EFFICIENCY AND ADAPTIVENESS OF MULTIPLE SCHOOL-TAUGHT STRATEGIES IN THE DOMAIN OF SIMPLE ADDITION

Joke Torbeyns [1 2], Lieven Verschaffel [2], and Pol Ghesquière [2]

[1] Research Assistant of the Fund for Scientific Research (F.W.O.)--Flanders

[2] University of Leuven, Belgium

This study investigated the fluency with which first-graders with strong, moderate, or weak mathematical abilities apply the decomposition-to-10 and tie strategy on almost-tie sums with bridge over 10. It also assessed children's memorized knowledge of additions up to 20. Children's strategies were analysed in terms of Lemaire and Siegler's model of strategic change, using the choice/no-choice method. Results showed that the children applied both the decomposition-to-10 and the tie strategy efficiently and adaptively. Furthermore, the first-graders had already memorized the correct answer to more than half of the tie sums. Finally, children with strong mathematical abilities applied the different strategies more efficiently but not more adaptively than their mathematically weaker peers.

INTRODUCTION

During the last decade, the goals and content of elementary mathematics education have changed internationally (Kilpatrick, Swafford, & Findell, 2001; Verschaffel & De Corte, 1996). With respect to the goals of elementary mathematics education, the adherents of the reform movement argue that instruction should foster the development of children's "adaptive expertise", i.e. children's ability to solve mathematical problems flexibly and creatively by means of meaningfully acquired strategies (Baroody & Dowker, 2003). This change at the level of instructional goals is reflected in an increased emphasis on new arithmetic procedures and skills, including flexible use of a rich variety of mental calculation strategies. However, the adherents of the reform movement still acknowledge and even stress the importance of older, well-established aims and contents, such as the good mastery of number concepts and mathematical skills in the domain of adding and subtracting up to 20 in the early grades of elementary school (TAL-team, 2001). But these concepts and skills should be taught in a way that supports the development of children's adaptive--instead of routine--expertise. This implies, for instance, that instruction in the domain of additions with bridge over 10 (like 8+9) should no longer focus on the perfect mastery of one calculation strategy, namely decomposition-to-10 ("8+9 = …; 10 = 8+2; 9 = 2+7; so 8+9 = 8+2+7 = 10+7 = 17"). In contrast, instruction should allow and stimulate children to apply a diversity of counting, calculation, and retrieval strategies. As illustrated in detail in Van Eerde, Van den Bergh and Lit (1992), children can apply diverse counting strategies on additions with bridge over 10, like counting all or counting on from the larger addend. Examples of calculation strategies that can be used to solve such additions, are decomposition-to-10, the tie strategy

("8+9 = 8+8+1 = 16+1 = 17"), and the one-less-than-10 strategy ("8+9 = 8+10-1 = 18-1 = 17"). Retrieval involves the (quasi-) automatic activation of the answer to the addition in long-term memory ("8+9 = (immediately) 17"). According to the adherents of the reform movement, instruction should further support children to gradually transform their informal counting strategies into more efficient calculation and retrieval strategies. Moreover, children should be stimulated to solve the additions efficiently, adaptively and mindfully on the basis of their individual strategy knowledge and skills.

Despite the increasing international acceptance of these reform ideas, this change in instructional orientation towards more strategy flexibility raises many, thus far unanswered questions. One of these questions concerns its desirability and feasibility for children of the weakest mathematical ability level (see, a.o., Mercer & Miller, 1992; Miller & Mercer, 1997). Research on the effectiveness of the new instructional approach for children of the weakest mathematical ability level did not yet result in a clear consensus (see, a.o., Woodward, Monroe, & Baxter, 2001). Therefore, we conducted a study to deepen our understanding of the fluency with which children of different mathematical ability level apply diverse school-taught strategies in the domain of simple arithmetic. We aimed to address two issues. First, we wanted to analyse the characteristics of the decomposition-to-10 and tie strategy on almost-tie sums with bridge over 10 for children who had received explicit instruction in both calculation strategies, in terms of Lemaire and Siegler's model of strategic change and with special attention for the strategy performances of mathematically weak children. We explicitly focused on almost-tie sums with bridge over 10, i.e. sums where the difference between the two addends equals only one unit (like 8+7), since (a) both the decomposition-to-10 and the tie strategy can be applied on this type of sums, which is not the case for other sums with bridge over 10, like 7+4, where the difference between the two addends is more than one unit, and the tie strategy is thus much harder to apply, and (b) the authors of textbooks in which multiple calculation strategies are taught (see below) generally assume the tie strategy to be a highly efficient strategy to solve almost-tie sums with bridge over 10. The second aim of the study was to examine children's memorized knowledge of additions up to 20, which is assumed to be enhanced by this type of instruction (Baroody, 1985).

We used Lemaire and Siegler's model of strategic change (1995) to analyse children's strategies. This model distinguishes four parameters of strategy competence. The first parameter, strategy repertoire, refers to the types of strategies children apply to solve a series of additions. The second parameter, strategy distribution, involves the frequency with which each strategy is used. The accuracy and speed of strategy execution belong to the third parameter, strategy efficiency. The fourth parameter, strategy selection, refers to the adaptiveness of individual strategy choices, defined as the selection of the strategy that leads fastest to an accurate answer to the addition.

We examined the efficiency and adaptiveness of strategy execution by means of the choice/no-choice method, which has so far been used rarely in previous studies in the domain of simple arithmetic (Torbeyns, Verschaffel, & Ghesquière, 2002). The choice/no-choice method requires testing each subject under two types of conditions. In the choice condition, subjects can freely choose which strategy they use to solve each problem. In the no-choice condition(s), the researcher forces them experimentally to solve all problems with one particular strategy. As argued convincingly by Siegler and Lemaire (1997), the efficiency data gathered in the choice condition can be biased by selection effects. In contrast, the forced application of one particular strategy on all items in the no-choice condition(s), makes selective assignments of strategies impossible, and thus yields unbiased data about strategy efficiency. Moreover, comparison of the data about the efficiency of the different strategies in the no-choice conditions with the strategy choices made in the choice condition allows the experimenter to assess the adaptiveness of individual strategy choices accurately: Does the subject solve each item (in the choice condition) with the strategy that leads fastest to an accurate answer to this item, as evidenced by the data obtained in the no-choice conditions?

METHOD

Participants

We selected 97 first-graders who had received instruction in both the decomposition-to-10 and the tie strategy on almost-tie sums with bridge over 10. All children were administered a standardized achievement test (Rekenen Eind Eerste Leerjaar [Arithmetic End First Grade or AE1], Dudal, 2000) to assess their general mathematical abilities. Furthermore, they all solved a series of 10 additions with bridge over 10 in a choice condition. Only those children who were able to solve the additions in the latter condition beyond the level of counting were also tested in the three no-choice conditions, and thus included in our final analyses. Consequently, 14 first-graders who still solved the majority of additions by counting were excluded from the sample.

The remaining 83 first-graders were divided in three groups on the basis of their general mathematical abilities: (a) children with strong mathematical abilities (n=31), i.e. children with a score at or above Pc75 on the AE1; (b) children with moderate mathematical abilities (n=20), i.e. children with a score between Pc50 and Pc74 on the AE1; (c) children with weak mathematical abilities (n=32), i.e. children with a score below Pc50 on the AE1. In line with our criteria, we observed group differences in score on the AE1, $F(2, 80) = 173.10$, $p < .0001$. The strong first-graders scored higher on the AE1 than the first-graders with moderate mathematical abilities, who received a higher score on the AE1 than the weak first-graders.

All children were tested in the month of May 2003, i.e. nearly at the end of the first grade. At that moment, they all had been taught additions with bridge over 10 for five to seven weeks. All teachers used the same mathematical textbook to instruct this specific topic to the children. A careful textbook analysis and a structured interview with the teachers revealed that all children first had practiced the tie sums with bridge over 10 with a view to memorize these answers. Afterwards, children had been taught how to solve additions with bridge over 10 with the decomposition-to-10 strategy. Finally, children had learned to answer almost-tie sums with bridge over 10 with the tie strategy. Special attention was paid to the relation between the different types of additions with bridge over 10, namely (a) tie sums, (b) almost-tie sums and (c) other additions, and the specific type of strategy that--according to the authors of the textbook--is most efficient to solve them, resp. (a) the retrieval, (b) the tie, and (c) the decomposition-to-10 strategy. But none of the teachers forced the children to effectively solve each addition with the strategy that was considered as most efficient on the addition. They rather allowed each child to solve each addition with his or her own preferential strategy.

Materials

All children solved a series of five experimental items, i.e. five almost-tie sums with bridge over 10 (6+7, 7+6, 7+8, 8+7, 9+8), in four different conditions. To stimulate the children to choose effectively between the decomposition-to-10 and the tie strategy in the choice condition, these five experimental items were mixed with five buffer items, i.e. additions with bridge over 10 that can not be classified as almost-tie or tie sums. To examine children's memorized knowledge of the number combinations up to 20, we added a series of 15 extra retrieval items, i.e. three additional almost-tie sums with bridge over 10, four tie sums with bridge over 10, and eight additions up to 10 (three tie sums up to 10, five non-tie sums up to 10), to the series of five experimental items in the retrieval condition.

Conditions

All children were tested individually in one choice and three no-choice conditions. In the choice condition, children were asked to solve the experimental items and the buffer items with either the tie or the decomposition-to-10 strategy. Children could choose between the two strategies by means of pictures, which contained a visual representation of the strategies (Figure 1). The identification of the strategies in the choice condition was based on the pictures filled in by the children.

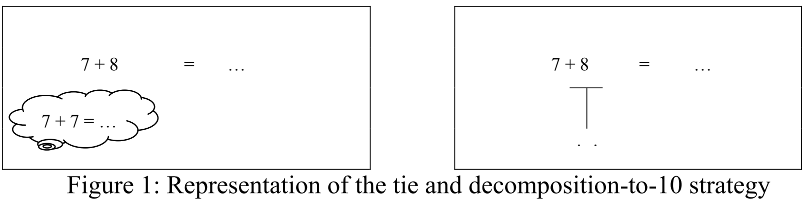

Figure 1: Representation of the tie and decomposition-to-10 strategy

In the decomposition-to-10 and tie condition, children were forced to solve the experimental items with resp. the decomposition-to-10 and the tie strategy by means of the pictures offered. In the retrieval condition, children were forced to retrieve the answer to the experimental items and the extra retrieval items by including a time limit of two seconds. On the first day, all children solved the items in the choice condition. On the second day, items were offered in the decomposition-to-10 and tie condition. On the third day, all children answered items in the retrieval condition. The experimenter registered the answer and the reaction time per child and per problem in each condition.

RESULTS

Characteristics of the Decomposition-to-10 and Tie Strategy

Strategy repertoire. The three groups of children applied the decomposition-to-10 and tie strategy at least once to solve the five almost-tie sums in the choice condition. Moreover, the two types of strategies were used at least once on each almost-tie sum. We observed group differences in the repertoire of strategies used, $\chi^2(4) = 19.8082$, $p = .0005$. A larger number of strong and moderate first-graders than of weak first-graders applied both the decomposition-to-10 and tie strategy, whereas the number of strong and moderate first-graders who exclusively relied on decomposition-to-10 was smaller than the number of weak ones. The number of children who solved all sums with the tie strategy did not differ between the three groups.

Strategy distribution. The frequency with which the children applied the decomposition-to-10 and tie strategy on the almost-tie sums in the choice condition is presented in Table 1. As shown in Table 1, the children used the tie strategy as frequently as the decomposition-to-10 strategy in the choice condition, $F(1, 80) = 0.81$, $p = .3701$. We observed group differences in the frequency of strategy use, $F(2, 80) = 8.50$, $p = .0004$. The weak first-graders applied decomposition-to-10 more frequently, and the tie strategy less frequently, than the moderate first-graders.

	Decomposition-to-10			Tie		
	Frequency	Accuracy	Speed	Frequency	Accuracy	Speed
Strong	47.74	0.97	10.69	52.26	0.92	07.63
Moderate	32.00	0.96	16.49	68.00	0.95	11.15
Weak	61.88	0.95	16.28	36.88	0.85	13.42
All	49.40	0.96	14.24	50.12	0.90	10.71

Note. Frequency = frequency of strategy use in the choice condition, expressed in percentages. Accuracy = accuracy of strategy execution in the decomposition-to-10 and the tie condition, expressed in proportion correct. Speed = speed of strategy execution in the decomposition-to-10 and the tie condition, expressed in seconds.

Table 1: Frequency, accuracy, and speed of the decomposition-to-10 and tie strategy

Strategy accuracy. We observed no group differences in the accuracy with which the almost-tie sums were answered in the decomposition-to-10 and tie condition, $F(2, 744) = 1.20$, $p = .3010$. As can be derived from the data in Table 1, children did not perform equally well in the decomposition-to-10 and the tie condition, $F(1, 744) = 11.73$, $p = .0006$. When they were forced to solve all sums with the tie strategy, more errors were made than when they had to apply decomposition-to-10. Finally, the interaction between the variables group and condition was not statistically significant, $F(2, 744) = 1.59$, $p = .2053$. The weak first-graders solved the sums as accurately as the moderate and strong first-graders in both the decomposition-to-10 and the tie condition.

Strategy speed. The strong, moderate, and weak first-graders did not answer the almost-tie sums with the same speed in the decomposition-to-10 and tie condition, $F(2, 664) = 8.90$, $p = .0002$. Overall, the strong first-graders answered the almost-tie sums faster in these two no-choice conditions than the moderate and weak first-graders (resp., M = 09.16s, M = 13.82s, and M =14.85s). The speed of responding also differed between the decomposition-to-10 and tie condition, $F(1, 80) = 12.79$, $p = .0006$. As shown in Table 1, children solved the almost-tie sums faster in the tie than in the decomposition-to-10 condition. Finally, the above-mentioned group differences in speed of responding were observed in the decomposition-to-10 as well as in the tie condition, $F(2, 664) = 0.49$, $p = .6101$. The strong first-graders answered the almost-tie sums faster than the moderate and weak first-graders in both the decomposition-to-10 and the tie condition.

Strategy selection. In line with Lemaire and Siegler's definition of an adaptive strategy choice as choosing the strategy that leads fastest to an accurate answer to the problem, we scored a strategy choice as adaptive if the child solved the almost-tie sum in the choice condition with the strategy that led fastest to an accurate answer to the same almost-tie sum in the decomposition-to-10 and tie condition. These analyses revealed that the strong as well as the moderate and weak first-graders took into account strategy efficiency characteristics while choosing a strategy: The proportion of adaptive strategy choices exceeded the chance level in the group of strong (M = 0.58, $p = .0472$), moderate (M = 0.65, $p = .0035$), and weak first-graders (M = 0.66, $p = .0001$). Furthermore, we observed no group differences in the adaptiveness of individual strategy choices in the choice condition, $F(2, 330) = 1.14$, $p = .3201$.

Accuracy of Task Performance in the Retrieval Condition

In the retrieval condition, we scored all additions that were answered inaccurately and/or not answered within the time limit of two seconds as incorrect. Subsequent analyses revealed group differences in the accuracy of task performance in the retrieval condition, $F(2, 1549) = 7.66$, $p = .0005$. The strong first-graders answered more additions accurately than their moderate and weak peers (resp., M = 0.41, M = 0.24, and M = 0.23). Next, children did not answer the different types of additions with the same accuracy in the retrieval condition, $F(3, 1549) = 141.36$, $p < .0001$. Children answered the tie sums up to 10 most accurately (M = 0.78). They made less retrieval errors on tie sums with bridge over 10 (M = 0.48) than on almost-tie sums with bridge over 10 (M = 0.12) and non-tie sums up to 10 (M = 0.13). Finally, we observed group differences in the accuracy with which the different types of additions were answered, $F(6, 1549) = 2.21$, $p = .0394$. The strong children answered all types of additions more accurately than the moderate and weak first-graders. The weak first-graders answered the tie sums with bridge over 10, the tie sums up to 10, and the non-tie sums up to 10 as accurately as the moderate first-graders, but were less accurate than the latter on retrieval of almost-tie sums with bridge over 10.

DISCUSSION

From a theoretical point of view, our study deepened our insight in children's calculation strategies on additions up to 20. More specifically, it improved our understanding of the quantitative and qualitative characteristics of the tie strategy, which were left largely unexplored in our--and others'--previous work (Torbeyns et al., 2001, 2002, in press). The focus on the strategy performances of children of different mathematical ability level further revealed clear differences in the calculation strategies and number fact knowledge of mathematically strong and mathematically weak children, favouring the former, which is in line with the results of earlier work on the strategy characteristics of mathematically weak children in the domain of simple arithmetic (for an overview of studies, see Geary & Hoard, 2003).

From a methodological viewpoint, our study showed the usefulness of the choice/no-choice method to examine young children's calculation strategies in the domain of simple addition. As documented above, the choice/no-choice method was applied successfully to study first-graders' use of the decomposition-to-10 and tie strategy on almost-tie sums with bridge over 10. It allowed us to gather unbiased data about each strategy's efficiency and proved necessary to unravel the level of adaptiveness of children's strategy choices in the choice condition.

From an instructional point of view, our study indicates that children, even the mathematically weak ones, are able to apply multiple school-taught calculation strategies efficiently and adaptively on simple additions. However, it should be noted that these results can not be generalised to children of the weakest mathematical ability level, who were excluded from our sample on the basis of their immature strategy performances in the choice condition. Although our study provided new and important insights in the strategy characteristics of first-graders in the early stage of the formal mathematics curriculum, we think it is very important that future studies try to unravel the developmental changes that are important to become "adaptive experts" by longitudinally assessing children's strategies throughout the entire mathematics curriculum, providing building blocks to optimise our learning and teaching approaches in the mathematical domain.

References

Baroody, A. J. (1985). Mastery of basic number combinations: Internalization of relationships or facts? *Journal for Research in Mathematics Education, 16*, 83-98.

Baroody, A. J., & Dowker, A. (2003). *The development of arithmetic concepts and skills: Constructing adaptive expertise*. Mahwah, N.J.: Lawrence Erlbaum Associates.

Dudal, P. (2000). *Leerlingvolgsysteem. Rekenen: Toetsen 1-2-3. Basisboek* [System for following pupils. Arithmetic: Tests 1-2-3. Textbook]. Leuven, Belgium: Garant.

Geary, D. C., & Hoard, M. K. (2003). Learning disabilities in basic mathematics: Deficits in memory and cognition. In J.M. Royer (Ed.), *Mathematical cognition* (pp. 93-115). Greenwich: Information Age Publishing.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up. Helping children learn mathematics*. Washington, D.C.: National Academy Press.

Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General, 124*, 83-97.

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education, 13*(3), 19-35.

Miller, S. P., & Mercer, C. D. (1997). Educational aspects of mathematics disabilities. *Journal of Learning Disabilities, 30*, 47-56.

Siegler, R. S., & Lemaire, P. (1997). Older and younger adults' strategy choices in multiplication: Testing predictions of ASCM using the choice/no-choice method. *Journal of Experimental Psychology: General, 126*, 71-92.

TAL-team (2001). *Children learn mathematics. A learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers in primary school*. Groningen, The Netherlands: Wolters Noordhoff.

Torbeyns, J., Verschaffel, L., & Ghesquière, P. (2001). Investigating young children's strategy use and task performance in the domain of simple addition, using the "choice/no-choice" method. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* (Volume 4, pp 273-278). Amersfoort, The Netherlands: Wilco.

Torbeyns, J., Verschaffel, L., & Ghesquière, P. (2002). Strategic competence: Applying Siegler's theoretical and methodological framework to the domain of simple addition. *European Journal of Psychology of Education*, 17, 275-291.

Torbeyns, J., Verschaffel, L., & Ghesquière, P. (in press). Strategy development in children with mathematical disabilities: Insights from the choice/no-choice method and the chronological-age/ability-level-match design. *Journal of Learning Disabilities*.

Van Eerde, D., Van den Berg, W., & Lit, S. (1992). *Kwantiwijzer voor leerkrachten. Werkboek 4: overbruggen van 10 (optellen)* [Kwantiwijzer for teachers. Workbook 4: The bridge over 10 (addition)]. Tilburg, The Netherlands: Zwijsen.

Verschaffel, L., & De Corte, E. (1996). Number and arithmetic. In A. Bishop, K. Clements, C. Keitel, & C. Laborde (Eds.), *International handbook of mathematics education. Part I* (pp. 99-138). Dordrecht, The Netherlands: Kluwer.

Woodward, J., Monroe, K., & Baxter, J. (2001). Enhancing student achievement on performance assessments in mathematics. *Learning Disabilities Quarterly, 24* (Winter), 33-46.