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RANDIC TYPE ADDITIVE CONNECTIVITY ENERGY OF A GRAPH

K. V. Madhusudhan[1], P. Siva Kota Reddy[2] and K. R. Rajanna[3]

[1] ATME College of Engineering, Mysore 570 028, Karnataka, India;
[2] Siddaganga Institute of Technology, B. H. Road, Tumkur 572 103, Karnataka, India;
[3] Acharya Institute of Technology, Bangalore 560 107, Karnataka, India

E-mail: E-mail: kvmadhu13@gmail.com; reddy_math@yahoo.com, pskreddy@sit.ac.in;
rajanna@acharya.ac.in

Abstract. The Randic type additive connectivity matrix of the graph G of order n and size m is defined as $RA(G) = (R_{ij})$, where $R_{ij} = \sqrt{d_i} + \sqrt{d_j}$ if the vertices v_i and v_j are adjacent, and $R_{ij} = 0$ if v_i and v_j are not adjacent, where d_i and d_j be the degrees of vertices v_i and v_j respectively. The purpose of this paper is to introduce and investigate the Randic type additive connectivity energy of a graph. In this paper, we obtain new inequalities involving the Randic type additive connectivity energy and presented upper and lower bounds for the Randic type additive connectivity energy of a graph. We also report results on Randic type additive connectivity energy of generalized complements of a graph.

Key words: Randic type additive connectivity energy, Randic type additive connectivity eigenvalues.

Mathematical Subject Classification (2010): 05C50.

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1. Introduction

Let G be a simple, finite, undirected graph. The energy $E(G)$ is defined as the sum of the absolute values of the eigenvalues of its adjacency matrix. Basically energy of graph is originated from chemistry. In For more details on energy of graphs (see [1, 2]).

In chemistry, we can represent the conjugated hydrocarbos by a molecular graph. Each edge between the carbon-carbon atoms can be represented by an edge. Here we will neglect the hydrogen atoms. Now a days energy of graph attracting more and more researchers due its significant applications. The Randic type additive connectivity matrix $RA(G) = (R_{ij})_{n\times n}$ is given by

$$RA_{ij} = \begin{cases} \sqrt{d_i} + \sqrt{d_j}, & v_i \sim v_j, \\ 0, & \text{otherwise.} \end{cases}$$

The characteristic polynomial of $RA(G)$ is denoted by $\phi_R A(G, \lambda) = \det(\lambda I - RA(G))$. Since the Randic type additive connectivity matrix is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1 > \lambda_2 > \cdots > \lambda_n$. The minimum dominating Randic energy is given by

$$RAE(G) = \sum_{i=1}^{n} |\lambda_i|. \tag{1}$$

Definition 1.1. The spectrum of a graph G is the list of distinct eigenvalues $\lambda_1 > \lambda_2 > \cdots > \lambda_r$, with their multiplicities $m_1, m_2, \ldots, m_r$, and we write it as

$$\mathrm{Spec(G)} = \begin{pmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_r \\ m_1 & m_2 & \cdots & m_r \end{pmatrix}.$$

In [3, 4], the authors defined the minimum covering Randic energy of a graph and minimum dominating Randic energy of a graph and presented the upper and lower bounds on these new energies.

This paper is organized as follows. In the Section 3, we get some basic properties of Randic type additive connectivity energy of a graph. In the Section 4, Randic type additive connectivity energy of some standard graphs are obtained.

2. Some basic properties of Randic type additive connectivity energy of a graph

Let us define the number K as

$$K = \sum_{i<j} \left(\sqrt{d_i} + \sqrt{d_j}\right)^2.$$

Then we have

Proposition 2.1. *The first three coefficients of the polynomial $\phi_{RA}(G, \lambda)$ are as follows:*
(i) $a_0 = 1$,
(ii) $a_1 = 0$,
(iii) $a_2 = -K$.

$\lhd$ (i) By the definition of $\Phi_{RA}(G, \lambda) = \det[\lambda I - RA(G)]$, we get $a_0 = 1$.

(ii) The sum of determinants of all 1×1 principal submatrices of $RA(G)$ is equal to the trace of $RA(G)$ implying that

$$a_1 = (-1)^1 \times \text{the trace of } \ RA(G) = 0.$$

(iii) By the definition, we have

$$(-1)^2 a_2 = \sum_{1\leqslant i<j\leqslant n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix} = \sum_{1\leqslant i<j\leqslant n} a_{ii}a_{jj} - a_{ji}a_{ij} = \sum_{1\leqslant i<j\leqslant n} a_{ii}a_{jj} - \sum_{1\leqslant i<j\leqslant n} a_{ji}a_{ij} = -K. \ \rhd$$

Proposition 2.2. *If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the Randic type additive connectivity eigenvalues of $RA(G)$, then*

$$\sum_{i=1}^{n} {\lambda_i}^2 = 2K.$$

◁ It follows as

$$\sum_{i=1}^{n}\lambda_i^2=\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}a_{ji}=2\sum_{i<j}(a_{ij})^2+\sum_{i=1}^{n}(a_{ii})^2=2\sum_{i<j}(a_{ij})^2=2P. \triangleright$$

Using this result, we now obtain lower and upper bounds for the Randic type additive connectivity energy of a graph:

Theorem 2.1. *Let G be a graph with n vertices. Then*

$$RA(G)\leqslant\sqrt{2nK}.$$

◁ Let $\lambda_1,\lambda_2,\dots,\lambda_n$ be the eigenvalues of $RA(G)$. By the Cauchy–Schwartz inequality we have

$$\left(\sum_{i=1}^{n}a_ib_i\right)^2\leqslant\left(\sum_{i=1}^{n}{a_i}^2\right)\left(\sum_{i=1}^{n}{b_i}^2\right).$$

Let $a_i=1$, $b_i=\mid\lambda_i\mid$. Then

$$\left(\sum_{i=1}^{n}|\lambda_i|\right)^2\leqslant\left(\sum_{i=1}^{n}1\right)\left(\sum_{i=1}^{n}|\lambda_i|^2\right)$$

implying that

$$[RAE]^2\leqslant n\cdot 2K$$

and hence we get

$$[RAE]\leqslant\sqrt{2nK}$$

as an upper bound. ▷

Theorem 2.2. *Let G be a graph with n vertices. If $R=\det RA(G)$, then*

$$RAE(G)\geqslant\sqrt{2K+n(n-1)R^{\frac{2}{n}}}.$$

◁ By definition, we have

$$(RAE(G))^2=\left(\sum_{i=1}^{n}\mid\lambda_i\mid\right)^2=\sum_{i=1}^{n}\mid\lambda_i\mid\sum_{j=1}^{n}\mid\lambda_j\mid=\left(\sum_{i=1}^{n}\mid\lambda_i\mid^2\right)+\sum_{i\neq j}\mid\lambda_i\mid\mid\lambda_j\mid.$$

Using arithmetic-geometric mean inequality, we have

$$\frac{1}{n(n-1)}\sum_{i\neq j}\mid\lambda_i\mid\mid\lambda_j\mid \geqslant \left(\prod_{i\neq j}\mid\lambda_i\mid\mid\lambda_j\mid\right)^{\frac{1}{n(n-1)}}.$$

Therefore,

$$\begin{aligned}[RA(G)]^2 &\geqslant \sum_{i=1}^{n}\mid\lambda_i\mid^2+n(n-1)\left(\prod_{i\neq j}\mid\lambda_i\mid\mid\lambda_j\mid\right)^{\frac{1}{n(n-1)}}\\ &\geqslant \sum_{i=1}^{n}\mid\lambda_i\mid^2+n(n-1)\left(\prod_{i=1}^{n}\mid\lambda_i\mid^{2(n-1)}\right)^{\frac{1}{n(n-1)}}\\ &= \sum_{i=1}^{n}\mid\lambda_i\mid^2+n(n-1)R^{\frac{2}{n}}=2K+n(n-1)R^{\frac{2}{n}}.\end{aligned}$$

Thus,

$$RAE(G) \geqslant \sqrt{2K + n(n-1)R^{\frac{2}{n}}}. \rhd$$

Let λ_n and λ_1 are the minimum and maximum values of all $\lambda_i' s$. Then the following results can easily be proven by means of the above results:

Theorem 2.3. *For a graph G of order n,*

$$RAE(G) \geqslant \sqrt{2Kn - \frac{n^2}{4}(\lambda_1 - \lambda_n)^2}.$$

Theorem 2.4. *For a graph G of order n with non-zero eigenvalues, we have*

$$RAE(G) \geqslant \frac{2\sqrt{\lambda_1 \lambda_n}\sqrt{2Kn}}{(\lambda_1 + \lambda_n)^2}.$$

Theorem 2.5. *Let G be a graph of order n. Let $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant \ldots \geqslant \lambda_n$ be the eigenvalues in increasing order. Then*

$$RAE(G) \geqslant \frac{|\lambda_1||\lambda_n|n + 2K}{|\lambda_1| + |\lambda_n|}.$$

3. Randic type additive connectivity energy of Some Standard Graphs

Theorem 3.1. *The Randic type additive connectivity energy of a complete graph K_n is $RE^D(K_n) = 4(n-1)^{\frac{3}{2}}$.*

$\lhd$ Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$. The Randic type additive connectivity matrix is

$$RA(K_n) = \begin{bmatrix} 1 & 2\sqrt{n-1} & 2\sqrt{n-1} & \ldots & 2\sqrt{n-1} & 2\sqrt{n-1} \\ 2\sqrt{n-1} & 0 & 2\sqrt{n-1} & \ldots & 2\sqrt{n-1} & 2\sqrt{n-1} \\ 2\sqrt{n-1} & 2\sqrt{n-1} & 0 & \ldots & 2\sqrt{n-1} & 2\sqrt{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2\sqrt{n-1} & 2\sqrt{n-1} & \ldots & 2\sqrt{n-1} & 0 & 2\sqrt{n-1} \\ 2\sqrt{n-1} & 2\sqrt{n-1} & \ldots & 2\sqrt{n-1} & 2\sqrt{n-1} & 0 \end{bmatrix}.$$

Hence, the characteristic equation is

$$\left(\lambda + 2\sqrt{n-1}\right)^{n-1}\left(\lambda - 2(n-1)^{\frac{3}{2}}\right) = 0$$

and the spectrum is

$$\operatorname{Spec}_R^D(K_n) = \begin{pmatrix} 2(n-1)^{\frac{3}{2}} & -2\sqrt{n-1} \\ 1 & n-1 \end{pmatrix}.$$

Therefore, we get $RAE(K_n) = 4(n-1)^{\frac{3}{2}}$. $\rhd$

Theorem 3.2. *The Randic type additive connectivity energy of star graph $K_{1,n-1}$ is*

$$RAE(K_{1,n-1}) = 2\left[\sqrt{n-1} + (n-1)\right].$$

◁ Let $K_{1,n-1}$ be the star graph with vertex set $V = \{v_0, v_1, \ldots, v_{n-1}\}$. Here v_0 be the center. Randic type additive connectivity matrix is

$$RA(K_{1,n-1}) = \begin{bmatrix} 1 & \sqrt{n-1}+1 & \sqrt{n-1}+1 & \ldots & \sqrt{n-1}+1 & \sqrt{n-1}+1 \\ \sqrt{n-1}+1 & 0 & 0 & \ldots & 0 & 0 \\ \sqrt{n-1}+1 & 0 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sqrt{n-1}+1 & 0 & 0 & \ldots & 0 & 0 \\ \sqrt{n-1}+1 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix}.$$

The characteristic equation is

$$\lambda^{n-2}\big(\lambda + \sqrt{n-1} + (n-1)\big)\big(\lambda - \big(\sqrt{n-1} + (n-1)\big)\big) = 0$$

and the spectrum would be

$$\operatorname{Spec}_R^D(K_{1,n-1}) = \begin{pmatrix} \sqrt{n-1}+(n-1) & 0 & -\sqrt{n-1}+(n-1) \\ 1 & n-2 & 1 \end{pmatrix}.$$

Therefore, $RAE(K_{1,n-1}) = 2[\sqrt{n-1} + (n-1)]$. ▷

Theorem 3.3. *The Randic type additive connectivity energy of Crown graph S_n^0 is*

$$RAE(S_n^0) = 8(n-1)^{\frac{3}{2}}.$$

◁ Let S_n^0 be a crown graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The Randic type additive connectivity matrix is

$$RAE(S_n^0) = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 & 2\sqrt{n-1} & \ldots & 2\sqrt{n-1} \\ 0 & 0 & \ldots & 0 & 2\sqrt{n-1} & 0 & \ldots & 2\sqrt{n-1} \\ 0 & 0 & \ldots & 0 & 2\sqrt{n-1} & \ldots & 2\sqrt{n-1} & 2\sqrt{n-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & 2\sqrt{n-1} & \ldots & 2\sqrt{n-1} & 0 \\ 0 & 2\sqrt{n-1} & \ldots & 2\sqrt{n-1} & 1 & 0 & \ldots & 0 \\ 2\sqrt{n-1} & 0 & \ldots & 2\sqrt{n-1} & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2\sqrt{n-1} & 2\sqrt{n-1} & \ldots & 2\sqrt{n-1} & 0 & 0 & \ldots & 0 \\ 2\sqrt{n-1} & 2\sqrt{n-1} & \ldots & 0 & 0 & 0 & \ldots & 0 \end{bmatrix}.$$

Hence, the characteristic equation is

$$\left(\lambda + 2\sqrt{n-1}\right)^{n-1}\left(\lambda - 2\sqrt{n-1}\right)^{n-1}\left(\lambda - 2(n-1)^{\frac{3}{2}}\right)\left(\lambda + 2(n-1)^{\frac{3}{2}}\right) = 0$$

and spectrum is

$$\operatorname{Spec}_{RA}(S_n^0) = \begin{pmatrix} 2(n-1)^{\frac{3}{2}} & -2(n-1)^{\frac{3}{2}} & 2\sqrt{n-1} & -2\sqrt{n-1} \\ 1 & 1 & n-1 & n-1 \end{pmatrix}.$$

Therefore, $RAE(S_n^0) = 8(n-1)^{\frac{3}{2}}$. ▷

Theorem 3.4. *The Randic type additive connectivity energy of complete bipartite graph $K_{n,n}$of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$ is*

$$RAE(K_{m,n}) = 2(\sqrt{mn})(\sqrt{m} + \sqrt{n}).$$

$\lhd$ Let $K_{m,n}$ be the complete bipartite graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The Randic type additive connectivity matrix is

$$R^D(K_{m,n}) = \begin{bmatrix} 0 & 0 & 0 & \dots & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} \\ 0 & 0 & 0 & \dots & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} \\ 0 & 0 & 0 & \dots & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} \\ 0 & 0 & 0 & \dots & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \dots & 0 & 0 & 0 \\ \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \dots & 0 & 0 & 0 \\ \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \sqrt{m}+\sqrt{n} & \dots & 0 & 0 & 0 \end{bmatrix}.$$

Hence, the characteristic equation is

$$\lambda^{n-2}[\lambda - (\sqrt{mn})(\sqrt{m} + \sqrt{n})][\lambda + (\sqrt{mn})(\sqrt{m} + \sqrt{n})] = 0.$$

Hence, spectrum is

$$\mathrm{Spec}_{RA}(K_{m,n}) = \begin{pmatrix} (\sqrt{mn})(\sqrt{m}+\sqrt{n}) & 0 & -(\sqrt{mn})(\sqrt{m}+\sqrt{n})] \\ 1 & m+n-2 & 1 \end{pmatrix}.$$

Therefore, $RAE(K_{m,n}) = 2(\sqrt{mn})(\sqrt{m} + \sqrt{n})$. $\rhd$

Theorem 3.5. *The Randic type additive connectivity energy of Cocktail party graph $K_{n\times 2}$ is*

$$RAE(K_{n\times 2}) = \frac{4n-6}{n-1}.$$

$\lhd$ Let $K_{n\times 2}$ be a Cocktail party graph of order $2n$ with vertex set $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$. The Randic type additive connectivity matrix is

$$RA(K_{n\times 2}) = \begin{bmatrix} 0 & 2\sqrt{2n-2} & 2\sqrt{2n-2} & \dots & 0 & 2\sqrt{2n-2} & 2\sqrt{2n-2} & 2\sqrt{2n-2} \\ 2\sqrt{2n-2} & 0 & 2\sqrt{2n-2} & \dots & 2\sqrt{2n-2} & 0 & 2\sqrt{2n-2} & 2\sqrt{2n-2} \\ 2\sqrt{2n-2} & 2\sqrt{2n-2} & 0 & \dots & 2\sqrt{2n-2} & 2\sqrt{2n-2} & 0 & 2\sqrt{2n-2} \\ 2\sqrt{2n-2} & 2\sqrt{2n-2} & 2\sqrt{2n-2} & \dots & 2\sqrt{2n-2} & 2\sqrt{2n-2} & 2\sqrt{2n-2} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 2\sqrt{2n-2} & 2\sqrt{2n-2} & \dots & 0 & 2\sqrt{2n-2} & 2\sqrt{2n-2} & 2\sqrt{2n-2} \\ 2\sqrt{2n-2} & 0 & 2\sqrt{2n-2} & \dots & 2\sqrt{2n-2} & 0 & 2\sqrt{2n-2} & 2\sqrt{2n-2} \\ 2\sqrt{2n-2} & 2\sqrt{2n-2} & 0 & \dots & 2\sqrt{2n-2} & 2\sqrt{2n-2} & 0 & 2\sqrt{2n-2} \\ 2\sqrt{2n-2} & 2\sqrt{2n-2} & 2\sqrt{2n-2} & \dots & 2\sqrt{2n-2} & 2\sqrt{2n-2} & 2\sqrt{2n-2} & 0 \end{bmatrix}.$$

Hence, the characteristic equation is

$$\lambda^n\big(\lambda + 4\sqrt{2n-2}\big)^{n-1}\big(\lambda - 4(n-1)\sqrt{2n-2}\big) = 0$$

and the spectrum is

$$\mathrm{Spec}_{RA}(K_{n\times 2}) = \begin{pmatrix} 4(n-1)\sqrt{2n-2} & 0 & -4\sqrt{2n-2} \\ 1 & n & n-1 \end{pmatrix}.$$

Therefore, $RAE(K_{n\times 2}) = 8(n-1)\sqrt{2n-2}$. $\rhd$

4. Randic type additive connectivity energy of complements

DEFINITION 4.1 [5]. Let G be a graph and $P_k = \{V_1, V_2, \ldots, V_k\}$ be a partition of its vertex set V. Then the k-complement of G is denoted by $\overline{(G)}_k$ and obtained as follows: For all V_i and V_j in P_k, $i \neq j$, remove the edges between V_i and V_j and add the edges between the vertices of V_i and V_j which are not in G.

DEFINITION 4.2 [5]. Let G be a graph and $P_k = \{V_1, V_2, \ldots, V_k\}$ be a partition of its vertex set V. Then the $k(i)$-complement of G is denoted by $\overline{(G)}_{k(i)}$ and obtained as follows: For each set V_r in P_k, remove the edges of G joining the vertices within V_r and add the edges of $\overline{G}$ (complement of G) joining the vertices of V_r.

Here we investigate the relation between some special graph classes and their complements in terms of the Randic type additive connectivity energy.

Theorem 4.1. *The Randic type additive connectivity energy of the complement $\overline{K_n}$ of the complete graph K_n is*

$$RAE(\overline{K_n}) = 0.$$

$\lhd$ Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$. The Randic type additive connectivity matrix of the complement of the complete graph K_n is

$$RA(\overline{K_n}) = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix}.$$

Characteristic polynomial is

$$RA(\overline{K_n}) = \begin{vmatrix} \lambda & 0 & 0 & \ldots & 0 & 0 \\ 0 & \lambda & 0 & \ldots & 0 & 0 \\ 0 & 0 & \lambda & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & \lambda & 0 \\ 0 & 0 & 0 & \ldots & 0 & \lambda \end{vmatrix}.$$

Clearly, the characteristic equation is $\lambda^n = 0$ implying

$$RAE(\overline{K_n}) = 0. \rhd$$

Theorem 4.2. *The Randic type additive connectivity energy of the complement $\overline{K_{n\times 2}}$ of the cocktail party graph $K_{n\times 2}$ of order $2n$ is*

$$RAE\overline{(K_{n\times 2})} = 4n.$$

$\lhd$ Let $K_{n\times 2}$ be the cocktail party graph of order $2n$ having the vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The corresponding Randic type additive connectivity matrix is

$$RA(\overline{K_{n\times 2}}) = \begin{bmatrix} 0 & 0 & 0 & 0 & \ldots & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 2 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & \ldots & 0 & 0 & 0 & 0 \end{bmatrix}$$

Characteristic polynomial is

$$RA(\overline{K_{n\times 2}}) = \begin{vmatrix} \lambda & 0 & 0 & 0 & \ldots & -2 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & \ldots & 0 & -2 & 0 & 0 \\ 0 & 0 & \lambda & 0 & \ldots & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & \lambda & \ldots & 0 & 0 & 0 & -2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ -2 & 0 & 0 & 0 & \ldots & \lambda & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & \ldots & 0 & \lambda & 0 & 0 \\ 0 & 0 & -2 & 0 & \ldots & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & -2 & \ldots & 0 & 0 & 0 & \lambda \end{vmatrix}$$

and the characteristic equation becomes

$$(\lambda + 2)^n (\lambda - 2)^n = 0$$

implying that the spectrum would be

$$\mathrm{Spec}_{RA}\,(\overline{K_{n\times 2}}) = \begin{pmatrix} 2 & -2 \\ n & n \end{pmatrix}.$$

Therefore,

$$RAE(\overline{K_{n\times 2}}) = 4n. \triangleright$$

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References

1. *Gutman, I.* The Energy of a Graph, *Ber. Math. Stat. Sekt. Forschungsz. Graz*, 1978, vol. 103, pp. 1–22.
2. *Gutman, I.* The Energy of a Graph: Old and New Results, *Algebraic Combinatorics and its Applications* / eds. Betten, A., et al., Berlin, Springer-Verlag, 2001, pp. 196–211.
3. *Prakasha, K. N., Siva Kota Reddy, P. and Cangül, I. N.* Minimum Covering Randic Energy of a Graph, *Kyungpook Math. J.*, 2017, vol. 57, no. 4, pp. 701–709.
4. *Siva Kota Reddy, P., Prakasha, K. N. and Siddalingaswamy, V. M.* Minimum Dominating Randic Energy of a Graph, *Vladikavkaz. Mat. J.*, vol. 19, no. 1, pp. 28–35. DOI 10.23671/VNC.2017.2.6506.
5. *Sampathkumar, E., Pushpalatha, L., Venkatachalam, C. V. and Pradeep Bhat*, Generalized Complements of a Graph, *Indian J. Pure Appl. Math.*, 1998, vol. 29, no. 6, pp. 625–639.

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Krishnarajapete Venkatarama Madhusudhan
ATME College of Engineering,
Mysore 570 028, Karnataka, India,
Assistant Professor
E-mail: kvmadhu13@gmail.com;

Polaepalli Siva Kota Reddy
Siddaganga Institute of Technology,
B. H. Road, Tumkur 572 103, Karnataka, India,
Associate Professor
E-mail: reddy_math@yahoo.com, pskreddy@sit.ac.in;

Karpenahalli Ranganathappa Rajanna
Acharya Institute of Technology,
Bangalore 560 107, Karnataka, India,
Professor and Head of Mathematics
E-mail: rajanna@acharya.ac.in

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ЭНЕРГИЯ АДДИТИВНОЙ СВЯЗНОСТИ ТИПА РАНДИКА ГРАФА

К. В. Мадхусудхан[1], П. Сива Кота Редди[2], К. Р. Раджанна[3]

[1] Инженерный колледж, Майсур 570 028, Картанака, Индия;
[2] Сиддаганга технологический институт, Тумкур 572 103, Картанака, Индия;
[3] Технологический институт Ачарьи, Бангалор 560 107, Карнатака, Индия

E-mail: kvmadhu13@gmail.com; reddy_math@yahoo.com, pskreddy@sit.ac.in; rajanna@acharya.ac.in

Аннотация. Матрица аддитивной связности типа Рандика $RA(G) = (R_{ij})_{n\times m}$ задается равенствами $R_{ij} = \sqrt{d_i} + \sqrt{d_j}$, если вершины v_i и v_j смежны, $R_{ij} = 0$, в противном случае, где d_i и d_j — степени вершин v_i и v_j соответственно. Целью данной статьи является исследование энергии аддитивной связности типа Рандика. В данной статье мы получили новые неравенства, включающие энергию аддитивной связности типа Рандика, и представили ее верхнюю и нижнюю границы. Мы также получили результаты по энергии аддитивной связности типа Рандика обобщенных дополнений графа.

Ключевые слова: энергия аддитивной связности типа Рандика, собственные значения аддитивной связности типа Рандика.

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