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Mathematics Classrooms that Promote Understanding

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1. Aims and the target readership of the book

The aim of the book is to promote a certain way of teaching mathematics which, according to the authors, is conducive to the learning of this school subject “with understanding”, and to persuade the teachers, teacher educators and educational policy makers that this goal is, indeed, attainable. The book could be classified as belonging to the “energizers of practice” category of publication in mathematics education (Sierpinska et al. 1993).

2. The content

The book has three parts. The first part presents the general assumptions of a reform of teaching mathematics, from kindergarten to the secondary school, with regard to the contents, the aims and the modes of teaching mathematics in those grades. This part also comprises a description of what the authors understand by “learning mathematics with understanding” and “teaching mathematics for understanding”. The second part is a collection of vignettes from innovative classrooms which made it possible to reach the aims of the reform. In the third part some conditions for the possibility of developing such classrooms are identified and discussed.

Part I is devoted to the presentation of the fundamental assumptions of the reform

What does it mean to “learn mathematics with understanding”?

The authors of the first chapter in Part I, entitled *Mathematics worth teaching, mathematics worth understanding* state that “school mathematics should be viewed as a human activity that reflects the work of mathematicians – finding out why given techniques work, inventing new techniques, justifying assertions, and so forth” (p. 5). On the other hand, the authors of the second chapter, *Teaching and learning with understanding*, distance themselves from defining understanding in mathematics on the basis of mathematicians’ views on that matter. They say that they choose, instead, to ground their definition in the research on the teaching and learning of mathematics (p. 19). Thus two concepts are proposed: “learning with understanding” and “teaching for understanding”.

The first characterization of “learning with understanding” is formulated in terms of the outcome of the process: Students have learned some mathematics with understanding if and only if they are able to apply this knowledge to learning new topics and to solving new and unfamiliar topics (p. 19). This is called the “generative” property

of knowledge, learned with understanding. Next, a characterization in terms of mental activities involved in the process is proposed. Five such activities are identified and described: constructing relationships, extending and applying mathematical knowledge, reflecting about experiences, articulating (formulating) what one knows, making mathematical knowledge one’s own (p. 20–23).

Let me stop the description of the contents for a while at this point, and ask the question: Is the point of view of the authors of this chapter indeed different, as they claim it is, from the mathematicians’? The authors do not elucidate what they mean by “notions of understanding derived from ways that mathematicians understand and teach mathematics” (p. 19), so let me speculate.

A mathematician would probably agree that these activities are, indeed, necessary for the process of “understanding” in mathematics. But the mathematician may not consider them specific to mathematics (understanding *anything* requires these activities). Abstraction and generalization of pattern and spatial relationships, and their representation in a symbolic notation, instrumental in the creation of a calculus, would probably be mentioned by a mathematician as mental activities characteristic of mathematics. These are specific forms of what the authors have called “constructing relationships”. But abstraction appears to be important for the authors as well; it is seen as a result of the activity of reflection: “As students reflect on the use of [manipulatives], the manipulations of the physical materials become abstracted. Eventually students no longer have to actually manipulate the physical tools themselves; they can think directly about more abstract symbolic representations of the tools ...” (p. 27).

The characterization of learning with understanding by the “generative” property of the knowledge thus learned appears to contain what mathematicians call “intuition” or “the ability to transfer ideas from one place to another” which allows one to choose, among many logically equivalent paths, the most promising strategy of approaching a problem (Shimshon Amitzur, in Sfard 1998, p. 446).

The list of mental activities necessary for “learning mathematics with understanding” does not contain conditions related to the quality of the mathematical contents of this understanding. This approach thus appears to differ from that of mathematicians who are interested mainly in “good understanding” and tend to have quite precise ideas about what it means to have such a “good understanding” of this or that piece of mathematics. However, the authors do take a stance with respect to the contents of understanding, albeit not explicitly and not in their definition of “learning with understanding”. We can infer it from Chapter 1, Part I, totally devoted to the description of the mathematics “worth teaching and understanding” and from the examples of teaching particular topics, given in the chapters of Part II. Interestingly, in Chapter 1 of Part I, the authors support their view of mathematics with a reference to a mathematician’s opinion, namely to William Thurston’s well known metaphor of mathematics as a “banyan tree”.

The authors include mental activities generating the personalization of knowledge by students as basic in learning with understanding : “[students shouldn’t] perceive their

knowledge simply as something that someone else has told them or explained to them; they need to adopt a stance that knowledge is evolving and provisional. They will not view knowledge in this way, however, if they see it as someone else's knowledge, which they simply assimilate through listening, watching, and practicing" (p. 23). This concern is quite justified from the point of view of the teaching of mathematics, as is the "reflection" on one's mathematical experiences instead of just "routine application of skills", as well as the "ability to communicate" one's mathematical ideas using various semiotic means such as ordinary language, symbolic notation, diagrams, or models (p. 22). There would be a difference here with the point of view of a mathematician for whom these conditions of learning mathematics are usually taken for granted, unworthy of mentioning and discussing.

What does it mean to "teach mathematics for understanding"?

The authors define teaching mathematics for understanding by characterizing three axes of instruction: tasks, tools and normative practices.

It is not important, the authors say at one point, to have specially designed tasks in the teaching for understanding; any task will do as long as it is "engaged in for the purpose of fostering understanding, not for the purpose of completing the task" (p. 24): "the most challenging tasks can be taught so that students simply follow routines, and the most basic computational skills can be taught to foster understanding of fundamental mathematical concepts" (ibid.). However, a few pages later, they come back on their word and say that "the selection and sequencing of tasks and tools is critical (p. 27)". It is important, they say, that tasks be chosen so as to build on students' informal mathematical knowledge. Moreover, unlike the traditional teaching where problem solving is seen as an application of skills acquired through imitation and practice, the reform-based teaching encourages an integration of problem solving with the learning of basic skills and concepts.

The "tools" dimension refers, for the authors, to representations: standard symbolic representations, manipulatives, representations used in calculators and computers. Teaching mathematics for understanding means that many different representations are used, and connections between them are made; representations are not only given by the teacher but also invented by the students; students are engaged in explicit reflection on the characteristics of different representations (p. 25).

The axis of the "normative practices" appears to have something in common with Brousseau's "didactic contract" (1997) and Voigt's "socio-mathematical norms" (1995). The authors say that "the norms in a particular class determine how students and the teacher are expected to act or respond to a particular situation. Normative practices form the basis for the way tasks and tools are used for learning, and they govern the nature of the arguments that students and teachers use to justify mathematical conjectures and conclusions" (p. 25–6). The authors attribute an important role to this axis in the teaching for understanding. It is the normative practices in a particular classroom that will or will not allow the students to en-

gage with the tasks and the available tools in ways that encourage learning with understanding. Classrooms that promote understanding should resemble "discourse communities in which all students discuss alternative strategies ... [S]tudents expect that the teacher and their peers will want explanations as to why their conjectures and conclusions make sense and why a procedure they have used is valid for a given problem ... [M]athematics becomes a language for thought rather than merely a collection of ways to get answers" (p. 26).

According to the authors, an important condition for the preferred normative practices to be implemented in the mathematics classrooms is that the teachers understand both mathematics and students' mathematical thinking: "Understanding mathematics for instruction involves more than understanding mathematics taught in university mathematics content courses. It entails understanding how mathematics is reflected in the goals of instruction and in different instructional practices. Knowledge of mathematics must also be linked to knowledge of students' thinking, so that teachers have conceptions of typical trajectories of student learning and can use this knowledge to recognize landmarks of understanding in individuals. Teachers need to reflect on their practices and on ways to structure their classroom environment so that it supports students' learning with understanding" (p. 31).

The contents of teaching mathematics for understanding

The chapter on "Mathematics worth teaching, mathematics worth understanding" starts with a very emotional portrayal of the so-called traditional mathematics curriculum and the organization of its teaching. The division of the mathematical content into arithmetic, algebra, geometry and pre-calculus and its distribution over the years of elementary and secondary school, called the "layer-cake structure" by the authors, is blamed for "the tendency to design each course primarily to meet the prerequisites of the next course", and held responsible for "the unacceptably high attrition from mathematics that plagues our schools" (p. 4). Traditional school mathematics is depicted as an unconnected collection of techniques, learned for their own sake. The organization of the teaching into three-segment lessons, composed of 1) correction of homework, 2) introduction of new material by the teacher and solution of a few exemplary exercises, 3) students working on an assignment for the following day, is characterized as a "mechanistic approach to instruction" which "isolates mathematics from its uses and from other disciplines", and makes it appear to students as "a tedious, uninteresting path to follow ... bear[ing] little resemblance to what a mathematician or a user of mathematics does" (p. 4–5).

The authors then propose a vision of school mathematics that better reflects the work of mathematicians and users of mathematics, "finding out why given techniques work, inventing new techniques, justifying assertions ..., investigating problem situations, deciding on variables, ... ways to quantify and relate the variables, carry out calculations, make predictions, and verify the utility of the predictions" (p. 5).

Finally some more concrete proposals are made concerning the curriculum. It is proposed that the content of teach-

ing be divided into “content domains” or “strands”, each strand being characterized by a number of “big ideas”. The content domains which the authors identified as regarded by the society as worthy of being “learned with understanding by all students” are: “number, quantities, algebra, geometry, and statistics” (p. 8).

It is not quite clear to me what the authors meant by the “big ideas”. The examples given on p. 8 are not, unfortunately, very helpful. One example of a “big idea” related to the operations of addition and subtraction of whole numbers was formulated as follows:

“symbolic statements that characterize the domain (e.g., $a + b = c$ and $a - b = c$, where a , b , and c are natural numbers)”.

Maybe there is a misprint, and the example should read, “... $a + b = c$ and $c - b = a$...” and the “big idea” is that subtraction of a number is an operation inverse to the operation of addition of the same number?

In general, the examples do not read as formulations of ideas but rather as a list of skills and abilities which are, by the way, well known from the traditional elementary school curriculum in some countries:

- representing symbolically the properties of operations in natural numbers; representing relations between three numbers a , b , c , in the form of $a + b = c$ or $a - b = c$;
- given two of these numbers, to find the third;
- using rules and procedures such as counting, addition tables (known under the name of “basic facts” in the US), and symbolic transformations;
- performing addition and subtraction of natural numbers using place value decomposition (e.g. $234 = 2 * 100 + 3 * 10 + 4 * 1$) as well as grouping according to the needs of a particular situation (e.g. $234 - 40 = 240 - 6 - 40 = 200 - 6 = 194$);
- solving word problems involving finding and using relationships between the various concepts, rules and procedures.

In the proposed curriculum, these common activities are required to be embedded in tasks and “situations” that would motivate the students to engage in “investigations” or “inquiries”, construction of mathematical models, formulation of mathematical justifications, and invention rather than just use of given routines or formulas (p. 10–12). Use of technology is also mentioned: the tasks for the students should involve “flexible use of technologies” (p. 12). In particular, the strand labeled “geometry” is meant to refer not just to the ordinary static Euclidean geometry, but to a domain enriched by the “dynamic geometry” learned in interaction with some dynamic geometry software. The authors stress the fact that, in the second half of the twentieth century, technology has transformed parts of mathematics in an important way: Statistics is used as one example, study of iterative processes (dynamical systems) as another. This, they claim, can no longer be ignored in the teaching of mathematics even at the elementary and secondary school level (p. 15).

Part II contributes “existence proofs” for the possibility of developing mathematics classrooms that promote understanding

Examples of situations provided in this part of the book are taken from the teaching of basic concepts related to natural numbers and geometry in primary school, rational numbers in middle grades, and statistics and algebra in secondary school.

The chapter on the teaching of number concepts presents four research programs whose aim is to develop approaches supporting a concurrent development of computational skills and understanding: Cognitively Guided Instruction, Conceptually Based Instruction, Supporting Ten-Structured Thinking, Problem-Centered Mathematics Project. Except for the last one, established in South Africa, all programs have been developed in the US. The discussion of the convergence and differences between the programs is organized along the characteristics of the “learning with understanding” laid out in the first part of the book: “constructing relationships”, “extending and applying mathematical knowledge”, “reflection and articulation”, “ownership”. The authors make explicit statements about how these postulated characteristics have been realized in each of the four programs. This format of analysis is followed in every chapter in this part of the book.

The chapter on the development of children’s mathematical understanding of space refers to a research conducted in collaboration with a small group of primary school teachers. The research was based on the assumption that although children have a broad experience of material space, special activities have to be designed for the development of these “intuitions” into mathematical reasoning about space. A detailed account of the activities designed by the authors and their realization in the classrooms is provided, with focus on the gradual transformation of the students’ spontaneous ideas about space into more systematic geometric thinking, mediated by two-dimensional representations of physical objects (e.g. boxes, parts of a city) in the form of diagrams, figures and maps.

The authors of the chapter on the teaching of rational numbers in middle grades are mainly concerned with the teachers’ understanding of the mathematics involved in this topic. The authors imply that this understanding appears to be quite insufficient among teachers and go on to show the difference it makes when “teachers have reconceptualized the mathematics they teach”. Two classroom episodes are presented. In one of the episodes the teacher was helping the students to construct for themselves a meaning for the rule for dividing fractions. In the other episode the teacher made attempts to help the students develop the so-called “multiplicative reasoning” and overcome a tendency to apply “additive reasoning” in situations calling for the identification of ratios of quantities and their comparison. Both episodes illustrate “teaching for understanding” and, indeed, in both, all students were involved in “constructing relationships, extending and applying mathematical knowledge, reflecting about mathematical experiences, articulating what one knows, and making mathematical knowledge one’s own” (p. 97).

However, not all students constructed the intended meanings for the mathematical situations proposed by the teachers.

The next chapter in this part of the book provides a description of a research and development activity in the area of the teaching of statistics in secondary school. The activity involved the design and experimentation of a teaching unit for Grade 8, which lasted 2 weeks, and was composed of a phase of “knowledge acquisition” followed by a “production phase” or a phase of statistical investigations with the use of technology. The knowledge acquisition phase comprised a tutorial and a set of concrete examples or models of statistical investigations, called the “Library of Exemplars”. So far the design looks like a very traditional approach to teaching where students have to absorb some ready made knowledge and acquire some skills by practice before they can apply them in problem solving. However, the interesting feature of the “Library of Exemplars” is that the examples are not presented as model investigations done by an expert which have to be simply imitated by the students, but as projects done by other students. The exemplars also contain information about how these other students were evaluated on their projects. By analyzing the exemplars, the students thus learn, not only from the achievements of others, but also from their failures. There are other features of the “Authentic Statistics Project” that allow the author to claim that it actually satisfies the definitions of “learning with understanding” and “teaching for understanding” adopted in the book.

The chapter related to the teaching of algebra is entitled “Teaching and Learning a *New Algebra*” and contains statements reminiscent of the rhetoric of the famous New Math reforms of the sixties. It starts with a criticism of the “traditional school algebra” as taught in the United States, which is blamed for “failing in virtually all the dimensions of understanding” assumed by the authors of the book. In particular, the traditional algebra, with its “memorization of procedures known only as operations on strings of symbols” and “notoriously artificial applications” is accused of turning away many students from mathematics (pp. 133–134). This dark picture is then contrasted with high praise for the “[true] algebraic reasoning ... and the use of algebraic representations such as graphs, tables, spreadsheets and traditional formulas” which afford one of the “most powerful tools that our civilization has developed”. The chapter then continues with a plea for a long-term reform of the teaching of algebra which should begin much earlier than it is done presently (in the US), and should involve “infusing algebra throughout the mathematics curriculum from the very beginning of school” in the form of activities of generalization and of expressing the generalizations “using increasingly formal languages” (p. 134). Several examples of such activities in elementary grades are given.

Part III is devoted to some general questions related to the implementation of the reforms

There are two chapters in this part of the book, one concerned specifically with assessment and the other bearing upon questions of a more general nature. The key ex-

pression of the chapter on assessment is “domain-based approach”, which appears to be used as a label for the way of teaching mathematics promoted in the book. It is surprising to find it only in one of the last chapters of the book. Its explicit definition is not given. It is possible that the word “domain” refers to the organization of the content of teaching into “content domains” or “strands” and “substrands” of which the authors have spoken in the first chapter of Part I, but the expression “domain-based approach” was not used there.

The postulates found in this chapter will be familiar to those readers who are acquainted with the “NCTM Assessment Standards”. In brief, assessment restricted to formal end-of-unit and end-of term written tests is rejected, and assessment integrated with instruction, based on multiple sources of evidence of students’ work and progress in learning is promoted. Several examples of assessment items are provided in the chapter and analyzed from the point of view of their fitting with the definition of “learning with understanding” as proposed in the first part of the book.

The last chapter is a synthesis of the previous ones written from the perspective of the question: “It has been shown that classrooms that promote understanding exist. In what ways can many more such classrooms be developed?”.

3. Evaluation

In his book on *Ethics and Language*, C.L. Stevenson (1944) gives the following example of a conversation between two people, A and B, about a mutual friend:

A: He has but little formal education, as is plainly evident from his conversation. His sentences are often roughly cast, his historical and literary references rather obvious, and his thinking is wanting in that subtlety and sophistication which mark a trained intellect. He is definitely lacking in culture.

B: Much of what you say is true, but I should call him a man of culture notwithstanding.

A: Aren’t the characteristics I mention the antithesis of culture, contrary to the very meaning of the term?

B: By no means. You are stressing the outward forms, simply the empty shell of culture. In the true and full sense of the term, “culture” means imaginative sensitivity and originality. These qualities he has; and so I say, and indeed with no little humility, that he is a man of far deeper culture than many of us who have had superior advantages in education. (Stevenson 1944, p. 211).

Let me offer the following paraphrase of the above conversation.

A: He rarely if at all refers to the definition of matrix of a linear transformation when he needs to find one or reason about it. A couple of days ago, to write a matrix, in a given basis, of a linear transformation of the vector plane, he found the image of one vector, put the coordinates of the vector in the first column, the coordinates of the image in the second column of a matrix and claimed that this 2×2 matrix is the matrix of the transformation. In general, he disregards definitions in learning a concept and invests a lot of his intellectual effort in making sense of the defined term by generalizing from the examples that the teacher gives after the definition. This is how he developed his very own idea that a linear transformation is always some combination of a rotation and a dilation. And this is one of the reasons which explains his problems with the concept of matrix of a linear

transformation. He definitely does not understand this concept.
 B: Much of what you say is true, but I should say that he has been learning with understanding nevertheless.

A: Aren't the characteristics I mention the antithesis of understanding, contrary to the very meaning of the term?

B: By no means. You are stressing the formal aspects of the concept and the procedural skills in learning mathematics, simply the empty shell of mathematics. In the true and full sense of the word, understanding mathematics, as a human activity, involves constructing relationships, extending and applying mathematical knowledge, reflecting about experiences, articulating what one knows and making mathematics one's own. Into these activities he has fully engaged himself; and so I say, and indeed with no little humility, that he has learned the concept of matrix of linear transformation with more understanding than many of us who have had to follow the traditional formal education.

The point that Stevenson is trying to make by his "conversation" is to illustrate how "persuasive definitions", as opposed to "descriptive definitions", work. "Culture", just as "understanding", is a word rich in laudatory emotive meaning, but at the same time its descriptive meaning is rather vague, and thus it may be easy to alter the latter so that the change appears "natural". The emotive meaning remaining unaltered, the hearer may not even notice that he or she is being influenced (Stevenson *ibid.*, p. 212).

In my paraphrase, the interlocutor B, who is meant to represent the standpoint of the book, is not so much altering the sense of the verb "to understand" as replacing it with "to learn with understanding". But this subtle alteration is not explicitly discussed in the book and, indeed, some authors confuse the two notions. For example, in the concluding chapter, a statement is made that "The [described] classrooms were dramatically different from traditional ones, and students who studied in them did come to *understand* mathematics" (p. 185). This statement is not quite legitimate because all that the authors of the chapters in Part II were able to show was that students were "learning mathematics with understanding", according to the definition of this term proposed in Part I of the book. Therefore, the lack of explicit discussion of the difference between the meaning of the two expressions contributed to the persuasive character of the definition.

It may be all right to write in defense of a cause and use persuasive rhetoric to achieve its goals. It is undeniable that, in the human society, one can "do things with words". But researchers should beware of the confusion between persuasive rhetoric and scientific argument. The distinction is not easy in the domain of mathematics education where the line between being a researcher and being a reformer is extremely fine: After all, the ultimate goal of our research is the "improvement of the teaching practices" which lines our work with thinner or thicker layers of militating for a cause. The researcher can easily get carried away by the wishful thinking of the reformer and forget that some crucial elements of the reality he or she is studying may not lend themselves to change or removal.

It is interesting to note that the vision of school mathematics proposed by some authors in the book resembles that of the famous "New Math" reforms in the 1960s and 70s. The failure of these reforms was spectacular. Mathematics education has learned a lot from this fail-

ure (see e.g., Jonnaert 1999). It has learned, in particular, that school mathematics does not and cannot function as mathematicians' mathematics and that we cannot shape the school mathematics and the ways teachers and students act according to our whims and dreams. The school institution and school mathematics is a reality which is subject to certain laws that must be studied and understood before or during new attempts to influence it (Chevallard 1985). Mathematics classrooms do not exist in a void; they are parts of larger educational systems, social systems, political systems and cultures. These systems change, but changes in one system cannot be made independently from changes in the other systems; there exist complex relationships. It is the job of the researcher to investigate these relationships and explain why and in what ways a reform of mathematics teaching can fail or succeed in remaining faithful to its objectives. (Such research already exists with respect to the reform movement promoted in the book: see Saxe 1999). This knowledge could be useful to the reformer in designing "reasonable" changes or changes satisfying the "initial conditions" of achieving the aims of a reform.

In a recent issue of the ZDM, Thomas Romberg criticized the publication "Rethinking the mathematics curriculum" for its parochial character: "the majority of the papers – he wrote – were set in a British context". After substitution of "American" for "British", the same can be said with respect to the presently reviewed book. Although the authors may have had the ambition to describe and promote some world wide "reform movement" – shifts in curriculum documents from Australia, Japan, United Kingdom and the US are evoked in one place (p. 6) – in actual fact, the reference is to a reform movement that has been taking place in North America for some time now, inspired by the activity of the NCTM (National Council of Teachers of Mathematics) and supported, in the US, by the NCRMSE (National Center for Research in Mathematical Sciences Education) (p. 7). The authors make little or no reference to non-American publications. The only reference to models of understanding in mathematics, other than their own, mentions the work of Colburn, Brownell and Van Engen (p. ix). These must be household names in the US, for they do not appear in the Author Index, and cannot be found in any list of references in the book.

There is, of course, nothing wrong about the fact that American mathematics educators wrote a book for American teachers, but, if the book is considered to have some value also for the non-American readership, (as its being reviewed in an international journal would suggest), then it should contain enough information about the American context, the historical roots of the reform movement, its ideology, the nature and political status of the NCRMSE, the mathematical background of US elementary school mathematics teachers, the variety of cultures of the mathematics classrooms, etc.

This being said, let me acknowledge the value that the book may have for teachers and teacher educators, especially through the examples of "good practice" in the second part of the book. Clearly and vividly presented, these examples are analyzed at just the right level to inspire and

guide teachers in reflecting on their own practice. It is an interesting book to read.

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