# SOME APPLICATIONS OF GRAPH THEORY

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A brief survey of basic concepts and results in graph theory is presented with applications to various branches of mathematics and related fields. The main attention is focused on realizations of graphs in the form of objects with additional geometrical, topological or metrical structure (cf. [1]; see also [2], [3] and [4]).

The notion of combinatorial symmetry is introduced and examined for planar graphs and mosaics of different types. In particular, some asymmetric (in the geometrical sense) mosaics are considered and it is shown that they possess the combinatorial symmetry.

An application of graph theory to mathematical crystallography is mentioned especially.

For every natural number $i \geq 3$ and for every convex (more generally, simple) polyhedron $P$ in the 3-space, denote by $F_i(P)$ the family of all those faces of $P$ which have exactly $i$ sides (equivalently, $i$ vertices). The following statement is valid.

**Theorem.** *For any natural number $n$, there exists a natural number $m = m(n)$ possessing the following property: if a simple polyhedron $P$ in the 3-space has at least $m$ faces (vertices), then there is a natural number $i \in \{3, 4, 5, 6\}$ such that there are at least $n$ faces of $P$ which all belong to $F_i(P)$ and no two of them are neighbouring (i.e. no two of them have a common side).*

The proof of this statement is essentially based on the classical Ramsey theorem [5] and on the well-known fact that the Kuratowski graph $K_5$ (i.e. the complete graph with 5 vertices) is not planar.

The problem can be also generalized to the three-dimensional polyhedra with a given Euler characteristics.

Some other interconnections between graph theory and the theory of three-dimensional convex polyhedra are discussed.

## References

[1] Harary F., Graph Theory. Izdat. Mir, Moscow, 1973, (in Russian; translated from the English).

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[4] Kharazishvili A.B., $\varepsilon$-shifts and realization of graphs. Bull. Acad. Sci. GSSR, vol. 131, no. 2, 1988 (in Russian).

[5] Ramsey F., On a problem of formal logic. Proc. London Math. Soc., vol. 30, 1930.