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——————— ЗАМЕТКИ ———————

ON A QUESTION ON BANACH–STONE THEOREM

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We present a very simple and elementary proof of the main theorem of [l]. This also gives an answer to a conjecture in [1].

In this paper we use the standard terminology and notations of the Riesz spaces theory (see [2]). The Banach lattice of the continuous functions from a compact Hausdorff space into a Banach lattice E is denoted by $C(K, E)$. If $E = \mathbb{R}$ then we write $C(K)$ instead of $C(K, E)$. $\mathbf{1}$ stands for the unit function in $C(K)$.

One version of the Banach–Stone theorem states that:

Theorem 1. *Let X and Y be compact Hausdorff spaces. Then $C(X)$ and $C(Y)$ are Riesz isomorphic if and only if X and Y are homeomorphic.*

An elementary proof of this theorem can be found in [2]. This theorem is generalized in [l] as follows.

Theorem 2. *Let X and Y be compact Hausdorff spaces and E be a Banach lattice. If $\pi : C(X, E) \to C(Y)$ is a Riesz isomorphism such that $\pi(f)$ has no zeros whenever f has no zero, then X and Y are homeomorphic and E is Riesz isomorphic to $\mathbb{R}$.*

A quite difficult and long proof of the previous theorem is given without using Theorem 1 in [2] and it is conjectured that Theorem 2 follows from Theorem 1. In this paper we give an answer to this conjecture with an elementary proof as follows.

$\lhd$ Proof of Theorem 2. Clearly E is nonzero. Let $\in Y$ be fixed and $\pi_y : E \to \mathbb{R}$ be defined by $\pi_y() = \pi(1 \otimes e)(y)$, where $1 \otimes e(x) = e$. It is obvious that π_y is one-to-one and Riesz homomorphism. So, E is Riesz isomorphic onto a nonzero Riesz subspace of $\mathbb{R}$, As E is nonzero and dimension of $\mathbb{R}$ is one, E is Riesz isomorphic to $\mathbb{R}$. This complete the proof and answers to the conjecture in [1]. $\rhd$

References

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