ABOUT THE DIMENSIONALITY OF THE SETS OF NUMBERS

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This study draws on insights gleaned from recent educational research on children's intuitions and representations about the sets on numbers. We explore to what extent there is a correlation between the dimensions of some mathematical concepts and children's representations about these concepts. We discuss findings concerning students' conceptions, strategies and beliefs about two-dimensional sequences. A two-dimensional sequence supposes two recursive alternate rules.

The methodology we used consists in a series of specific questionnaires applied to students in grades 1 to 4, followed by interviews with selected students. The items contained two-dimensional sequences represented both in a mono-dimensional way (as a "linear" sequence) and in a two-dimensional manner (where the two rules are visualized on the two dimensions of the plane). Besides the content questions, the questionnaires also contained some meta-cognitive questions mostly asking students to express their preference regarding specific tasks.

We remarked that the students generally understand the set of natural numbers through sequences in which the recursion follows a unique rule, while the set of rationals is perceived through sequences in which the recursion follows a double rule. We identified various testimonies for students' linear perception of **N** and for the bi-dimensional perception of **Q**. For instance, many students have chosen a translation as a way to continue a geometrical sequence. When the same type of task was represented both mono-dimensionally and two-dimensionally, some students used the translation for the linear representation and, in the same time, the correct rule to continue the pattern, for the bi-dimensional representation. Moreover, when asked about their preferences, most of the students decided that the sequences with a plane distribution are preferred or are used as a base for reasoning in the "linear" cases.

From these considerations, a possible hypothesis is derived: the mental representation of children's algebraic concepts depends on the dimensionality of these concepts. In addition, children tend to use geometrical bi-dimensional drawings to underlie some algebraic ideas. This is why those concepts that have a natural multidimensional (2 or 3) structure (e.g.: the set of rational numbers, the functions) seem to be easier internalized at an intuitive level than the ones with a one-dimensional structure (e.g.: the set of natural numbers, the geometrical line). Therefore, the dimensionality of mathematical objects needs to be taken into account more seriously in curriculum development and teaching practice.