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Implications of the Ganea Condition

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Abstract Suppose the spaces X and $X \times A$ have the same Lusternik-Schnirelmann category: $\mathrm{cat}(X \times A) = \mathrm{cat}(X)$. Then there is a *strict* inequality $\mathrm{cat}(X \times (A \rtimes B)) < \mathrm{cat}(X) + \mathrm{cat}(A \rtimes B)$ for every space B, provided the connectivity of A is large enough (depending only on X). This is applied to give a partial verification of a conjecture of Iwase on the category of products of spaces with spheres.

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Introduction

The product formula $\mathrm{cat}(X \times Y) \leq \mathrm{cat}(X) + \mathrm{cat}(Y)$ [1] is one of the most basic relations of Lusternik-Schnirelmann category. Taking $Y = S^r$, it implies that $\mathrm{cat}(X \times S^r) \leq \mathrm{cat}(X) + 1$ for any $r > 0$. In [5], Ganea asked whether the inequality can ever be strict in this special case. The study of the 'Ganea condition' $\mathrm{cat}(X \times S^r) = \mathrm{cat}(X) + 1$ has been, and remains, a formidable challenge to all techniques for the calculation of Lusternik-Schnirelmann category. In fact, it was only recently that techniques were developed which were powerful enough to identify a space which does not satisfy the Ganea condition [8] (see also [9, 12]). It is still not well understood exactly which spaces X do not satisfy the Ganea condition, although it has been conjectured that they are precisely those spaces for which $\mathrm{cat}(X)$ is not equal to the related invariant $\mathrm{Qcat}(X)$ (see [14, 17]).

Since the failure of the Ganea condition appears to be a strange property for a space to have, it is reasonable to expect that such failure would have useful and interesting implications. In this paper we explore some of the implications of the equation $\mathrm{cat}(X \times A) = \mathrm{cat}(X)$ for general spaces A, and for $A = S^r$ in particular.

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A brief look at the method of the paper [8] will help to put our results into proper perspective. The new techniques begin with the following question: if $Y = X \cup_f e^{t+1}$, the cone on $f : S^t \to X$, then how can we tell if $\mathrm{cat}(Y) > \mathrm{cat}(X)$? It is shown (see [9, Thm. 5.2] and [12, Thm. 3.6]) that, if $t \geq \dim(X)$, then $\mathrm{cat}(Y) = \mathrm{cat}(X) + 1$ if and only if a certain Hopf invariant $\mathcal{H}_s(f)$ (which is a set of homotopy classes) does not contain the trivial map $*$. It is also shown [9, Thm. 3.8] that if $* \in \Sigma^r \mathcal{H}_s(f)$, then $\mathrm{cat}(Y \times S^r) \leq \mathrm{cat}(X) + 1$. Thus Y does not satisfy Ganea's condition if $* \notin \mathcal{H}_s(f)$, but there is at least one $h \in \mathcal{H}_s(f)$ such that $\Sigma^r h \simeq *$.

Of course, if $\Sigma^r h \simeq *$, then $\Sigma^{r+1} h \simeq *$ as well, and this suggests the following conjecture (formulated in [8, Conj. 1.4]):

Conjecture *If* $\mathrm{cat}(X \times S^r) = \mathrm{cat}(X)$, *then* $\mathrm{cat}(X \times S^{r+1}) = \mathrm{cat}(X)$.

In this paper we prove that this conjecture is true, provided r is large enough.

Theorem 1 *Suppose X is a $(c-1)$-connected space and let $r > \dim(X) - c \cdot \mathrm{cat}(X) + 2$. If $\mathrm{cat}(X \times S^r) = \mathrm{cat}(X)$, then*

$$\mathrm{cat}(X \times S^t) = \mathrm{cat}(X)$$

for all $t \geq r$.

The conjecture remains open for small values of r.

Our main result is much more general: it shows how the equation $\mathrm{cat}(X \times A) = \mathrm{cat}(X)$ governs the Lusternik-Schnirelmann category of products of X with a vast collection of other spaces.

Theorem 2 *Let X be a $(c-1)$-connected space and let A be $(r-1)$-connected with $r > \dim(X) - c \cdot \mathrm{cat}(X) + 2$. If $\mathrm{cat}(X \times A) = \mathrm{cat}(X)$ then*

$$\mathrm{cat}(X \times (A \rtimes B)) < \mathrm{cat}(X) + \mathrm{cat}(A \rtimes B)$$

for every space B.

Here $A \rtimes B = (A \times B)/B$ is the half-smash product of A with B. When A is a suspension, the half-smash product decomposes as $A \rtimes B \simeq A \vee (A \wedge B)$ (see, for example, [12, Lem. 5.9]), so we obtain the following.

Corollary *Under the conditions of Theorem 2, if A is a suspension, then*

$$\mathrm{cat}(X \times (A \wedge B)) = \mathrm{cat}(X)$$

for every space B.

Our partial verification of the conjecture is an immediate consequence of this corollary: it the special case $A = S^r$ and $B = S^{t-r}$.

Organization of the paper In Section 1 we recall the necessary background information on homotopy pushouts, cone length and Lusternik-Schnirelmann category. We introduce an auxiliary space and establish its important properties in Section 2. The proof of Theorem 2 is presented in Section 3.

1 Preliminaries

In this paper all spaces are based and have the pointed homotopy type of CW complexes; maps and homotopies are also pointed. We denote by $*$ the one point space and any nullhomotopic map. Much of our exposition uses the language of homotopy pushouts; we refer to [11] for the definitions and basic properties.

1.1 Homotopy Pushouts

We begin by recalling some basic facts about homotopy pushout squares. We call a sequence $A \to B \to C$ a *cofiber sequence* if the associated square

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ \downarrow & & \downarrow \\ * & \longrightarrow & C \end{array}$$

is a homotopy pushout square. The space C is called the *cofiber* of the map f. One special case that we use frequently is the *half-smash product* $A \rtimes B$, which is the cofiber of the inclusion $B \to A \times B$.

Finally, we recall the following result on products and homotopy pushouts.

Proposition 3 *Let X be any space. Consider the squares*

$$\begin{array}{ccc} A & \longrightarrow & B \\ \downarrow & & \downarrow \\ C & \longrightarrow & D \end{array} \qquad \text{and} \qquad \begin{array}{ccc} X \times A & \longrightarrow & X \times B \\ \downarrow & & \downarrow \\ X \times C & \longrightarrow & X \times D. \end{array}$$

If the first square is a homotopy pushout, then so is the second.

Proof This follows from Theorem 6.2 in [15]. □

1.2 Cone Length and Category

A *cone decomposition* of a space Y is a diagram of the form

$$\begin{array}{ccccccccc} L_0 & & L_1 & & & & L_{k-1} & & \\ \downarrow & & \downarrow & & & & \downarrow & & \\ Y_0 & \longrightarrow & Y_1 & \longrightarrow & \cdots & \longrightarrow & Y_{k-1} & \longrightarrow & Y_k \end{array}$$

in which $Y_0 = *$, each sequence $L_i \to Y_i \to Y_{i+1}$ is a cofiber sequence, and $Y_k \simeq Y$; the displayed cone decomposition has *length* k. The *cone length* of Y, denoted $\mathrm{cl}(Y)$, is defined by

$$\mathrm{cl}(Y) = \begin{cases} 0 & \text{if } Y \simeq * \\ \infty & \text{if } Y \text{ has no cone decomposition, and} \\ k & \text{if the shortest cone decomposition of } Y \text{ has length } k. \end{cases}$$

The Lusternik-Schnirelmann category of X may be defined in terms of the cone length of X by the formula

$$\mathrm{cat}(X) = \inf\{\mathrm{cl}(Y) \,|\, X \text{ is a homotopy retract of Y}\}.$$

Berstein and Ganea proved this formula in [3, Prop. 1.7] with $\mathrm{cl}(Y)$ replaced by the strong category of Y; the formula above follows from another result of Ganea — strong category is equal to cone length [7]. It follows directly from this definition that if X is a homotopy retract of Y, then $\mathrm{cat}(X) \leq \mathrm{cat}(Y)$. The reader may refer to [10] for a survey of Lusternik-Schnirelmann category.

The category of X can be defined in another way that is essential to our work. Begin by defining the 0^{th} Ganea fibration sequence $F_0(X) \longrightarrow G_0(X) \xrightarrow{p_0} X$ to be the familiar path-loop fibration sequence $\Omega(X) \longrightarrow \mathcal{P}(X) \longrightarrow X$. Given the n^{th} Ganea fibration sequence

$$F_n(X) \longrightarrow G_n(X) \xrightarrow{p_n} X,$$

let $\overline{G}_{n+1}(X) = G_n(X) \cup CF_n(X)$ be the cofiber of p_n and define $\overline{p}_{n+1} : \overline{G}_{n+1}(X) \to X$ by sending the cone to the base point of X. The $(n+1)^{\text{st}}$ Ganea fibration $p_{n+1} : G_{n+1}(X) \to X$ results from converting the map $\overline{p}_{n+1}$ to a fibration. The following result is due to Ganea (cf. Svarc).

Theorem 4 *For any space X,*

(a) $\mathrm{cl}(G_n(X)) \leq n$,

(b) *the map $p_n : G_n(X) \to X$ has a section if and only if* $\mathrm{cat}(X) \leq n$, *and*

(c) $F_n(X) \simeq (\Omega(X))^{*(n+1)}$, *the* $(n+1)$*-fold join of* ΩX *with itself.*

Proof Assertion (a) follows immediately from the construction. For parts (b) and (c), see [6]; these results also appear, from a different point of view, in [16]. □

2 An Auxilliary Space

Let $\widetilde{G}_n$ denote the homotopy pushout in the square

$$\begin{array}{ccc} G_{n-1}(X) & \xhookrightarrow{i_1} & G_{n-1}(X) \times A \\ \downarrow & & \downarrow \\ G_n(X) & \longrightarrow & \widetilde{G}_n. \end{array}$$

The maps $p_n : G_n(X) \to X$ and $1_A : A \to A$ piece together to give a map $\widetilde{p}_n : \widetilde{G}_n \to X \times A$. The space $\widetilde{G}_n$ and the map $\widetilde{p}_n$ play key roles in the forthcoming constructions; this section is devoted to establishing some of their properties.

2.1 Category Properties of $\widetilde{G}_n$

We begin by estimating the category of $\widetilde{G}_n$.

Proposition 5 *For any noncontractible* A *and* $n > 0$, $\mathrm{cat}(\widetilde{G}_n) < n + \mathrm{cat}(A)$.

Proof For simplicity in this proof, we write F_i for $F_i(X)$ and G_i for $G_i(X)$. Let $\mathrm{cat}(A) = k$, so A is a retract of another space A' with $\mathrm{cl}(A') = k$. Let $\widetilde{G}'_n = G_n \cup G_{n-1} \times A'$; clearly $\widetilde{G}_n$ is a homotopy retract of $\widetilde{G}'_n$ and so it suffices to show that $\mathrm{cl}(\widetilde{G}'_n) < n + k$. Let

$$\begin{array}{ccccccccc} L_0 & & L_1 & & & & L_{k-1} & & \\ \downarrow & & \downarrow & & & & \downarrow & & \\ A'_0 & \longrightarrow & A'_1 & \longrightarrow & \cdots & \longrightarrow & A'_{k-1} & \longrightarrow & A'_k \end{array}$$

be a cone decomposition of A'. We will also use the cone decomposition of G_n given by the cofiber sequences $F_{i-1} \to G_{i-1} \to G_i$. According to a result of Baues [2] (see also [13, Prop. 2.9]), for each i and j there is a cofiber sequence

$$F_{i-1} * L_{j-1} \longrightarrow G_i \times A'_{j-1} \cup G_{i-1} \times A'_j \longrightarrow G_i \times A'_j.$$

Now define subspaces $W_s \subseteq \widetilde{G}'_n$ by the formula

$$W_s = \begin{cases} \bigcup_{i+j=s} G_i \times A'_j & \text{if } s \leq n \\ G_n \times A'_0 \cup \left(\bigcup_{\substack{i+j=s \\ i<n}} G_i \times A'_j \right) & \text{if } s > n \end{cases}$$

with the understanding that $A'_j = A'_k$ for all $j \geq k$. The cofiber sequences guaranteed by Baues' theorem can be pieced together with the given cone decompositions of A' and G_n to give the cofiber sequences

$$F_s \vee L_s \vee \left(\bigvee_{\substack{i+j=s-1 \\ i<n-1}} F_i * L_j \right) \longrightarrow W_s \longrightarrow W_{s+1}$$

for each $s < \min\{n, k\}$; when $s \geq n$ we alter the cobase of the cofiber sequence by removing the F_s summand, and when $s \geq k$ we must remove the summand L_s. Since $\widetilde{G}'_n = W_{n+k-1}$, we have the result. □

Next, we show that the map $\widetilde{p}_n : \widetilde{G}_n \to X \times A$ has one of the category-detecting properties of $p_n : G_n(X \times A) \to X \times A$.

Proposition 6 *If* $\mathrm{cat}(X \times A) = \mathrm{cat}(X) = n$, *then* $\widetilde{p}_n$ *has a homotopy section.*

Proof We follow [4] (see also [8, Thm. 2.7]) and define

$$\widehat{G}'_n(X \times A) = \bigcup_{i+j=n} G_i(X) \times G_j(A).$$

There is a natural map $h : \widehat{G}'_n(X \times A) \to X \times A$ induced by the Ganea fibrations over X and A. According to [4, Thm. 2.3], $\mathrm{cat}(X \times A) = n$ if and only if h has a homotopy section.

Each map $G_i(X) \times G_j(A) \to X \times A$ (with $j > 0$) factors through $G_i(X) \times A$ and these factorizations are compatible because p_{i+1} extends p_i. So h factors as $\widehat{G}'_n(X \times A) \to \widetilde{G}_n \to X \times A$. Therefore, if $\mathrm{cat}(X \times A) = n$, then h, and hence $\widetilde{p}_n$, has a section. □

2.2 Comparison of $\widetilde{G}_n$ with $G_n(X) \times A$

Let $j : \widetilde{G}_n \to G_n(X) \times A$ denote the natural inclusion map.

Proposition 7 *Assume that* X *is* $(c-1)$*-connected and that* A *is* $(r-1)$*-connected. Then the homotopy fiber* F *of the map* j *is* $(nc+r-2)$*-connected.*

Proof There is a cofiber sequence

$$\widetilde{G}_n \xrightarrow{j} G_n(X) \times A \longrightarrow \Sigma F_{n-1}(X) \wedge A.$$

Therefore the homotopy fiber of j has the same connectivity as the space $\Omega(\Sigma F_{n-1}(X) \wedge A) \simeq \Omega(\Omega(X)^{*n} * A)$, namely $nc + r - 2$. $\square$

Corollary 8 *Assume* $\dim(Z) < nc+r-2$ *and let* $f, g : Z \to \widetilde{G}_n$. *Then* $f \simeq g$ *if and only if* $jf \simeq jg$.

The proof is standard, and we omit it.

2.3 New Sections from Old Ones

Suppose that $\mathrm{cat}(X) = \mathrm{cat}(X \times A) = n$. By Proposition 6 there is a section $\sigma : X \times A \to \widetilde{G}_n$ of the map $\widetilde{p}_n : \widetilde{G}_n \to X \times A$. Define a new map $\sigma' : X \to G_n(X)$ by the diagram

$$\begin{array}{ccccc}
X & & \xrightarrow{\quad\sigma'\quad} & & G_n(X) \\
\big\downarrow{\scriptstyle i_1} & & & & \big\uparrow{\scriptstyle \mathrm{pr}_1} \\
X \times A & \xrightarrow{\ \sigma\ } & \widetilde{G}_n & \stackrel{j}{\hookrightarrow} & G_n(X) \times A.
\end{array}$$

We need the following basic properties of σ'.

Proposition 9 *If* $\mathrm{cat}(X \times A) = \mathrm{cat}(X) = n$, *then*

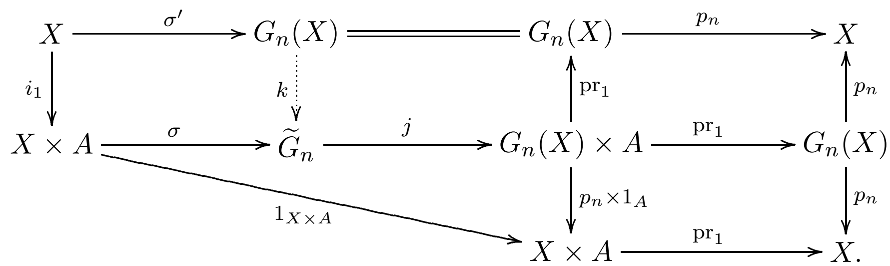
(a) σ' *is a homotopy section of the projection* $p_n : G_n(X) \to X$, *and*

(b) *if* X *is* $(c-1)$*-connected and* A *is* $(r-1)$*-connected with* $r > \dim(X) - nc + 2$, *then the diagram*

$$\begin{array}{ccc}
X & \xrightarrow{\ \sigma'\ } & G_n(X) \\
\big\downarrow{\scriptstyle i_1} & & \big\downarrow{\scriptstyle k} \\
X \times A & \xrightarrow{\ \sigma\ } & \widetilde{G}_n
\end{array}$$

commutes up to homotopy.

Proof First consider the diagram



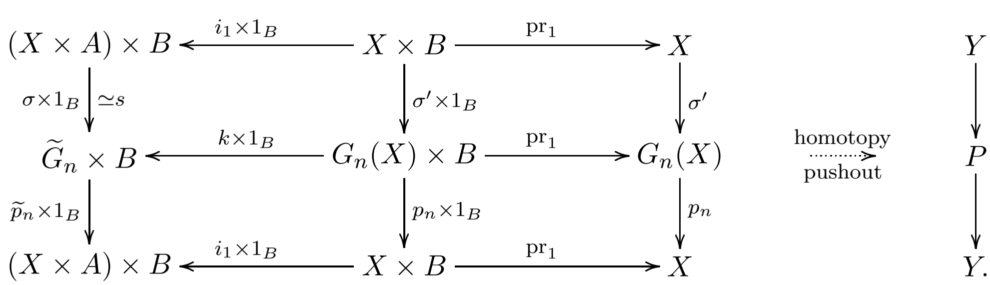
The diagram of solid arrows is evidently commutative. Therefore, we have $p_n \circ \sigma' \simeq \mathrm{pr}_1 \circ 1_{X\times A} \circ i_1 \simeq 1_X$, proving (a).

To prove (b) we have to show that two maps $X \to \widetilde{G}_n$ are homotopic. Since $\dim(X) < nc + r - 2$, it suffices by Corollary 8 to show that $j \circ (\sigma \circ i_1) \simeq j \circ (k \circ \sigma')$. Since $\mathrm{pr}_2 \circ j \circ (\sigma \circ i_1) \simeq * \simeq \mathrm{pr}_2 \circ j \circ (k \circ \sigma')$, it remains to show that $\mathrm{pr}_1 \circ j \circ (\sigma \circ i_1) \simeq \mathrm{pr}_1 \circ j \circ (k \circ \sigma')$. But both of these maps are homotopic to σ'. □

3 Proof of the Main Theorem

Proof of Theorem 2 We have $n = \mathrm{cat}(X) = \mathrm{cat}(X \times A)$ by hypothesis. It follows from Proposition 6 that there is a section $\sigma : X \times A \to \widetilde{G}_n$ of the map $\widetilde{p}_n : \widetilde{G}_n \to X \times A$. We then get the section $\sigma' : X \to G_n(X)$ that was constructed and studied in Section 2.3.

Consider the following diagram and the induced sequence of maps on the homotopy pushouts of the rows



Proposition 9 implies that the upper left square commutes up to homotopy. Since $i_1 \times 1_B$ is a cofibration, we can apply homotopy extension and replace the map $\sigma \times 1_B : (X \times A) \times B \to \widetilde{G}_n \times B$ with a homotopic map s which makes that square strictly commute. All other squares are strictly commutative as they stand.

Since the composites $(\widetilde{p}_n \times 1_B)\circ(\sigma' \times 1_B)$ and $p_n \circ \sigma'$ are the identity maps and $(\widetilde{p}_n \times 1_B)\circ s$ is a homotopy equivalence, each vertical composite in the modified diagram is a homotopy equivalence. Thus Y is a homotopy retract of P, and consequently $\mathrm{cat}(Y) \leq \mathrm{cat}(P)$.

The space Y is the homotopy pushout of the top row in the diagram, which is the product of the homotopy pushout diagram

$$\begin{array}{ccc} B & \longrightarrow & * \\ \downarrow & & \downarrow \\ A \times B & \longrightarrow & A \rtimes B \end{array}$$

with the space X. Therefore $Y \simeq X \times (A \rtimes B)$ by Proposition 3. Since Y is a homotopy retract of P, it follows that

$$\mathrm{cat}(X \times (A \rtimes B)) \leq \mathrm{cat}(P),$$

the proof will be complete once we establish that $\mathrm{cat}(P) < \mathrm{cat}(X) + \mathrm{cat}(A \rtimes B)$. This is accomplished in Lemma 10, which is proved below. ◻

Lemma 10 *The space P constructed in the proof of Theorem 2 satisfies* $\mathrm{cat}(P) \leq \mathrm{cl}(P) < \mathrm{cat}(X) + \mathrm{cat}(A \rtimes B)$.

Proof The space $\widetilde{G}_n$ is defined by the homotopy pushout square

$$\begin{array}{ccc} G_{n-1}(X) & \longrightarrow & G_n(X) \\ \downarrow & & \downarrow \\ G_{n-1}(X) \times A & \longrightarrow & \widetilde{G}_n. \end{array}$$

Take the product of this square with the space B and adjoin the homotopy pushout square that defines P to obtain the diagram

$$\begin{array}{ccccc} G_{n-1}(X) \times B & \longrightarrow & G_n(X) \times B & \longrightarrow & G_n(X) \\ \downarrow & & \downarrow & & \downarrow \\ G_{n-1}(X) \times A \times B & \longrightarrow & \widetilde{G}_n \times B & \longrightarrow & P. \end{array}$$

By [11, Lem. 13], the outer square

$$\begin{array}{ccc} G_{n-1}(X) \times B & \longrightarrow & G_n(X) \\ \downarrow & & \downarrow \\ G_{n-1}(X) \times A \times B & \longrightarrow & P \end{array}$$

is also a homotopy pushout square. The top map is the composite

$$G_{n-1}(X) \times B \xrightarrow{\mathrm{pr}_1} G_{n-1}(X) \hookrightarrow G_n(X),$$

and so we have a new factorization into homotopy pushout squares:

$$\begin{array}{ccccc} G_{n-1}(X) \times B & \xrightarrow{\mathrm{pr}_1} & G_{n-1}(X) & \longrightarrow & G_n(X) \\ \downarrow & & \downarrow & & \downarrow \\ G_{n-1}(X) \times A \times B & \longrightarrow & L & \longrightarrow & P. \end{array}$$

To identify the space L, observe that the left square is simply the product of the space $G_{n-1}(X)$ with the homotopy pushout square

$$\begin{array}{ccc} B & \longrightarrow & * \\ \downarrow & & \downarrow \\ A \times B & \longrightarrow & A \rtimes B. \end{array}$$

By Proposition 3, $L \simeq G_{n-1}(X) \times (A \rtimes B)$. Hence the right-hand square is the homotopy pushout square

$$\begin{array}{ccc} G_{n-1}(X) & \longrightarrow & G_n(X) \\ \downarrow & & \downarrow \\ G_{n-1}(X) \times (A \rtimes B) & \longrightarrow & P. \end{array}$$

Therefore $\mathrm{cl}(P) \leq \mathrm{cat}(X) + \mathrm{cat}(A \rtimes B)$ by Proposition 5. □

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