but they were not able to *re-present* it. Or better, they were able to re-present some of the properties of a hexagon, but not all, which again fits with van Hiele level 1+. The two tasks also show the characteristics of *synthetic* and *analytic* tasks, as these terms often are used in arithmetic to describe the difference between a task like "4 + 2 = " (synthetic) and "how can you partition these six elements: ♦ ♦ ♦ ♦ ♦ ♦?" (analytic) (Olsson 2000).

In the last episode the setting is that two pupils are sitting back to back, with access to the same geometrical objects (plastic pieces). One is making a figure out of these pieces without the other seeing it, and the task is to describe the figure so that the other can copy it. In the first task the figure is shown in Figure 4.

We enter the conversation when the rhombus, second piece from the top, shall be put in its place. Pupil 1 is going to describe what piece to pick and how to place it.



Protocol I

P1: "Long, white, kind of quadrilateral, really long, thickest in the middle, thinnest on the tips."

P2 (immediately choosing the right piece): "What way?"

P1: "The lower tip on top of the triangle."

P2 (not understanding): "What lower tip?"

P1: "Both of the longest sides outwards."

P2 puts the piece down correctly.

Figure 4

P1 does not have a precise word for this type of object, a rhombus, but still he is able to describe it very well: It is a quadrilateral, although only 'kind of', because for children of this age quadrilateral (NO: firkant) is very often used only for the square. It seems that this boy is starting to develop a more general concept of a quadrilateral by allowing the rhombus to be 'kind of quadrilateral'. When P2 does not understand which way he shall place the figure, P1 is able to use one more characteristic property of this rhombus, namely that the diagonals are not equally long. He expresses this by saying that the sides are not equally long, which they of course are, but it is clear what he means, and it immediately makes P2 choose the correct piece. In fact P1 uses the property with the non-equal diagonals already in his description in the beginning when he says that the figure is 'thickest in the middle, thinnest on the tips'. Again I propose that the level of development fits with van Hiele level 2 or 1+.

In the next task we see in Figure 5 the original object to the right and the copy to the left. Here is the conversation:

Protocol II

P1: “First a round piece. Thick. Then a rectangle.”

P2: “On top?”

P1: “Yes.”

P2: “What way?”

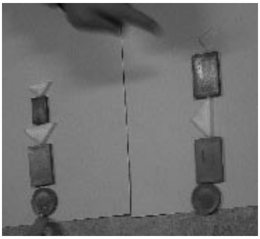
P1: “Not like outwards, but upwards. Then almost a triangle, but not quite. Slightly longer.”

P2: “What way?”

P1: “The tip at the bottom.”

P2: “What tip? The upper or the lower?”

P1: “The longest side downwards. Then a red, thick rectangle placed like the blue one.”



The main confusion here is about placing the triangle. When P2 does not understand how to place the tip, P1 changes his strategy and starts to talk about the direction of the longest side. Still, it does not give the desired result. We notice also that the last rectangle is not correct because P1 failed to say that it should be a red, thick, *big* rectangle.

Figure 5

The most interesting in the last conversation, I find, is the way the triangle is described. P1 says that it is ‘almost a triangle, but not quite’. This is the same boy that in the previous task described a rhombus as being ‘kind of quadrilateral’. When talking to the boy afterwards he emphasized his opinion by saying that “it has three edges (NO: tre kanter), but it is not a triangle (NO: trekant)”. I did not get a clear statement about what it takes to be a triangle, but I assume that he meant a regular triangle. If so this is the first time I have heard the word triangle reserved for regular triangles, whereas the corresponding phenomenon with quadrilaterals is very common.

CONCLUSION

From the observations made in this study I find that the pupils at an early stage start to identify properties of geometrical objects, although they are not able to see all properties of the object, let alone define the object. Sometimes they are also not able to describe the properties correctly, as in the case with the rhombus whose position was described as ‘having both the longest sides outwards’ (Protocol I). Hence, my findings support the revised van Hiele level 1 as suggested by Clements *et al.* (1997).

To get insight into the pupils' level of development I argue that it is important to perform tasks of both synthetic and analytic nature. In the example with the hexagon the second task gave more specific information about the pupils' level of development than the first one, information that is valuable for the teacher's further work. The study also suggests that the language that the pupils develop may be sensible to the interaction of the teacher. If the language can be developed in a relatively free manner, it might give the teacher valuable information to take into consideration when designing further teaching for the pupils.

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