RF04: CONTRASTING COMPARATIVE RESEARCH ON TEACHING AND LEARNING IN MATHEMATICS

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International comparative research in mathematics education is a growing field. Experiences from recent and ongoing studies seem to have huge impact on both the field of research and the field of practise. The very idea of both grasping and making use of diversity lies in the heart of all comparative approaches. However there is an ongoing need for enlightened discussion on how the character of these results relate to the research methods and techniques used and the theoretical and analytical perspectives enacted in the research. The main focus of the forum is how these different comparative approaches, and the consequent and profound differences in project outcomes, can inform our individual and collective ways of understanding learning and teaching in mathematics.

GENERAL FRAMEWORK

The idea is to contrast and discuss different approaches, and to discuss both differences and similarities, especially in the character of what we can learn about the learning and teaching of mathematics in classrooms from these studies. What are the possibilities and limitations associated with different approaches? The different types of comparative research that are represented in this forum are:

OECD-PISA, Organisations doing large scale studies with questionnaires and tests

IEA-TIMSS, Organisations doing large scale video studies

LPS, Researchers doing large studies on their own initiatives

Small scale comparative studies

SIMILARITY AND DIFFERENCE IN INTERNATIONAL COMPARATIVE RESEARCH

Schmidt, McKnight, Valverde, Houang and Wiley (1997) investigated the mathematics curricula of the “almost 50” countries participating in the Third International Mathematics and Science Study (TIMSS). The documented differences in curricular organisation were extensive. Even within a single country differentiated curricular catered to communities perceived as having different needs. Countries differed in the extent of such differentiation, in the complexity or uniformity of their school systems, and in the distribution of educational decision-making responsibility within those school systems. Given such diversity, the identification of any curricular similarity with regard to mathematics should be seen as significant. And there were significant similarities. There were similarities of topic, if not of curricular location; broad correspondences of grade level and content that became differences if you looked more closely; differences in the range of content addressed at a particular grade level, but which repeated particular developmental sequences where common content was addressed over several grade levels. In another international study of mathematics curricula, the OECD study of thirteen countries' innovative programs in mathematics, science and technology found that, "Virtually everywhere, the curriculum is becoming more practical" (Atkin & Black, 1997, p. 24). Yet, despite this common trend, the same study found significant differences in the reasons that prompted the new curricula (Atkin & Black, 1996). These interwoven similarities and differences are the signature of international comparative research in mathematics education (Clarke, 2003).

Schmidt, McKnight, Valverde, Houang, and Wiley (1997) reported that differences in the characterization of mathematical activity were extreme at the Middle School level; from 'representing' situations mathematically, 'generalizing' and 'justifying' to 'recalling mathematical objects and properties' and 'performing routine procedures.' Despite the apparent diversity, it was the latter two expectations that were emphasised in the curricula studied. Given the documented diversity, it is the occurrence of similarity that requires explanation. Some curricular similarities may be the heritage of a colonial past. Others may be the result of more recent cultural imperialism or simply good international marketing.

In attempting to tease out the patterns of institutional structure and policy evident in international comparative research (particularly in the work of LeTendre, Baker, Akiba, Goesling, and Wiseman, 2001), Anderson-Levitt (2002) noted the "significant national differences in teacher gender, degree of specialization in math, amount of planning time, and duties outside class" (p. 19). But these differences co-exist with similarities in school organization, classroom organization, and curriculum content. Anderson-Levitt (2002, p. 20) juxtaposed the statement by LeTendre et al. that "Japanese, German and U.S. teachers all appear to be working from a very similar 'cultural script'" (2001, p. 9) with the conclusions of Stigler and Hiebert (1999) that U.S. and Japanese teachers use different cultural scripts for running lessons. The apparent conflict is usefully (if partially) resolved by noting with Anderson, Ryan and Shapiro (1989) that both U.S. and Japanese teachers draw on the same small repertoire of "whole-class, lecture-recitation and seatwork lessons conducted by one teacher with a group of children isolated in a classroom" (Anderson-Levitt, 2002, p.21), but they utilise their options within this repertoire differently.

LeTendre, Baker, Akiba, Goesling and Wiseman (2001) claim that "Policy debates in the U.S. are increasingly informed by use of internationally generated, comparative data" (p.3). LeTendre and his colleagues go on to argue that criticisms of international comparative research on the basis of "culture clash" ignore international isomorphisms at the level of institutions (particularly schools). LeTendre et al. report yet another interweaving of similarity and difference.

> We find some differences in how teachers' work is organised, but similarities in teachers' belief patterns. We find that core teaching practices and teacher beliefs show little

national variation, but that other aspects of teachers' work (e.g., non-instructional duties) do show variation (LeTendre, Baker, Akiba, Goesling & Wiseman, 2001, p. 3)

These differences and the similarities are interconnected and interdependent and it is likely that policy and practice are best informed by research that examines the nature of the interconnection of specific similarities and differences, rather than simply the frequency of their occurrence. This Forum uses brief presentations relating to five different research projects, each representing a very different approach to international comparative research in mathematics education, as a catalyst for discussion of how such research might best inform theory and practice in mathematics teaching and learning.

KEY QUESTIONS

What can be said about the teaching and learning of mathematics in our own countries and how can results be used to reach better performance within our own educational systems?

We have invited researchers that are responsible for very different studies that draw on different paradigms and use different methodological approaches. Furthermore, in order to give a background to the overarching question above, each contribution will address the following questions in relation to their respective study.

What are the goals of the various international comparative studies?

By studying reports and other documents from the studies above we see different aims in comparing countries. Why do we do it? Is it an effort in trying to find good examples of teaching or organisational aspects such as "Lesson study" and implement them in our own country? Are other countries' practises used as mirrors in the quest of trying to understand the practise of our own country? These two approaches can be related to different ways of interpreting your data. Hence producing results of different character.

What is being studied and how does this relate to teaching and learning?

The object of research varies between studies. The "what" we are trying to understand can be exemplified with: Lesson structure, teacher scripts, , negotiation of meaning, object of learning, patterns of interaction and learner practises.

What are the methods of data collection and analysis employed and with what adequacy do they document teaching and learning and their interrelationship?

The perspectives we adopt in our interpretations of these objects also varies. E.g. some studies take their point of departure in the students' perspective others in the teachers'. Furthermore, the theoretical positions are different. They vary both in type (pragmatic, socio-cultural, constructivist, phenomenographical) and in explicitness. The methods and techniques used in producing data vary considerably. Among the group doing classroom research we find examples of studies using audiotape only and some use video recording. Among those using video recording, the number of cameras used varies between one and up to three. Other studies use interviews both as a principal source of information and as a complement to video recordings. This is also true with regard to the use of test and questionnaires as well. There are studies where test and/or questionnaires are the only way of collecting data, in others they are used to collect supplementary information.

GOALS

The forum is intended to deepen the discussion on international comparative studies in mathematics education and their potential contribution to theorising mathematics teaching and learning. This Forum aims to problematise some of the more superficial readings of international comparative research in mathematics education (e.g. league tables of national performance) and move discussion within the community towards a collective and qualitatively more sophisticated reading and utilisation of the results of current and recent comparative studies. Those of us concerned with advancing theory in regard to mathematics teaching and learning must develop strategies to realise the potential of international comparative research in mathematics education to enhance both theory and practice, both in research and in our educational systems.

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WHAT IS COMPARED IN COMPARATIVE STUDIES OF MATHEMATICS EDUCATION?

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INTRODUCTION

Our aim is to discuss what is compared in international comparisons in Mathematics Education. It goes without saying that what is compared constrains what conclusions that are possible to draw from these comparisons. More precisely, presumptions about the phenomenon in focus govern our theoretical understanding as well as the qualities of facts that are collected. That is trivial from a scientific point of view, but not trivial when dealing with comparative studies in Maths Education. In order to penetrate Maths Education comparisons we need to describe what is compared in well known and significant comparative studies in mathematics education. We have chosen the PISA studies, the TIMSS studies and the TIMSS-R studies.

WHAT IS COMPARED?

In most international comparisons of Mathematics Education (ME) it is achievement in terms of test results that is compared. From such *outcome comparisons* we can conclude that students in some countries are doing better than students in other countries. Why this is the case is impossible to tell without further information. But we might also collect data about the prerequisites for learning mathematics, such as the size of per student investments in education in different countries, class size, number of hours in mathematics teaching etc. If the correlation between achievement and prerequisites variables, like those above, were high, we could possibly come up with conjecture, such as one country could boost achievement in mathematics by increasing its investments in education, reducing class size, increasing the number of class hours etc. But such correlation evidence is extremely scarce. If outcome comparisons have such limitations the next move is in a way self-evident, since we need to know what is happening in the *teaching process* in order to understand the outcomes of this process. And this was exactly what the TIMSS-99 did in the most advanced attempt to produce plausible explanations of differences in Maths achievement between different countries. One hundred year 8 classes were selected by random sampling in seven countries. In each class one lesson was video-recorded. When all the data were collected and analysed the results were published on the internet. We could compare the different countries with regard to, for instance: Length of lesson, Time devoted to mathematical work, Time devoted to problem segments, Percentage of time devoted to independent problems, Time per independent problem, Time devoted to practising new content, Time devoted to public interaction, Number of problems assigned as homework, Number of outside interruption, Number of problems of moderate complexity, Number of problems that included proofs, Number of problems using real life connections, Number of problems requiring the students to make connections, Time devoted to repeating procedures, Number of words said by teacher, Number of words said by students, Number of lessons during which chalkboard was used, Number of lessons during which computational calculators was used. Now, it would not be unreasonable to expect several of such factors be correlated with differences in achievement between countries, given that more or less the same Mathematical content has been covered in different countries. But as should be obvious from table 1 below, this was not the case. This means that not only factors like those presented above, referring to *how* Mathematics is taught varied between the countries but also that the content covered, i.e. *what* was taught in Mathematics varied between the countries as well. This in turn means that the characteristics of ME referred to different things in different ways. Small wonder that basically no correlations with achievement were found!

TABLE 1 (4.1.in original) Average percentage of problems per eighth-grade mathematics lesson within each major category and sub-category topic area, by country: TIMSS-1999 (http://nces.ed.gov/pubs2003/2003013.pdf) p 69.

Topic area	AU	CZ	HK	JP2	NL	SW	US
Number	36	27	18	‡	16	42	30
Whole numbers, fractions, decimals	15	13	5	‡	6	20	17
Ratio, proportion, percent	19	4	10	‡	6	19	6
Integers	2	9	3	‡	4	3	8
Geometry	29	26	24	84	32	33	22
Measurement (perimeter and area)	10	6	3	11	9	12	13
Two-dimensional geometry (polygons, angles, lines)	14	15	17	73	15	17	4
Three-dimensional Geometry	5	6	5	‡	9	4	5
Statistics	9	3	2	‡	10	2	6
Algebra	22	43	40	12	41	22	41
Linear expressions	7	16	11	‡	6	5	6
Solutions and graphs of linear equations and inequalities	15	21	23	12	33	14	27
Higher-order functions	6	6	‡	3	3	8	‡
Trigonometry	‡	‡	14	‡	‡	‡	‡
Other	‡	1	‡	‡	‡	1	1

‡Reporting standards not met. Too few cases to be reported. AU=Australia; CZ=Czech Republic; HK=Hong Kong SAR; JP=Japan; NL=Netherlands; SW=Switzerland; and US=United States.

COMPARING DIFFERENT WAYS OF DEALING WITH THE SAME THING

We believe that these results are extremely important because they show what is not critical. So what is critical?

In order to find pedagogically interesting correlations between what the students learn (outcomes) and what happens in the classroom (teaching process), we would need to keep one of the two aspects of teaching invariant and making statements about e.g. how the same thing was taught (instead of making statements how the different things were taught. As a matter of fact Karen Givven – using data from TIMSS-99 – gives, in this research forum, an excellent example that while differences between countries in the frequency of a certain category (problems requiring students to make connections) was not correlated with differences in achievement (for instance, while the best performing country, Hong Kong, had the lowest frequency of such problems; the next best performing country, Japan had the highest frequency) , differences in the way the same kind of problem were dealt with was correlated with differences between countries in achievement.

Another example of keeping the *what-aspect*, or the object of learning invariant and looking at differences in how the same object of learning is dealt with and thus producing pedagogically interesting characterizations of differences between classrooms, we see in Ulla Runesson's and Ida Mok's contribution to this symposium. They found that in the Hong Kong class in their study more different aspects of fractions were taught at the same time compared to Swedish classrooms, where fewer aspects were brought out in a more sequential (than simultaneous) manner. This means that there were different things possible to learn in the Hong Kong classroom as compared to the Swedish classrooms.

We take a third example from our Learner's Perspective Study. The Swedish team was interested in making comparisons of ME in different countries when the same topic was taught. Emanuelsson & Sahlström (2004) compared, for instance, how the geometrical representation of functions was introduced in a Swedish and in an American class.

In the Swedish class the teacher presents three equations:

$K = 15x$; $K = 10x$, $K = 2x$

and the corresponding graphs. The three lines are drawn in three different colours and the three functions are written in colours corresponding to the graphs.

Here, the contrast is between the lines (notably their steepness) and the corresponding differences between the written expressions (notably the coefficient of inclination). It is basically this correspondence that is possible for the students to learn (and even this was made more difficult by using matching colours of equations and lines).

Transcript 1. Student initiatives in relation to a teacher question.

1. Teacher:	WHAT CAN YOU in other words find out from that term or that- that number that stands in relation to X in a formula like this (3.5)
2. Student:	what
3. Student:	what
4. Student:	what are you on about
5. Student:	what
6. Students:	((laughs))
7. Teacher:	we have (.) three formulas
8. Martina:	yes
9. Teacher:	what can one then say on fifteen (.) ten (.) two. (.) directly when you get a such formula then you can say something about these here (.) anyway on their mutual relation ((points to graph)) (1.0)
10. Student:	eh
11. Martina:	in what order they are or
12. Teacher:	yes- no not what order but how they slope
13. Johan:	high low or eh … in between
14. Teacher:	sure (.)

In this case we are seeing the mathematical content as something that is negotiated in classroom interaction. Thus, in order to understand the meaning of the content we need to understand the meaning(s) of classroom interaction. In the US class the students are expected to "compare and contrast", i.e. to describe similarities and differences within five pairs of equations: $y=3x+2$ and $y=-3x-2$; $0x+3y=6$ and $2x+0y=6$; $y=x^2$ and $y=1/x$; $y=1-2x$ and $y=1-x^2$; $2y=x$ and $y=2x$.

Also in this case the students have the opportunities of learning about the correspondence between slope and coefficient of inclination but this relationship is extended to the case of zero slope (horizontal lines) and no slope (vertical lines). Furthermore the students have the opportunity of forming the concept of linear functions through the contrast with non-linear functions. So to the extent that the students learn different things in the Swedish and in the American class, the simplest explanation would be that their opportunities to learn these things have differed.

CONCLUDING COMMENTS

The close relationship between what the students have the opportunity to learn and what they actually learn is logically necessary and has also been empirically demonstrated (Marton & Morris, 2002; Marton, Tsui et al, 2004) and this relationship has been taken as a point of departure for improving learning in ME. By finding out the necessary conditions for a certain group of students to appropriate a certain object of learning and by bringing those necessary conditions about the likelihood of learning is most considerably enhanced (Lo, Marton, Pang & Pong, 2004). So, if we want to understand differences in achievement in ME between students in different countries we must explore to what extent the objects of learning reflected in the achievement test have been possible at all to appropriate. And in order to do so we have to look at how the same objects of learning have been handled in classes in different countries. Now, to us it seems that the international comparative studies such as TIMSS-R och TIMSS are not designed to be comparative in essence, since they show little interest in e.g. keeping the content invariant. So what are they then? We put forwards two kinds of understanding. First, an important side-effect is making of what is important in Maths Education by means of the items that are used in order to measure knowledge in Mathematics. International comparisons such as the PISA or the TIMSS are not only producing data for comparisons, they also produce conceptions of what is important and of value in Maths Education. They are not only comparing, they are participating in the social construction of curricula in Maths Education. This thought is well developed in the work of Ian Hacking (1999). From this point of view, international comparisons are about homogenisation of Maths Education. Second, going back to the correlations between different variables that are sought in international comparisons we find another thought and that is that given a certain correlation it is predicted that some fact will have an impact on another fact. Given what we know about correlations on one side and explanations on the other side such conclusions are of course problematic. But we think that, on a pragmatic level, even the search for correlations between facts is problematic. What we find is an instrumentalistic system of reason, that construct technical directives (von Wright, 1972) based on abstract numerical relations instead of e.g. didactical arguments. Stated otherwise, what is compared in international comparisons of preconditions and outcomes are educational phenomena shrunk to fit an instrumentalistic system of reason. In a word, international comparisons carried out in this way are examples of intellectual thrift of content as well as of educational reason.

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VIDEO SURVEYS: HOW THE TIMSS STUDIES DREW ON THE MARRIAGE OF TWO RESEARCH TRADITIONS AND HOW THEIR FINDINGS ARE BEING USED TO CHANGE TEACHING PRACTICE

Karen Givvin

LessonLab

If our aim is to improve performance in our educational systems, we must first obtain an accurate picture of those systems as they currently exist. To paint a picture of teaching practices in eighth-grade mathematics classes in the United States (and elsewhere) we sought to document and describe average teaching experiences, not exemplary ones. The approach taken in the 1995 and 1999 TIMSS Video Studies was that of a video survey. The marriage of the two research traditions offers a way to resolve the tension between anecdotes (visual images) and statistics (Stigler et al., 2000). Bringing together the two research approaches allowed us to overcome some of the limitations of each. This, along with cross-national comparison, helped provide a detailed description of "typical" classroom teaching.

This forum presents an exciting opportunity to look closely at different approaches to comparative research on mathematics education. We've been challenged by conference organizers to focus on *what* is to be compared in comparative research on teaching and learning mathematics, and *why*. Only then should we focus on *how* comparisons can be done. The idea is that the nature of learning and teaching mathematics, as the substance of comparative studies, needs to come before a discussion of the means and processes of comparison. Beyond this, the goal is to examine what each approach can teach us about improving students' mathematics performance.

WHAT IS TO BE COMPARED?

I'm here to provide the perspective that guided the TIMSS Video Studies. The question of *what is to be compared* within the TIMSS Video Studies can be addressed at multiple levels. At one level, the goal was to examine "typical" teaching. That is, we weren't interested in documenting a particular approach to teaching nor did we set out to examine high- versus low-quality teaching. Likewise, we were not interested in the differential effects of teaching on different categories of students. What we wanted to capture was simply everyday practice as it is experienced by teachers and students in different countries.

The "what" question can also be asked in terms of the aspects of the classroom lessons we examined. The answer is that we coded for a wide array of variables. The variables were chosen and developed by mathematics educators and cultural insiders, and were guided by both the literature and the desire to adequately capture what was seen in the lessons we collected. The time and manpower we had available allowed us to reliably code more than 60 distinct aspects of the lesson, from codes such as interaction pattern, mathematical content activity, and activity purpose, to myriad codes about each mathematical problem (e.g., evidence of real life connections, graphic representations, procedural complexity, and student choice in solution methods), to judgments of student engagement, lesson coherence, and overall quality.

The “what” question can also be asked in terms of what we intended to describe when we reassembled the discrete classroom elements we examined. What we hoped for was to be able to describe systems of teaching. Our thinking was that the individual codes would come together in a coherent way, with particular codes acting to inform others and with a broad set being used to describe and give meaning to the system.

WHY MAKE COMPARISONS?

Because we regard teaching as a cultural activity we began the study with the assumption that many classroom activities would vary little within each country and would be so familiar to cultural insiders that they would become invisible (Geertz, 1984). To describe teaching fully requires exploring it in relation to that seen in other countries. Examining different cultures helps us see what is commonplace in our own classrooms (Stigler & Heibert, 1999; Stigler, Gallimore, & Heibert, 2000) and being forced to explain classroom events (or the absence of particular features) to cultural outsiders helps draw our attention to details that are otherwise transparent to us. Beyond this, examining practices across cultures can help us discover pedagogical alternatives. One might, for example, see unfamiliar ways to pose problems, to organize how students work on problems, or for teachers and students to interact. Discovering alternatives can in turn lead to a discussion of pedagogical choices. The TIMSS Video Studies were conducted with these goals in mind.

HOW WERE DATA AND RESULTS PRODUCED IN THE TIMSS VIDEO STUDIES?

With some *whats* and *whys* behind us, we may turn to *how* we approached the process of comparing mathematics teaching and learning. The approach we took was that of a video survey. As with traditional survey methods, and in order to arrive at an “average,” we began with large, nationally representative samples. Using a national sample provides information about students’ common experiences. It is important to know what teaching looks like, on average, so that national discussions of teaching focus on what most students experience. The survey quality of the research speaks to the theme of this year’s conference: inclusion and diversity. By conducting a national sample we made a best effort at capturing the full range of teaching, not intentionally limiting what we sampled. By applying to it a wide array of codes, we were poised to capture the diversity in teaching within and across countries.

Unlike traditional survey methods, we didn’t use a questionnaire as our primary data source. We instead used video. Videos offer the ability to conduct a detailed examination of complex activities from different points of view. They preserve classroom activity so it can be slowed down and viewed multiple times, by many people with different kinds of expertise, making possible in-depth descriptions of many classroom lessons. The marriage of the two research traditions offers a way to resolve the tension between anecdotes (visual images) and statistics (Stigler et al. 2000).

In the more recent and larger-scale of the two studies, we examined between 50 and 140 lessons (one per participating teacher) from each of seven countries. The TIMSS Video Studies were studies of teaching, so the primary of our two cameras focused on the teacher. The second, stationary camera was fixed on students and was, in the end, used for classroom analysis only rarely. The videos were supplemented by a teacher questionnaire. Items on it were sometimes used to clarify lesson events, but the questionnaire was more generally used to round out the picture of teachers in each country (e.g., years of experiences, education) and their perceptions of the videotaped lesson.

Not surprising based on the data collection design, the TIMSS Video Studies report statistically-based characterizations of the 'typical lesson.' For each of the codes examined we can explore the frequency of occurrence across lessons (or across mathematical problems) by country. Examined singly, the codes provide a fine-grained description of classroom practice. Organized by concept, they can paint a nuanced picture of teaching in each country – one that can then be compared across countries.

Although we feel strongly about the affordances of our research approach, we recognize the limitations of it as well. Foremost is the enormous cost of such an undertaking. With respect to the data collection procedure, we can say nothing of how teaching plays out over a series of lessons, how teachers of varying competence teach or the degree of variance within the practices of competent teachers, or of how classroom events are perceived by either the teacher or the students. (Fortunately, for some of these goals one can turn to David Clarke's Learner's Perspective Study.) The design of the TIMSS Video Studies also makes it impossible to make a direct link between classroom practice and student achievement.

WHAT CAN BE LEARNED FROM THIS APPROACH AND HOW CAN RESULTS BE USED?

With this approach, we were able to answer questions such as (1) whether teachers in all high-achieving countries teach as those do in Japan, (2) with Japan aside, whether teachers in high-achieving countries share a common pedagogy, and (3) what, if any, features most higher-achieving countries have in common. With regard to the last question, Lindblad and Marten correctly point out that we had difficulty finding lesson features that correlate with differences in achievement. There was at least one feature, however, that appeared to have such a correlation. I'd like to expand on it and on how we've begun to use the finding to make an impact on teaching and learning in the United States.

The code to which I'm referring is called "making connections." In the TIMSS 1999 Video Study, it was found that U.S. students, in typical classrooms, rarely had opportunities to engage in challenging work during their eighth-grade mathematics lessons. Although U.S. teachers posed problems with the potential for rich mathematical learning just as frequently as did teachers in the other, higher-achieving, countries, they almost never maintained the problems at this conceptual level as they were worked on and discussed.

Taking a detailed look, column B of Table 1 shows that the percentage of problems categorized as making connections varied across countries, and even among the high-achieving countries. While Japan exceeded all of the other countries on this dimension (54%), the presentation of making connection problems in the other high achieving countries looked more like that found in the U.S. This suggests it is not necessary to present a high percentage of rich problems in a single lesson in order to produce high levels of mathematics achievement.

Table 1. Types of Problem Presentation and Implementation in the TIMSS 1999 Video Study

	A	B	C	D
Country	Average TIMSS 1995 mathematics score	% of Problems Presented as Making Connections	% of Making-Connections Problems Implemented As Making Connections	% of Making-Connections Problems Converted to Lower-Level Problems
Japan	581	54	48	52
Hong Kong SAR	569	13	46	54
Czech Republic	546	16	52	48
Netherlands	529	24	37	63
Australia	519	15	8	92
United States	492	17	0	100

However, presenting a rich problem is one thing, exploiting it for rich learning opportunities is another. The data in Column B considers only how a problem is presented, not how the problem is implemented as the lesson unfolds. A making connections problem, for example, might be converted into a using procedures problem as a teacher works it through with the class. When we look at how problems were implemented, quite a different story emerged. In Column C, we present the percentage of making connections problems that were actually solved by making connections (i.e., not transformed into using procedures, stating concepts, or giving results only problems).

Comparing Columns B and C reveals a striking difference. The data in Column C reveal something that high-achieving nations have in common. It is not the percentage of rich problems presented but the way they are implemented in the lesson that distinguishes them from the United States (and, to some degree, Australia). Most of the making connections problems in the United States were converted into lower level problems (see Column D). Instead of using these problems as opportunities to explore and reason about mathematical concepts, U.S. teachers typically broke them into procedural elements and took students through the procedures step-by-step (Hiebert, et. al., 2003).

This pattern of results can be interpreted in various ways. First, it is possible that U.S. teachers lack the content knowledge that would be necessary for them to facilitate rich discussions of mathematics (Ma, 1999). Another hypothesis is that U.S. teachers have little experience – either as teachers or when they were students themselves – engaging in conceptually rich discussions of mathematical problems. Again, we argue that teaching is a cultural activity, varying more across cultures than within. If this is true, then it will be difficult for teachers to practice instructional strategies that are rare in their own culture, and thus less likely that they would have observed many examples of others doing so.

Whatever the merit of these interpretations, the making connections results from the TIMSS 1999 Video Study suggest a potentially cost-effective strategy for professional development of mathematics teachers. In brief, these data indicate that:

U.S. mathematics curricula already include rich problems that lend themselves to conceptually rich discussions, and that U.S. teachers typically present as many rich problems to their students as teachers in high achieving countries;

Gains in student mathematics achievement might be obtained if U.S. teachers more often implemented these rich problems by maintaining their complexity, rather than converting them to using procedures problems.

These results suggest a way to improve mathematics achievement using existing curricula and programs, and provide the rationale for our current intervention study. With a grant from the Institute for Education Sciences, we are attempting to teach teachers to identify and effectively implement mathematically rich problems in their pre-algebra lessons. Our plan is to assess the impact of the training on (1) teachers' knowledge of mathematical content for its use in the classroom (i.e., pedagogical content knowledge), (2) teachers' ability to present rich problems in their lessons and maintain high conceptual levels of implementation, and (3) students' mathematical achievement. During the first of two years of implementation, we will compare the teachers and students who receive our professional development training with those who do not. During the second year of implementation, the control group from the first year will receive our PD program as well. The plan will allow us to examine the effects of the program on two groups of teachers, as well as enable us to assess its effectiveness when used over two consecutive years.

SUMMARY

As the pendulum swings to and fro between pedagogical movements and with the comings and goings of popular practice and political policy, what often becomes overlooked is the need to obtain a clear picture of the state of everyday practice. The absence of such an understanding prevents programs from being adequately informed by teacher and student needs.

With the TIMSS Video Studies, our interest was to describe everyday teaching across different countries via a methodology we refer to as video survey. Results indicated possible ways of improving classroom teaching. We're currently pursuing one of those ideas – a focus on implementing problems with a conceptual focus – in our current intervention program.

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LEARNER'S PERSPECTIVE STUDY: DEVELOPING MEANING FROM COMPLEMENTARY ACCOUNTS OF PRACTICE

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By examining mathematics classroom practice over sequences of ten lessons, the Learner's Perspective Study provides data on the teacher's and learners' participation in the co-construction of the possible forms of participation through which classroom practice is constituted. The use of post-lesson video-stimulated interviews offers additional insight into participants' intentions, actions and interpretations. Complementarity of account is possible on at least three levels: Between study participants (teacher and students – through both videotaped classroom actions and post-lesson reconstructive interviews); Between project researchers (through parallel analyses of a common data set); and, Between projects (eg LPS and TIMSS video studies). All three are important.

INTERNATIONAL COMPARISONS OF CLASSROOM PRACTICE

The practices of classrooms are the most evident institutionalized means by which the policies of a nation's educational system are put into effect. Given this, the classroom seems a sensible place to look for explanations and consequences of the differences and similarities identified in international comparative studies of curriculum, teaching practice, and student achievement (see Clarke, 2003).

If we are to engage in international comparative research, there are two quite distinct methodological alternatives:

Alternative 1.

If two groups of objects are to be compared then one approach is to consider these two questions:

Difference – "What is the characteristic about which the *comparison* is to be made?"

Similarity – "How might each group of objects be separately *typified* with respect to that characteristic?" The international comparison of national norms of student achievement could be described as conforming to this approach.

The order in which these two questions are posed is a major methodological signature.

Alternative 2.

If two groups of objects are to be compared, consider these two questions:

Similarity – "Which characteristics appear to *typify* this collection of objects?"

Difference – "What *comparisons* can be made between these two groups of objects using the identified characteristics?"

Posing the questions as in Alternative 2 reduces the danger of constraining the data to a predetermined structure, but may lead to the typification of the two groups by different emergent characteristics, restricting the common bases on which comparison of the two groups might be made. Note: Alternative 2 assumes a domain within which comparison is sought, such as classroom practice or curricular policy.

For example, it might be that for one nation or culture there is no nationally characteristic structure to the lesson as a whole, but that particular types of idiosyncratic lesson events offer the most appropriate typification. For another nation or culture, there could be a high degree of regularity to the composition of lessons, or in the sequencing of particular types of instructional activity in the delivery of a topic. Such differences in the form of typification provide a basis for international comparison that reflects something more essential to each than the identification (imposition) of the same structural level as the basis for the comparison. The methodological choice of Alternative 1 makes the basis for comparison a matter of prescription based on either theory or on the prevailing educational priorities of the country conducting the study. Choice of Alternative 2 makes the identification of possible bases for comparison an empirical result of the research.

DATA IN THE LEARNER'S PERSPECTIVE STUDY

The Learner's Perspective Study documented sequences of ten lessons, using three video cameras, and supplemented by the reconstructive accounts of classroom participants obtained in post-lesson video-stimulated interviews, and by test and questionnaire data, and copies of student written material (Clarke, 1998, 2001, 2003). In each classroom, formal data collection was preceded by a one-week familiarization period in which the research team undertook preliminary classroom videotaping and post-lesson interviewing until such time as the teacher and students were accustomed to the classroom presence of the researchers and familiar with the research process. In each participating country, the focus of data collection was the classrooms of three teachers, identified by the local mathematics education community as competent, and situated in demographically different school communities within the one major city. For each school system (country), this design generates a data set of 30 'well-taught' lessons (three sequences of at least ten lessons), involving 120 video records, 60 student interviews, 12 teacher interviews, plus researcher field notes, test and questionnaire data, and scanned student written material. Data collection is complete in Australia, China, Germany, Hong Kong, Japan, the Philippines, South Africa, Sweden, and the USA; underway in Israel and Korea; and planned for the Czech Republic, England and Singapore. The teacher and student interviews offer insight into both the teacher's and the students' participation in particular lesson events and the significance and meaning that the students associated with their actions and those of the teacher and their classmates.

COMPLEMENTARITY AS ESSENTIAL

Complementarity Between Participant Accounts: Establishing the Co-Construction of Classroom Practice

Like Wenger (1998), Clarke's (2004) analysis of patterns of participation in classroom settings stresses the multiplicity and overlapping character of communities of practice and the role of the individual in contributing to the practice of a community (the class). Clarke (2001) has discussed the acts of interpretive affiliation, whereby the learners align themselves with various communities of practice and construct their participation and ultimately *their* practice through a customizing process in which their inclinations and capabilities are expressed within the constraints and affordances of the social situation and the overlapping communities that compete for the learner's allegiance and participation. By examining sequences of ten lessons, the Learner's Perspective Study provides data on the teacher's and learners' participation in the co-construction of the possible forms of participation through which classroom practice is constituted (cf. Brousseau, 1986). An example of utilizing the complementarity of teacher and student accounts can be found in Clarke (2004), which examines the legitimacy of the characterisation of kikan-shido (Between-Desks-Instruction) as a whole class pattern of participation, and to situate the actions of teacher and learners in relation to this pattern of participation. By drawing on classroom video evidence and juxtaposing teacher and student interview data, it is possible to demonstrate that while engaging in kikan-shido, the teacher and the students participate in actions that are mutually constraining and affording, and that the resultant pattern of participation can only be understood through consideration of the actions of all participants. A key characteristic of kikan-shido, as it is practiced in the Australian classrooms, is the implicit devolution of the responsibility for knowledge generation from the teacher to the student, while still institutionalizing the teacher's obligation to scaffold the process of knowledge generation being enacted by the students. Comparison with the enactment of kikan-shido in other classrooms (Hong Kong, Shanghai, and San Diego, for example) provides significant insight into the pedagogical principles underlying the practices of different classrooms internationally.

Complementarity Between LPS Researcher Accounts: A More Comprehensive Portrayal of Classroom Practice

Classrooms are complex social settings, and research that seeks to understand the learning that occurs in such settings must reflect and accommodate that complexity. This accommodation can occur if your data collection process generates a sufficiently rich data set. Such a data set can be adequately exploited only to the extent that the research design employs analytical techniques sensitive to the multifaceted and multiply-connected nature of the data . . . we need to acknowledge the multiple potential meanings of the situations we are studying by deliberately giving voice to many of these meanings through accounts both from participants and from a variety of "readers" of those situations. The implementation of this approach requires the rejection of consensus and convergence as options for the synthesis of these accounts, and instead accords the accounts “complementary” status, subject to the requirement that they be consistent with the data from which they are derived, but not necessarily consistent with each other, since no object or situation, when viewed from different perspectives, necessarily appears the same (Clarke, 2001, p. 1). In the LPS project, multiple, simultaneous analyses are being undertaken of the accumulated international data set from a variety of analytical perspectives. For example, while Ference Marton and his colleagues in Sweden and Hong Kong analyse the practices of classrooms in Shanghai from the perspective of Marton’s Theory of Variation, Clarke is undertaking analysis of the same lessons in relation to the Distribution of the Responsibility for Knowledge Generation. These two analytical approaches do not appeal to the same theoretical premises, but nor are they necessarily in conflict. They represent complementary analyses of a common body of data, aspiring to advance different theoretical perspectives and to inform practice in different ways.

Complementarity Between Project Accounts: Approaches to Studying Lesson Structure

Lesson structure can be interpreted in three senses:

- At the level of the whole lesson - regularity in the presence and sequence of instructional units of which lessons are composed;

- At the level of the topic – regularity in the occurrence of lesson elements at points in the instructional sequence associated with a curriculum topic, typically lasting several lessons;

- At the level of the constituent lesson events – regularity in the form and function of types of lesson events from which lessons are constituted.

A research design predicated on a nationally representative sampling of individual lessons, as in the TIMSS Video Studies (1995 and 1999), inevitably reports a statistically-based characterization of the representative lesson (the first of the alternatives listed above). The analysis of video data collected in the first TIMSS video study (Stigler and Hiebert, 1999) centred on the teacher’s adherence to a culturally-based “script.” Central to the identification of these cultural scripts for teaching were the “lesson patterns” reported by Stigler and Hiebert for Germany, Japan and the USA, and the contention that teaching in each of the three countries could be described by a “simple, common pattern” (Stigler & Hiebert, 1999, p. 82).

The characterisation of the practices of a nation’s or a culture’s mathematics classrooms with a single lesson pattern has been problematised by the results of the Learner’s Perspective Study (see www.edfac.unimelb.edu.au/DSME/lps). The recent report of the TIMSS 1999 Video Study (Hiebert et al., 2003) employed ‘lesson signatures’ rather than ‘lesson patterns’ to characterize differences between the practices of international mathematics classrooms internationally. These lesson signatures characterize national norms of practice in terms of the prevalence of different activity types at different points in the lesson. The resultant ‘signatures’ remain insensitive to the location of the sampled lesson(s) within a topic sequence. As such, they can give a misleading impression that the structure of any particular lesson is independent of whether it is the introductory lesson at the commencement of a topic, a consolidation or developmental lesson later in the topic sequence, or a summative lesson occurring towards the end of a topic. Nonetheless, the TIMSS data offers the opportunity to estimate the prevalence of a particular activity type identified as significant from LPS data. Similarly, activities identified in the TIMSS project as prevalent within a particular country can be evaluated from within the LPS data in relation to their capacity to stimulate specific responses in students, particularly learning outcomes. The complementarity of these two projects is acknowledged and valued by both research groups.

CONCLUDING REMARKS

This paper has offered complementarity of accounts as an essential methodological and theoretical stance, adopted by the Learner's Perspective Study, for the explication of mathematics teaching and learning in classroom settings, the advancement of theories relating to such settings, and the informing of practice in mathematics classrooms. This paper and the Research Forum of which it is a part embodies the PME conference theme of 'inclusion and diversity' in a very fundamental way.

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LESSONS FROM A SMALL-SCALE OBSERVATIONAL STUDY: AN EXAMPLE OF THE TEACHING OF FRACTIONS

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The aim of this paper is to demonstrate what a small-scale project can tell about features of teaching and learning in two different cultures. We argue that some features, which may not be easily observed within one culture, can become more visible in the contrast in order to get a better understanding of the teaching practice per se, even from a small scale project. We have studied the mathematics teaching in one classroom in Hong Kong and four in Sweden. Based on the assumption, that how the content is taught has an important implication on what students may possibly learn, we compared how the teaching of the same topic (fraction) may differ between the two places. Some profound differences regarding how the same topic was dealt with in the two countries were found. In the Hong Kong data several things were handled in one lesson at the same time whereas in the Swedish data this happened in a sequence of lessons spreading over a substantial period.

INTRODUCTION

Being in an environment constantly, one usually takes things for granted and fails to see the characteristics of the environment as special or different from the others. To bring about a better understanding of mathematics teaching itself is one argument for comparative studies (Stigler & Hiebert, 1999; Lopez-Real & Mok, 2002). However, comparison can be made at different levels and with different focus. Mostly, these studies are to different extent grounded in data from more extensive data sets (e.g. TIMSS Video Study and in the PISA-project). But, are these very expensive and extensive studies the only way to bring about insights about cultural differences? The study we will report here captures a small number of mathematics lessons in Hong Kong and Sweden. In Sweden five consecutive lessons from four different classrooms and in Hong Kong only one lesson were studied. Compared to the extensive studies mentioned, our study can appear to be too thin and insufficient to generalize anything about mathematics teaching in the two countries. However, our aim was to some extent different from these studies the overall aim of which was to compare the teaching practices in different cultures. This was not the goal for our study. Instead we hoped that some features, which may not be easily observed within one culture, would become more visible in the contrast in order to get a better understanding of the teaching practice *per se.* The question whether this is possible even within the frame of a small-scale project will be discussed in this paper.

THEORETICAL FRAMEWORK: A THEORY OF VARIATION

In this study, we approach classroom learning with a specific focus. Assuming learning is always learning of something – it has an object - we study how the object of learning is constituted in the classroom interaction, and with the particular interest in different possibilities to learn in different situations. What is possible to learn, has to do with those aspects of the object of learning that are possible for the learners to be aware of, or to discern. However, only that which is varying can be discerned (Bowden & Marton, 1998). So, the possibility to discern an aspect has to do with whether it is present as a dimension of variation or not (Marton & Booth, 1997; Marton, Runesson, & Tsui, 2004). If the particular aspect is present as a dimension of variation, it is likely discerned by the learner. And further, if the aspects are present as dimensions of variation at the same time, the learners likely discern them at the same time. So, what is studied is the pattern of simultaneous dimensions of variations related to the object of learning that are present to the learners in the situation (Runesson & Marton, 2002). And when studying the differences in possibilities to learn in different classrooms, it is the difference between the patterns of simultaneous dimensions of variations opened in the different classrooms that we describe.

THE STUDY

The current study has its origin in a previous study of Swedish mathematics classrooms, which aimed at finding differences between the teachers as regards how the topic was handled (Runesson, 1999). To shed new light on this data, a similar study in Hong Kong was conducted. The aim was to find differences between how the same topic was taught by contrasting mathematics teaching in two different cultures. However, to be able to see critical features in our own classrooms and one's own culture, we chose the same mathematical topic in order to see how the same topic can be handled in different cultural context. Therefore, the selection of the Hong Kong data set was made on the basis of matching up with the existing Swedish data as much as possible. The Hong Kong lesson was a primary four (age 10, grade 4) lesson on the topic "Comparing fractions". The lesson was carried out in Cantonese and videotaped. The Swedish data is drawn from a larger data set consisting 20 lessons from four different classrooms in grade six and seven. These lessons were audio taped and transcribed verbatim. Our aim was to be as close as possible with regards to the content of teaching. That is, when sections of the Swedish data were selected, this was done at the level of sub-constructs of fractions. The sub-constructs of fractions, which were available in the Hong Kong data, did appear in four of Swedish teachers' teaching. The analysis is grounded on data from all of these classrooms. Due to differences between the Swedish and the Hong Kong curriculum, we could not match the age of the pupils in the two countries. The topic was taught in grade six and seven (age 12 and 13 respectively) in Sweden and in grade four in Hong Kong (age 10). And although, we tried to come as close as possible to study the same content, some differences occurred. In the Hong Kong lesson the students worked with finding the common denominators of two fractions.

In the Swedish lessons the tasks was slightly different; the task was to find another fraction with the same value (e.g. 2/6=1/3). However, in both the Hong Kong and the Swedish lessons, comparison of fractions with different denominators was found. Unlike the Hong Kong data, which is drawn from one single lesson, the Swedish data consist of several lessons.

SUMMARY OF THE RESULTS: TWO DIFFERENT EXAMPLES OF SIMULTANEITY AND VARIATION

The analysis was with a particular focus on those aspects of the topic taught that were opened as dimensions of variation were identified. The Hong Kong lesson appears to have only one objective, i.e. comparing fractions with different denominators. Nevertheless, this objective was visited and revisited via several tasks, which were either in the form of questions in the worksheets or examples on the board. As a result of this, the Hong Kong lesson shows a pattern of variation, which consists many dimensions of variation. For example, some dimensions are: alternative representations of the method of amplification, the denominators, the fractional parts of different wholes and the contrast between the methods of comparison. Moreover, the intertwined relationship between these dimensions of variation forms a special arrangement or simultaneity of variation in a single lesson. Such experience is important because it provides a chance for “fusion” i.e., for the students to consider several aspects of the object of learning simultaneously (Marton, Runesson and Tsui, 2004). The Swedish lessons showed a very different pattern of variation. The most striking difference was perhaps that variation of methods was not opened. The students were presented to one method only, a diagrammatic method. Instead of varying the method, the teacher demonstrated a method on a couple of different examples. The other apparent difference was the sequential character identified in the Swedish lessons. We found that these sub-constructs were commonly never presented simultaneously in the Swedish lessons, but instead they were extended over time and presented as disjoint instances without any connection or reference to previous lessons. So, finding the bigger of two different fractions was taught in one lesson, and "fractions with different denominators but with the same value" was taught in another. The latter was taught with no reference to how this had been presented earlier although the two topics were indeed connected. In other words, since in the Hong Kong lesson several sub-constructs were presented and related to each other at the same time, the Hong Kong lessons were richer in terms of sub-constructs related at the same time.

Comparing to the Hong Kong lesson, the Swedish examples created a narrower space of variation, and in combination with the sequential character, accomplished a quite different space for learning in the Swedish lessons. From the theoretical position taken, we can assume that what was possible to discern of the same thing was different in Hong Kong and in Sweden. In other words, the students’ understanding of the two sub-constructs “comparison of fractions” and “fractions with the same value” are very likely to be different when the students from the two places experience such different space of learning. So, what we can say is, it was possible to discern different things in Hong Kong and Sweden. But, what that means for what the students actually learned, we cannot say, since this has not been studied.

WHAT COULD COME OUT OF A SMALL-SCALE OBSERVATIONAL STUDY?

The study presented here is in many respects a small-scale project, so what could possibly come out of such, as it seems, limited project?

The original purpose of this study was to shed new light on a study conducted earlier in Swedish classrooms. In line with the theoretical framework taken, discovering something new when revisiting the data would be easier if it was contrasted against something different, e.g. by contrasting mathematics classrooms and possibilities to learn in different school systems and educational traditions. The object of research in this study was not possibilities to learn in a general sense, but possibilities to learn *the same thing.* Therefore, it was important to study how the same topic was dealt with, i.e. to keep the content constant. This design has been used in a number of studies (Runesson, 1999; Marton & Morris, 2002; Marton, Tsui et al., 2004) However, it was in many ways a bit problematic to match up with a data set from Hong Kong to the existing Swedish data. From our point of view we wanted to delimit our definition of "the same topic" as much as possible. "The same topic" was defined in terms of how it appears in classroom practice, and on the level of tasks, so we asked the teacher to invite us to study a lesson when fractions with different denominators would be the topic taught. Although, we tried to come as close as possible to study the same content, some differences occurred. Being restrictive to having the same topic, it was not possible to study pupils of the same age, due to different curricula in the two countries. However, from this point of view the result is interesting. In the Hong Kong classroom the pupils were about three years younger than their counterparts, however a space of learning consisting of many simultaneous dimensions of variation was afforded to the learners, whereas for the older Swedish pupils dimensions of variation were brought out in sequence.

It could be argued that this sequential pattern of variation was a result of the longer period of observed lessons in the Swedish data, that the likelihood of such a finding is bigger if several consecutive lessons are observed. It could not be excluded that the sequential character of handling the object of learning, which was found in the Swedish data, would not appear in Hong Kong. This has never been claimed, and it was never the purpose of the study either, i.e. to say anything about the general in the two cultures. What we have described is two different ways of handling the same topic, or two different patterns of variation and simultaneity when teaching the same topic. This was found by comparing two different school cultures.

The way we worked in this study, implied doing a close and narrow analysis, but without the aim of finding more overall patterns or a more general character in the different classrooms. A main difference between, for instance, the TIMSS Video

Study and ours is what we were studying. To us the TIMSS Video Study was a study of teaching, whereas ours is a study of possibilities to learn the same thing. In our study we identified and described how the same object of learning could be dealt with differently by means of examples from different cultures. This was possible to do, even if only one single lesson from one teacher from each country was studied.

Necessarily a small-scale project like this touches the issue of representatives. Our aim was not to come up with something that could tell us something about the possibilities to learn about fractions with different denominators in Swedish and Hong Kong classrooms *in general,* or to explain differences in learning outcomes between the two countries. Instead we wanted this study to open our eyes to that, which is not easily seen within our own culture, so it would become visible, but without saying anything about the typical Swedish or the typical Hong Kong classroom. The most prominent coming out of this study is, that by seeing what could be done differently, what *could be the case*, new light has shed light on what *is* done and *what is the case* in some classrooms our own countries. When the characteristics identified from the two different data sets are used as a mirror, it gives us a better understanding of the practice in our countries. Surprisingly, such understanding could be achieved from a small-scale study like this.

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THE PISA-STUDY: DIFFERENTIATED ASSESSMENT OF "MATHEMATICAL LITERACY"

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More than some international comparative studies before, PISA (Programme for International Student Assessment) referred to conceptions from Mathematics Education. Specific backgrounds of PISA are

- the ideas behind realistic mathematics education as formulated by Freudenthal and de Lange, and

- in the German national option of PISA, the distinction of cognitive activities required in the item, esp. conceptual vs. procedural thinking (Hiebert).

According to that theoretical basis, PISA gives insight into the structure of mathematical achievement. These "profiles" bear messages which come closer to the needs of the development of mathematics education in the countries.

INTENTIONS OF THE INTERNATIONAL PISA-STUDY

PISA, the "Programme for International Student Assessment" is a study initiated by the Organization for Economic Cooperation and Development (OECD). PISA compares the achievement of 15 years old students in the (industrialized) OECD-countries in the domains Reading, Mathematics and Science (OECD 1999). The first test was held in 2000, followed by tests in 2003 and 2006. See OECD (2001) for results of PISA-2000; PISA-2003 will be released in Dec 04.

That PISA tests an age based sample of students, but not a grade based sample as many other studies did, has its origin in the political intentions of PISA. The OECD wanted to gain insight into the outcomes of the educational *systems* in the countries. Therefore, to choose the 15 years olds was decided according to the idea that this is the age of transition to vocational training or to an extended secondary school career. However, this decision once made has the consequence that a "core-curriculum" approach, as e.g. TIMSS has chosen, should not be appropriate. In apparent contrast, PISA focused on "mathematical literacy" which is intended to target the resulting abilities acquired in school up to the age 15, from whatever schooling it may come.

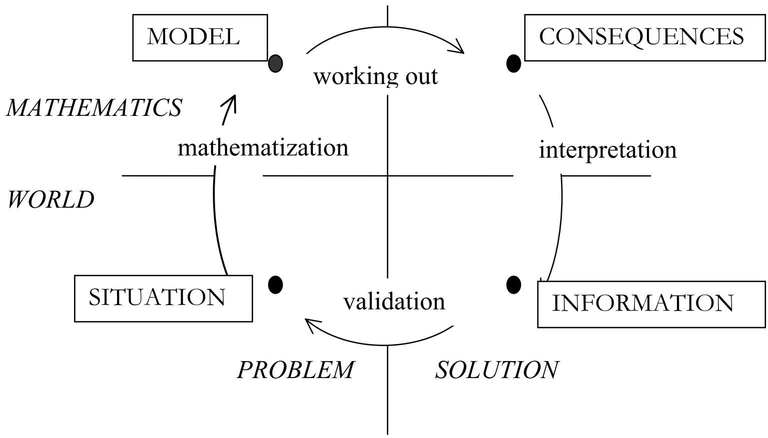
Mathematical literacy focuses on the "functional use of mathematics" (OECD 1999) in various situations, not only realistic ones, or as the framework says: "Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments, and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen." (OECD 1999)

MEETING THE FULL RANGE OF MATHEMATICS

Any serious test in mathematics or in other domains has first to clear up its views of the area tested, mathematics in our case. A literacy test as PISA is even stronger urged to do this. The PISA mathematics items are thus constructed as problems which call for using mathematics in context, i.e. mathematical modelling in a very broad sense, including not only contexts from daily life, but also components of the social and cultural embedding of mathematics (OECD 1999). It is not the place here to go into a deeper discussion of what "literacy" should mean: see e.g. Kilpatrick & al. (2001), Jablonka (2003), Neubrand (2003) for an extended discussion. However, one common idea is a kind of red thread through all these discourses: "literacy" is not fully captured if one does not try to meet the full range of mathematics.

A starting point for considerations of what mathematical activity can be is the well known picture of the (idealized) process of modelling (Fig.1):

Figure 1: The Process of Mathematical Modelling



This picture can be applied to the modelling of situations from outside of mathematics, as it was created for, but it may also describe mathematical problem solving when the situation one starts from is a challenging problem inside of mathematics. From the cognitive point of view, both activities have the same structure: Modelling in applications as well as working on intra-mathematical problem bearing situations is essentially characterized by the activity of transformation, translation, seeing in other connections etc., which is called mathematization in the classical sense, but equivalent to the activity of structuring an intra-mathematical situation.

The process of "working out" as in Fig. 1 can follow two cognitive ways: We distinct if the working-out process of an item needs more procedural or more conceptual thinking (Hiebert 1986). This idea was added to the international framework of PISA in the German national option to PISA-2000 (Neubrand et al 2001). Such a distinction essentially defines three classes of items (Knoche & al 2002) which we called *„The Three Types of Mathematical Activity"*:

(a) employing only techniques (abbreviated: „technical items"),

(b) modelling and problem solving activities which lead to use mathematical tools and procedures (abbr.: items with „procedural modelling")

(c) modelling and problem solving activities which need drawing connections and using mathematical conceptions (abbr.: „conceptual modelling").

What "technical items" are is very clear. These items consist of just "working out" in the picture of Fig. 1, and the whole lower half-plane in the picture does not even exist in that item. Regretfully, all too often knowing mathematics is considered as just to be able to run an algorithm when the starting point is given. All non-technical items, i.e. those items in which the lower half-plane in Fig.1 is relevant, are called here "modelling problems". Among the "modelling tasks", two kinds of "working out" (in the sense of Fig.1) make the difference from cognitive perspective: "Working out" can consist of solving an equation, doing a calculation, etc., to produce the result, as it is the case in the most "classical" textbook problems. These items are the „procedural modelling items". In contrast, there are items, which can be solved by giving a appropriate argument, or by connecting the situation to a mathematical concept and drawing conclusions from that connection, etc. In this cases we speak of „conceptual modelling" items.

The "full range of mathematics" is therefore defined by giving items from all three types of mathematical activity. It is this distinction between different aspects of mathematical thinking which rules the German national framework for the national option of PISA in Germany (Neubrand & al. 2001).

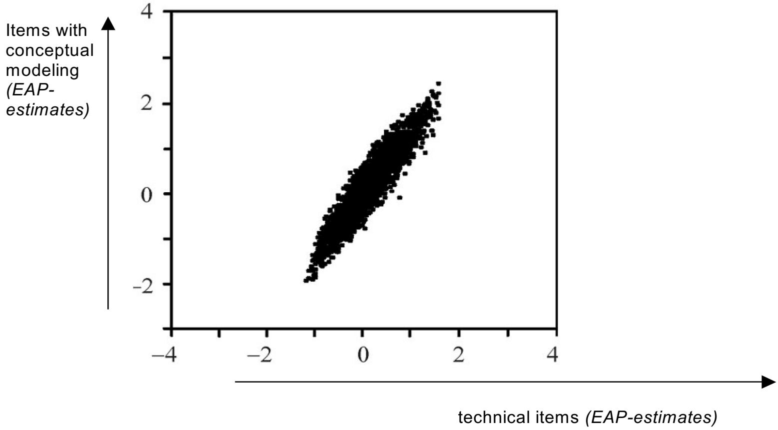
THE IDEA OF "PROFILES" OF MATHEMATICS ACHIEVEMENT

There is empirical evidence, that in some cases a differentiated analysis of mathematical achievement gives insight into the inner structure of mathematical achievement - and is possible on the basis of a full set of items distributed over the three Types of Mathematical Activity. We sketch only two instances of such "profiles" (cf. Neubrand & Neubrand 2004 for details).

(A) Individual differences on the three types of mathematical activities

Knoche & al. (2002) showed this scatter plot (Fig. 2). At least in the German population of PISA, there is not enough reason to conclude that good performance in the technical items is sufficient to show good performances on the conceptual modelling items.

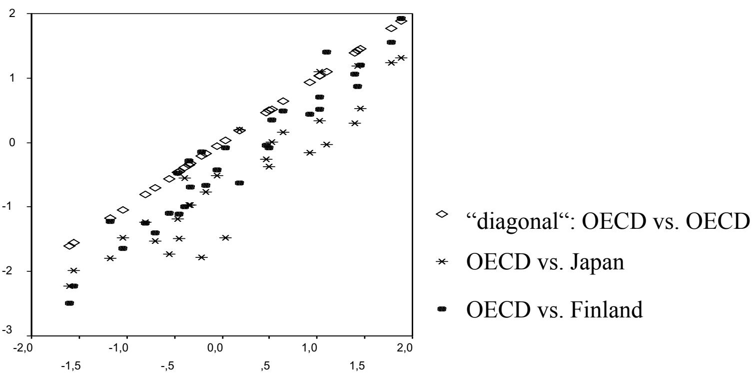
Figure 2: Achievement of German students on “technical” and “conceptual” items



(B) Differences between countries

The following picture (Neubrand & Neubrand 2004) shows the striking different behaviour of Japanese and Finish students in PISA. Against the achievement in the OECD average (the diagonal line) there are plotted the means for every international item in the respective country.

Figure 3: Different achievement of two high scoring countries, Japan and Finland



Apparently, the Japanese population in PISA behaves very different from both, the OECD average, and one other high achieving country, Finland. Furthermore, Finland shows a tendency to score very well at the easier items, but does not show substantial advantages at the harder items.

CONCLUSION: STRUCTURES OF ACHIEVEMENT POINT TO DIFFERENCES IN TEACHING AND LEARNING

Comparisons of mathematical achievement can, or at least should, used to indicate fields of further development of mathematics education. However, this requires a broad and theoretically based picture of the mathematics included in the test. In PISA this reflections were done extensively. The results shown here indicate empirical foundations e.g. that it is not a promising way to restrict mathematics education to a better performance of technical abilities (Fig. 2), nor that “high achievement” can be restricted to the one score point on the “horse race axis” (Fig.3). In both cases, the inner structure of mathematical achievement revealed areas where to act, in Germany on the improvement of conceptual capabilities, in Finland on stronger results also at the more challenging items.

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