A COMPARISON OF A VISUAL-SPATIAL APPROACH AND A VERBAL APPROACH TO TEACHING MATHEMATICS

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Despite mathematicians valuing the ability to visualize a problem and psychologists finding positive correlations of visual-spatial ability with success in mathematics, many educationists remain unconvinced about the benefits of visualization for mathematical understanding. This paper describes research that compared a 'visual' to a 'verbal' teaching approach through teaching a range of early secondary school mathematics topics to two classes using one or other approach. The two classes were compared through a post-intervention test of mathematical competency, on which the verbally taught class scored significantly higher. No interactions were found between teaching style and the learner's preferred style although the pupils identified as 'visualizers' did tend to perform more poorly.

INTRODUCTION

Understanding mathematics, visually and verbally

Educators would tend to agree that a major aim of teaching mathematics is for students to develop understanding, even if it can be difficult to conceptualize this understanding fully (Sfard, 1994: Sierpinska, 1994). Taking a broadly constructivist view of knowledge, as teachers are inclined to (Sfard, 1994), suggests the importance of individually constructed understanding. Yet this interpretation still leaves the problem of how teachers interact with learners' constructions and facilitate their building. The challenge of communication can lead beyond a concern with issues of vocabulary and clarity to a tendency to think that verbal descriptions constitute knowledge (Davis, 1984) and that mathematical abstraction is essentially verbal (Anghileri, 1999).

However, the history of the development of mathematical concepts points to the importance within the subject of its visual side (Sfard, 1991). Furthermore, it has been argued that students should be encouraged to develop this aspect within their own understanding (Davis, 1984). It seems necessary to see how a broadly visual approach compares in the classroom to a more verbal approach.

Visual-spatial thinking

In considering how the visual side of mathematics might influence teaching, it seems appropriate to look first at the nature of human visual-spatial cognition. Although there is plenty of debate about this, it is widely agreed that the broad idea of visual-spatial processes as distinct from verbal ones is valid (see Hunt, 1994 for a review).

Other researchers have then moved from this idea of distinctive skills, via observations about individual differences, to the concept of a distinction between people who seem to prefer to use verbal abilities and those who seem to prefer visual processing. Paivio (1971) argued that most information can be encoded visually or verbally and that, along with other factors, the visual/verbal tendency of the subject will affect which mode is used. Other researchers have developed this idea of 'visualizer' and 'verbalizer' thinking styles.

Mathematics and visual spatial thought

The idea that mathematics involves thought beyond the verbal is supported by the observations of mathematicians, who frequently emphasize the importance of visual reasoning to their thinking (Stylianou, 2002; Sfard 1994). In general, spatial ability predicts success in mathematics (Smith, 1964) and there is evidence that visual-spatial working memory may be important in supporting mathematical performance.

However, all this conviction, mainly in the literatures of mathematics and psychology, appears to be undermined by research in education. Krutetskii (1976), and then Presmeg (1986), found that secondary school students classified as visualizers do not tend to be among the most successful performers in mathematics. At the elementary level, it has been proposed that 'low achievers' experience qualitatively different mental representations, where 'numbers quickly become concrete objects' (Gray & Pitta, 2000), leading to difficulties with arithmetic.

Successful visualization but unsuccessful visualizers

The apparent contradiction between the positive associations of visual-spatial strength with mathematics and the tendency for 'visualizers' to struggle could arise for a number of reasons. An underlying theme is the need to consider individual differences. Firstly, there is the suspicion that visualizers may be failing in school mathematics because of a mismatch between their preferred learning style and the predominance of verbal teaching and assessment. By assessing participants' habitual learning styles, as visual or verbal, and using two teaching approaches, this research aimed to address this idea.

However, other explanations for the paradox of successful visualizing mathematicians and unsuccessful visualizing pupils suggest attention must be paid to differences between individual visual-spatial approaches, often implying that there are different sorts of visualizer. A major contention is that there is actually a distinction between visual and spatial processes (see e.g. Baddeley, 1997), which has consequences for reasoning (Knauff & Johnson Laird, 2002).

Presmeg (1992) drew attention to different forms of imagery on a 'continuum from specific to more general'. Similarly, in contrast to the concrete images they criticize, Gray and Pitta describe more abstract 'dynamic images of marble or dots'. The concrete type of image, which is found to be less helpful mathematically, can be seen as 'visual' while the abstract style of images seems to demand more 'spatial' skills. This idea has been developed by Kozhevnikov et al (2002), who argue that some visualizers tend to use pictorial images and suffer difficulties, while others succeed through using more abstract spatial representations. Visualizers, according to this theory, have either high or low spatial ability, leading to these distinct approaches.

Alternatively, it has been suggested that the problems of visualizers might arise because of a lack of balance between their visual and verbal understanding. Many writers have pointed out the importance of flexibility in mathematical thought (e.g. Sierpinska, 1994) and it is likely that the visual methods described by mathematicians are balanced and supported by a more verbal understanding (Sfard, 1994). In contrast, the visualizers identified by research into school mathematics are, by definition, unbalanced.

Both these explanations suggest the importance of scrutinizing the visualizers a project identifies, looking at other assessed abilities and observing details of mathematical performance.

METHOD

The aim of this research was to test a broad, visual-spatial based teaching style against teaching with an emphasis on mathematical vocabulary and verbal explanation, while also investigating possible interactions of the teaching styles with the pupils' learning styles. Furthermore, it was anticipated that that assessment and observation would reveal more about the nature of visualizer and verbalizer thinking styles.

It was intended to test the utility of the visual approach in a normal school environment so whole classes were taught by one person (the researcher) with the lesson content ranging over many standard Year 7 (children aged 11 to 12 years) areas, as they arose in the school's scheme of work. The visual ideas were derived from various sources, some of them having been suggested by teachers and researchers. The verbal lessons covered the same content area, using the same questions and investigations, and, where appropriate, identical teaching materials. The intervention lessons were taught once a week, for ten weeks.

The school involved was an 11-18 comprehensive school. Two Year 7 classes, containing children from roughly the lower achieving half of the school population, participated. These two classes were chosen so that the two experimental classes could comprise half from each ordinary class.

Assessment

The research made use of a number of assessment tools and observations, but only the ones referred to in the results section will be described here.

MidYIS test: This test, administered by the CEM Centre, Durham University, UK produces the scores 'Vocabulary', 'Maths.' and 'Non-verbal'.

Mathematics Competency Test (MCT): Vernon, Miller and Izard 1995: This was given to the participants before, and immediately after, the interventions.

Spatial memory test: A test of spatial memory (from the Kaufmann Battery) was administered before the interventions to most of the participants. This involved remembering the positions on a grid of an increasing number of pictorial items.

Recognition test: This was adapted from a procedure described by Richardson (1980) as a feasible method of indicating a person's coding preference, either verbal or visual, when remembering items. Each participant's visual/verbal ratio was calculated and the test-retest reliability of these was 0.478 (N=36; p=0.03).

RESULTS

Comparing the two interventions

The children were assigned to the two teaching groups in a broadly random way, but with an attempt being made to balance the distributions of MCT scores in the two groups. Subsequently, the decision was taken to remove from the analysis of MCT gain those individuals who had been absent for half or more of the intervention lessons. The scores of these modified groups did not differ significantly on the MCT or the other measures.

MCT improvement

After the intervention lessons, MCT scores were higher among the pupils who had received verbal style lessons (see Table 1). Since there was a good correlation (r=0.669) between pre and post intervention scores, a regression was completed, predicting post-intervention MCT score from pre-intervention MCT score, with the resulting standardized residuals used as a measure of improvement. Table 2 shows how these compare for the two teaching groups.

Table 1

	Intervention group	N	Mean	Std. Deviation	Sig.(2-tailed)
MCT-pre intervention	Visual	19	13.84	4.50	
	Verbal	17	14.65	4.78	0.606
MCT-post intervention	Visual	17	14.88	4.30	
	Verbal	19	19.32	7.27	0.035

Table 2

	Intervention group	N	Mean	Std. Deviation	Sig.(2-tailed)
MCT gain (residual)	Visual	17	-0.339	0.79	
	Verbal	17	0.339	1.06	0.043

There is a significant difference between the two teaching groups in their post-intervention performance and in individual MCT gain ($p<0.05$). This produces an effect size of 0.7, a fairly sizable effect due to the verbal style of teaching.

Nature of the MCT

It seemed worth considering whether the verbal group's superiority on the post-intervention MCT covered all the questions or was limited to a certain style of question. In particular, some of the questions made quite heavy demands on literacy skills. Therefore the test items were classified, according to literacy demands, into three types.

There was no significant difference between the teaching groups on two of the types of question but there was a significant difference in scores on the questions with heavy literacy demands. It is only on these that the verbal group's scores were significantly higher than the visual group's ($p<0.05$).

Profiles of MCT improvement in the two classes

The distributions of pre-intervention MCT scores within the two teaching groups were similar, but this was not the case with the post-intervention or the improvement scores. Although the verbal group generally improved, there was variation, with children at the lower end of the range doing little better than those in the other group.

Interactions between individuals and interventions

The correlations of the various measures with the MCT gain did not differ significantly between the two groups. This suggests that the pre-existing abilities and styles of the children were not interacting with the teaching approaches to produce differing patterns of outcome in the two groups.

To clarify whether there were any interactions between the styles of the children and the teaching approaches a series of two-way ANOVAs was conducted. For each measure that related to visual or verbal ability or thinking style the participants were classified as 'high' or 'low'. Two-way ANOVAS were then carried out, considering the influence of each measure together with the intervention group on the MCT improvement. No significant interactions were found between the teaching group and any of the visual-verbal indicators.

This analysis was repeated using the scores on the questions with heavy literacy demands as the dependent variable. Again, there were no significant interactions.

Individual style and mathematical improvement

Although there was no evidence of systematic interactions of lesson and pupil style, in both classes some children's MCT gain was much larger than others and it is worth questioning how this gain relates to pre-intervention measures (see Table 3).

Table 3

	MidYIS non-verbal	MidYIS maths.	Spatial memory	Visual/ verbal ratio	MCT gain (residual)
MidYIS vocabulary	.282	.604**	.056	-.123	.478**
MidYIS non-verbal		.437**	.433*	-.127	.354*
MidYIS maths.			.233	.108	.271
Spatial memory				.071	.268
Visual/verbal ratio					-.499**

** Correlation significant at the 0.01 level. * Correlation significant at the 0.05 level.

It is striking that the measure of visual tendency derived from the recognition test is negatively correlated with the measure of improvement on the MCT: the more 'visual' children tended to fail to improve their MCT score. Furthermore, the visual/verbal ratio does not correlate with any of the other assessment scores, which is consistent with previous research findings that verbalizer-verbalizer measures do not correlate with tests of spatial ability (Kozhevnikov et al 2002).

DISCUSSION

The research produced a wealth of observations on teaching and learning, which can be related to the conclusions possible from the quantitative results reported. A main conclusion is the success of the verbal teaching approach. However, the superiority of the verbally taught class only applied to the verbal style of assessment questions, which is a distinct limitation. Furthermore, the large range of results from the verbal class suggests that the approach only benefited some children, while the correlations with the MidYIS scores imply that these children were the generally more able. The finding that the verbal teaching was better preparation for the verbal assessment does show the importance of style of teaching and assessment, with the visual-verbal distinction appearing valid. However, the lack of any interaction with any measure of verbal style, or ability, might tend to suggest the lack of utility of the visual-verbal distinction, as applied to individuals.

This finding of no straightforward interactions between teaching and learning styles could mean that teaching and learning styles do not interact, or at least not in the simple way proposed. Alternatively, the results could be seen as indicating a failure in the assessment of either the teaching or learning styles. In particular, the visualizer-verbalizer assessment used was not found to be very reliable, although this could be a symptom of problems with such attempts at global style classifications. As Kozhevnikov et al (2002) point out, there has been long debate about the value of the visualizer-verbalizer distinction.

The visualizers who were identified did have in common their failure to improve their MCT scores, whichever teaching group they were in, and classroom observations suggested that they were struggling. However, there was no evidence of them having particularly low spatial ability, as proposed by Kozhevnikov et al, while the classroom observations and consideration of their various assessment scores failed to indicate any other defining characteristics. There might be a number of reasons why an individual was assessed as a visualizer by the tool used, some of which could be failings of the tool. However, this could also lend support to the idea of different sorts of visualizer. The idea that such differences could result from different reasons for preferring visual thinking is suggested by the finding that some of the visualizers had low MidYIS vocabulary scores; it is clearly different processing visually because of limited verbal ability to preferring visual thinking, given more even abilities.

Yet it should be noted that this idea of there being different sorts of visualizer does not preclude the importance of being cognitively balanced, flexibly using both visual and verbal thinking. Furthermore, this research can be seen as lending support to such an idea, which implies that visualizers defined in this way will tend to have problems. In addition to the observed difficulties of the visualizers identified, the visual teaching highlighted the distinctly visual mathematical thinking of one individual. This child was able to use visual ideas very effectively in class, but was not assessed as a visualizer by the tool used. The implication that his thinking was more balanced between the verbal and the visual was supported by his development of a visual proof, which he used to help him put his, initially visual, reasoning into words.

In conclusion, this project clearly does not support the idea of visual mathematics teaching as a panacea, while the restricted success of the verbal teaching perhaps suggests the limitations of any one style. Although it seems valid to distinguish between visual and verbal strategies, presentations and question styles, it appears more debatable to what extent the distinction can be usefully applied to individuals. If the distinction *is* made, it seems possible to identify varying sorts of visualizers, some with more mathematical problems than others, but also to suggest that lack of balance in visual and verbal thinking might generally be problematic.

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