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Research of the Stress State of Flexible Multilayered Corrugated Cylindrical Shells of Rotation According to a Refined Theory

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The problem of deformation of flexible layered corrugated cylindrical shells of rotation with orthotropic layers with constant thickness is considered. The study of the regularities of the shell deformation process is carried out on the basis of a refined nonlinear theory, which takes into account the nonhomogeneity of the lateral-shear deformations.

The obtained results are compared with the results obtained in accordance with the linear theory. Graphs and tables of solutions of the problem are given.

Keywords: Corrugated cylindrical shells, orthotropic layers, lateral-shear deformation.

AMS Subject Classification: 74B05.

1. Introduction

In the branches of modern technology, structures are used in the form of flexible shells of rotation in various configurations, assembled from several layers, which are under conditions of force and temperature effects.

For shells assembled from several layers, the mechanical characteristics of which differ significantly, it is essential to take into account the lateral-shear deformation. In some works [1, 2, 6, 8, 11] devoted to the theory and methods of calculation of layered shells, the effects due to the inhomogeneity of the layer material are taken into account to a certain extent. Accounting for lateral-shear deformations in each layer leads to the complication of the mathematical model and computational implementation [1, 2, 8].

One of the approaches in which the problem is simplified consider that each layer has of a local rotation angle due to lateral shear. This assumption makes it possible to express the displacements and rotation angles of whole layer package in terms of the parameters of one of layer and to derive equations whose order does not depend on the number of layers. Such an approach in the case of a linear problem was proposed in [5]. For a geometrically nonlinear problem of deformation of layered shells of revolution with layers of constant thickness, this approach was generalized in [3].

In this article, this approach is extended to a class of problems on the stress-strain state of flexible layered corrugated cylindrical shells of revolution with orthotropic layers with constant thickness.

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2. Basic equations

Let's write the main relations and equations of the theory of layered orthotropic shells in a geometrically nonlinear formulation with a quadratic approximation, taking into account the inhomogeneity of transverse shear deformations over layers [3, 5, 6].

In particular, for the tangential displacements have the following form

$$\begin{aligned} u_\alpha^{(i)} &= u + a_1^{(i)}\gamma_\alpha^{(0)} + \gamma(\psi_\alpha - a_2^{(i)}\gamma_\alpha^{(0)}),\\ u_\beta^{(i)} &= v + b_1^{(i)}\gamma_\beta^{(0)} + \gamma(\psi_\beta - b_2^{(i)}\gamma_\beta^{(0)}), \end{aligned} \tag{1}$$

where u and v are the tangential displacement of coordinate surface, ψ_α, ψ_β are the complete angles of rotation of the normal, $\gamma_\alpha^{(0)}$ and $\gamma_\beta^{(0)}$ are the lateral shears in the layer containing the coordinate surface, α and β are the orthogonal coordinates on the select surface. The formulas for calculation of coefficients $a_1^{(i)}$, $a_2^{(i)}$, $b_1^{(i)}$, $b_2^{(i)}$ are given [4, 6]. taking (1) into account, we represent the components of deformation as

$$\begin{aligned} &\varepsilon_{\alpha\alpha}^{(\gamma)} = \varepsilon_{\alpha\alpha}^{(i)} + \gamma\varkappa_{\alpha\alpha}^{(i)}, \quad &&\varepsilon_{\alpha\beta}^{(\gamma)} = \varepsilon_{\alpha\beta}^{(i)} + 2\gamma\varkappa_{\alpha\beta}^{(i)},\\ &\varepsilon_{\beta\beta}^{(\gamma)} = \varepsilon_{\beta\beta}^{(i)} + \gamma\varkappa_{\beta\beta}^{(i)}, \quad &&\varepsilon_{\alpha\gamma}^{(\gamma)} = \gamma_\alpha^{(i)},\\ &\varepsilon_{\gamma\gamma}^{(\gamma)} = 0, &&\varepsilon_{\beta\gamma}^{(\gamma)} = \gamma_\beta^{(i)}. \end{aligned} \tag{2}$$

In expressions (2) the components $\varepsilon_{\alpha\alpha}^{(i)}$, $\varepsilon_{\beta\beta}^{(i)}$,..., $\varkappa_{\beta\beta}^{(i)}$ are the same as in the work [4]. The quantities which characterizing the deformation of the coordinate surface of the shell are represented by the following form:

$$\begin{aligned} &\varepsilon_{\alpha\alpha} = \varepsilon_\alpha + \frac{1}{2}\theta_\alpha^2, \quad \varepsilon_{\beta\beta} = \varepsilon_\beta + \frac{1}{2}\theta_\beta^2, \quad \varepsilon_{\alpha\beta}^* = \varepsilon_{\alpha\beta} + \theta_\alpha\theta_\beta,\\ &\theta_\alpha = -\frac{1}{A}\frac{\partial\omega}{\partial\alpha} + k_1 u, \quad \theta_\beta = -\frac{1}{B}\frac{\partial\omega}{\partial\beta} + k_2 v,\\ &\gamma_\alpha^{(0)} = \psi_\alpha - \theta_\alpha, \quad \gamma_\beta^{(0)} = \psi_\beta - \theta_\beta, \end{aligned} \tag{3}$$

where ε_α, $\varepsilon_{\alpha\beta}$ and ε_β are given in [4, 6]

Having expressions (2) and applying the generalized Hooke's law, we have obtained the following expressions for elasticity relations:

$$\begin{aligned} N_\alpha &= C_{11}\varepsilon_{\alpha\alpha} + C_{12}\varepsilon_{\beta\beta} + K_{11}\varkappa_\alpha + K_{12}\varkappa_\beta + A_{11}\frac{\partial\gamma_\alpha^{(0)}}{\partial\alpha}\\ &\quad + A_{12}\gamma_\alpha^{(0)} + B_{11}\frac{\partial\gamma_\beta^{(0)}}{\partial\beta} + B_{12}\gamma_\beta^{(0)},\\ N_\beta &= C_{12}\varepsilon_{\alpha\alpha} + C_{22}\varepsilon_{\beta\beta} + K_{12}\varkappa_\alpha + K_{22}\varkappa_\beta + A_{21}\frac{\partial\gamma_\alpha^{(0)}}{\partial\alpha}\\ &\quad + A_{22}\gamma_\alpha^{(0)} + B_{21}\frac{\partial\gamma_\beta^{(0)}}{\partial\beta} + B_{22}\gamma_\beta^{(0)}, \end{aligned}$$

$$N_{\alpha\beta} = C_{66}\varepsilon^*_{\alpha\beta} + 2K_{66}\varepsilon_{\alpha\beta} + k_2(K_{66}\varkappa^*_{\alpha\alpha} + 2D_{66}\varkappa_{\alpha\beta}) + (A_{16} + k_2E_{16})\frac{\partial\gamma^{(0)}_\alpha}{\partial\beta}$$
$$+(A_{26} + k_2E_{26})\gamma^{(0)}_\alpha + (B_{16} + k_2F_{16})\frac{\partial\gamma^{(0)}_\beta}{\partial\alpha} + (B_{26} + k_2F_{26})\gamma^{(0)}_\beta,$$
$$N_{\beta\alpha} = C_{66}\varepsilon^*_{\alpha\beta} + 2K_{66}\varepsilon_{\alpha\beta} + k_1(K_{66}\varkappa^*_{\alpha\alpha} + 2D_{66}\varkappa_{\alpha\beta}) + (A_{16} + k_1E_{16})\frac{\partial\gamma^{(0)}_\alpha}{\partial\beta}$$
$$+(A_{26} + k_1E_{26})\gamma^{(0)}_\alpha + (B_{16} + k_1F_{16})\frac{\partial\gamma^{(0)}_\beta}{\partial\alpha} + (B_{26} + k_1F_{26})\gamma^{(0)}_\beta,$$
$$M_\alpha = K_{11}\varepsilon_{\alpha\alpha} + K_{12}\varepsilon_{\beta\beta} + D_{11}\varkappa_\alpha + D_{12}\varkappa_\beta + E_{11}\frac{\partial\gamma^{(0)}_\alpha}{\partial\alpha} + E_{12}\gamma^{(0)}_\alpha$$
$$+F_{11}\frac{\partial\gamma^{(0)}_\beta}{\partial\beta} + F_{12}\gamma^{(0)}_\beta, \qquad (4)$$
$$M_\beta = K_{12}\varepsilon_{\alpha\alpha} + K_{22}\varepsilon_{\beta\beta} + D_{12}\varkappa_\alpha + D_{22}\varkappa_\beta + E_{21}\frac{\partial\gamma^{(0)}_\alpha}{\partial\alpha} + E_{22}\gamma^{(0)}_\alpha$$
$$+F_{21}\frac{\partial\gamma^{(0)}_\beta}{\partial\beta} + F_{22}\gamma^{(0)}_\beta,$$
$$M_{\alpha\beta} = M_{\beta\alpha} = K_{66}\varepsilon^*_{\alpha\beta} + 2D_{66}\varkappa_{\alpha\beta} + E_{16}\frac{\partial\gamma^{(0)}_\alpha}{\partial\beta} + E_{26}\gamma^{(0)}_\alpha + F_{16}\frac{\partial\gamma^{(0)}_\alpha}{\partial\alpha} + F_{26}\gamma^{(0)}_\beta,$$
$$Q_\alpha = K_1\gamma^{(0)}_\alpha, \quad Q_\beta = K_2\gamma^{(0)}_\beta,$$

where N_α, N_β, $N_{\alpha\beta}$ and $N_{\beta\alpha}$ are the tangential forces, Q_α and Q_β are the cutting forces, M_α, M_β are the bending moments, $M_{\alpha\beta}$ and $M_{\beta\alpha}$ are the torque moments, C_{ij}, K_{ij}, D_{ij}, K_1, K_2 are the rigidity characteristics determined through the elastic parameters of the layers and their thickness, A_{11}, A_{12}, ..., F_{16}, F_{26} are quantities depending on the geometrical and mechanical parameters of the layers of the shells, k_1 and k_2 are the curvatures [4, 6].

The equilibrium equation of the shell has the form

$$\begin{gathered}
\frac{\partial BN_\alpha}{\partial\alpha} + \frac{\partial AN_{\beta\alpha}}{\partial\beta} + \frac{\partial A}{\partial\beta}N_{\alpha\beta} - \frac{\partial B}{\partial\alpha}N_\beta + ABk_1Q^*_\alpha + ABq_1 = 0,\\
\frac{\partial AN_\beta}{\partial\beta} + \frac{\partial BN_{\alpha\beta}}{\partial\alpha} + \frac{\partial B}{\partial\alpha}N_{\beta\alpha} - \frac{\partial A}{\partial\beta}N_\alpha + ABk_2Q^*_\beta + ABq_2 = 0,\\
\frac{\partial BQ^*_\alpha}{\partial\alpha} + \frac{\partial AQ^*_\beta}{\partial\beta} - ABk_1N_\alpha - ABk_2N_\beta + ABq_3 = 0,\\
\frac{\partial BM_\alpha}{\partial\alpha} + \frac{\partial AM_{\beta\alpha}}{\partial\beta} + \frac{\partial A}{\partial\beta}M_{\alpha\beta} - \frac{\partial B}{\partial\alpha}M_\beta - ABQ_\alpha = 0,\\
\frac{\partial AM_\beta}{\partial\beta} + \frac{\partial AM_{\alpha\beta}}{\partial\alpha} + \frac{\partial B}{\partial\beta}M_{\beta\alpha} - \frac{\partial A}{\partial\beta}M_\alpha - ABQ_\beta = 0,
\end{gathered} \qquad (5)$$

where

$$\begin{gathered}
Q^*_\alpha = Q_\alpha - (N_\alpha + k_1M_\alpha)\theta_\alpha - (N_{\alpha\beta} + k_1M_{\alpha\beta})\theta_\beta,\\
Q^*_\beta = Q_\beta - (N_{\beta\alpha} + k_2M_{\beta\alpha})\theta_\alpha - (N_\beta + k_2M_\beta)\theta_\beta.
\end{gathered} \qquad (6)$$

In equations (5), the quantities q_1, q_2, q_3 are projections of the surface load in the direction of the coordinate lines α, β, γ, respectively.

3. Statement and Solution of the Problem

we dwell on problems on the stress-strain state of the layered shells of rotation. Assuming that $\alpha = s$ is the arc length of the meridian, $\beta = \theta$ is the central angle in the parallel circle. From general equations (2)-(6) for the case of axisymmetric deformation of layered shells of rotation, we obtain the resolving system of differential equations in the form

$$\frac{d\overline{Y}}{ds} = A^*(s)\overline{Y} + \overline{F}(s,\overline{Y}) + \overline{f}_{(s)}, \quad \overline{Y} = \{N_s,\ Q_s^*,\ M_s,\ u,\ \omega,\ \psi_{\}}^T, \tag{7}$$

here, in the system of differential equations (7), the elements of the matrix $A^*(s)$, the components of the nonlinear vector function $\overline{F}(s,\overline{Y})$, and the components of the vector $\overline{f}_{(s)}$ are defined as in [3].

Adding the corresponding boundary conditions to equations (7), we obtain a nonlinear boundary value problem. The boundary conditions are given as follows: static through forces and moments in an integral form, kinematic - in a discrete number of points along the end face of the shell.

To solve the nonlinear boundary value problem for the system of equations (7), the linearization method and the stable numerical discrete-orthogonalization methods are applied [6, 7].

Based on this version of the refined nonlinear theory, as an example, consider the deformation of a three-layer orthotropic corrugated cylindrical shell with layers of constant thickness, rigidly fixed at the edges, under the action of a normal external pressure q_3. When solving problems, we assume that the coordinate surface passes in the middle of the middle layer of the shell.

When studying the regularities of the process of deformation of a corrugated cylindrical shell, we restrict ourselves to considering the deformation of one wave of the corrugation (Fig. 1).

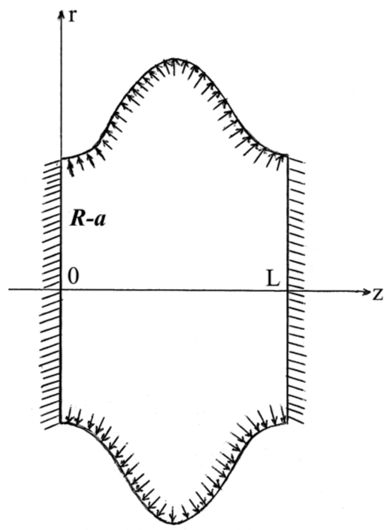

Figure 1.

The parametric equations of the meridian of the corrugated coordinate surface of the shell are written in the following form:

$$r = R - \alpha \cos \frac{2\pi}{L} z, \quad z = z, \quad (0 \le z \le L),$$

where r is the distance from rotation axis to the coordinate surface.

The geometric characteristics of the shell can be represented as follows:

$$\sin \varphi = \frac{1}{\gamma}, \quad \cos \varphi = \frac{2\pi a}{\gamma L} \sin \frac{2\pi}{L} z,$$

$$\frac{1}{R_s} = -\frac{a}{\gamma^3} \left(\frac{2\pi}{L}\right)^2 \cos \frac{2\pi}{L} z, \quad \gamma = \sqrt{1 + \left(\frac{2\pi a}{L} \sin \frac{2\pi}{L} z\right)^2},$$

where φ is the angle between the normal of the coordinate surface and the axis of rotation.

Let's denote by h_1, h_2, h_3 the thickness of the outer, middle and inner layers, respectively, by E_1^i, E_2^i the elasticity modulus according to the coordinate directions: s, θ; v_{12}^i, v_{21}^i are the Poisson coefficients; G_{13}^i is the shear modulus in the plane $\theta = const$, where $i = 1, 2, 3$ is the layer number. The problem was solved with the following values of the shell parameters:

$R = 20$; $\alpha = 1$; $L = 24$; $h_1 = 0.5$; $h_2 = 2$; $h_3 = 0.5$; $E_1^1 = 1.5 \cdot 10^5$; $E_2^1 = 3 \cdot 10^5$; $E_1^2 = 2 \cdot 10^4$; $E_2^2 = 3 \cdot 10^4$; $E_1^3 = 1.5 \cdot 10^5$; $E_2^3 = 3 \cdot 10^5$; $\gamma_{12}^1 = 0.2$, $\gamma_{21}^1 = 0.34$; $\gamma_{12}^2 = 0.1$; $\gamma_{21}^2 = 0.15$; $\gamma_{12}^3 = 0.2$; $\gamma_{21}^3 = 0.34$; $G_{13}^1 = 0.15 \cdot 10^5$; $G_{13}^2 = 0.15 \cdot 10^4$; $G_{13}^3 = 0.15 \cdot 10^5$.

On fig. 2 given graphs express the dependence between the bending w and the

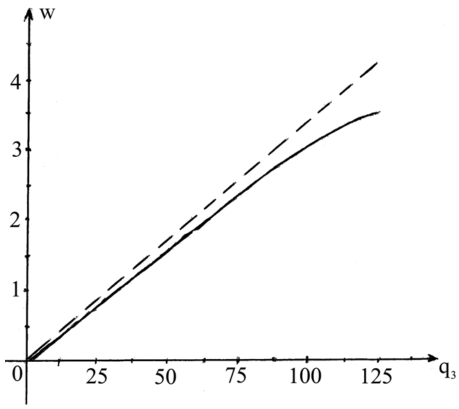

Figure 2.

surface normal load q_3 at the point $z = 6$. The solid line indicates the solution according to the nonlinear theory, the dashed line indicates the solution according to the linear one. On fig. 3 shows the distribution of bending along the axis of rotation under normal load $q_3 = 100$. The bending receives the greatest value at the points $z = 6$ and $z = 18$.

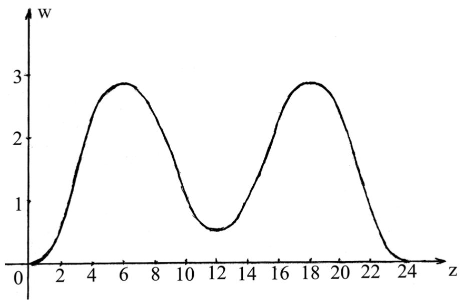

Figure 3.

Table 1

z	q_3: 25	50	75	100	125
	w				
0	0,0	0,0	0,0	0,0	0,0
	0,0	0,0	0,0	0,0	0,0
2	0,18560	0,37133	0,55699	0,74265	0,92832
	0,18595	0,37400	0,56533	0,76090	0,95988
4	0,59470	1,18942	1,7842	2,37881	2,97353
	0,58259	1,13721	1,6613	2,15483	2,61891
6	0,84993	1,69991	2,54981	3,39972	4,24972
	0,82292	1,58393	2,27932	2,91261	3,48994
8	0,63044	1,26093	1,89132	2,52182	3,15223
	0,61459	1,19672	1,74781	2,27061	2,76801
10	0,27106	0,54211	0,81317	1,08423	1,35536
	0,26687	0,52964	0,79296	1,05942	1,33014
12	0,15738	0,31477	0,47215	0,62953	0,78692
	0,15241	0,29838	0,44154	0,58351	0,72444
14	0,26949	0,53898	0,80847	1,07805	1,34753
	0,26532	0,52657	0,74205	1,05346	1,32264
16	0,62809	1,25621	1,88432	2,51233	3,14043
	0,61235	1,19262	1,74201	2,26341	2,75991
18	0,84997	1,69992	2,54993	3,39992	4,24981
	0,82294	1,58393	2,27942	2,91271	3,49013
20	0,59709	1,19423	1,79132	2,38843	2,98552
	0,58486	1,14154	1,66731	2,16231	2,62751
22	0,18726	0,37457	0,56177	0,74903	0,93629
	0,18752	0,37709	0,57008	0,76687	0,96723
24	0,0	0,0	0,0	0,0	0,0
	0,0	0,0	0,0	0,0	0,0

Table 1 shows the solution of the considered problem for the bending w at various points along the axis of rotation for various values of the normal surface load q_3. The solution of the problem was carried out both in linear and non-linear formulations. The first line of the table shows the solutions of the linear theory, and the second line shows the non-linear ones.

Table 2

z	σ_s^+	σ_s^-	σ_θ^+	σ_θ^-
0	$-0{,}1849 \cdot 10^4$ $-0{,}9378 \cdot 10^3$	$0{,}3483 \cdot 10^4$ $0{,}2826 \cdot 10^4$	$-0{,}6278 \cdot 10^3$ $0{,}3201 \cdot 10^3$	$0{,}1183 \cdot 10^4$ $0{,}9596 \cdot 10^3$
2	$-0{,}5876 \cdot 10^4$ $-0{,}4390 \cdot 10^4$	$0{,}8066 \cdot 10^4$ $0{,}8722 \cdot 10^4$	$0{,}8920 \cdot 10^4$ $0{,}9694 \cdot 10^4$	$0{,}1552 \cdot 10^5$ $0{,}1606 \cdot 10^5$
4	$-0{,}1611 \cdot 10^5$ $-0{,}1515 \cdot 10^5$	$0{,}1954 \cdot 10^5$ $0{,}2052 \cdot 10^5$	$0{,}2948 \cdot 10^5$ $0{,}2652 \cdot 10^5$	$0{,}4760 \cdot 10^5$ $0{,}4408 \cdot 10^5$
6	$-0{,}2288 \cdot 10^5$ $-0{,}2152 \cdot 10^5$	$0{,}2730 \cdot 10^5$ $0{,}2764 \cdot 10^5$	$0{,}4219 \cdot 10^5$ $0{,}3549 \cdot 10^5$	$0{,}6782 \cdot 10^5$ $0{,}5955 \cdot 10^5$
8	$-0{,}1698 \cdot 10^5$ $-0{,}1648 \cdot 10^5$	$0{,}2116 \cdot 10^5$ $0{,}2241 \cdot 10^5$	$0{,}3129 \cdot 10^5$ $0{,}2777 \cdot 10^5$	$0{,}5061 \cdot 10^5$ $0{,}4672 \cdot 10^5$
10	$-0{,}8184 \cdot 10^4$ $-0{,}7186 \cdot 10^4$	$0{,}1159 \cdot 10^5$ $0{,}1247 \cdot 10^5$	$0{,}1315 \cdot 10^5$ $0{,}1313 \cdot 10^5$	$0{,}2261 \cdot 10^5$ $0{,}2248 \cdot 10^5$
12	$-0{,}5760 \cdot 10^4$ $-0{,}3751 \cdot 10^4$	$0{,}8946 \cdot 10^4$ $0{,}8789 \cdot 10^4$	$0{,}7295 \cdot 10^4$ $0{,}7302 \cdot 10^4$	$0{,}1388 \cdot 10^5$ $0{,}1304 \cdot 10^5$
14	$-0{,}8150 \cdot 10^4$ $-0{,}7143 \cdot 10^4$	$0{,}1155 \cdot 10^5$ $0{,}1242 \cdot 10^5$	$0{,}1307 \cdot 10^5$ $0{,}1306 \cdot 10^5$	$0{,}2249 \cdot 10^5$ $0{,}2236 \cdot 10^5$
16	$-0{,}1692 \cdot 10^5$ $-0{,}1643 \cdot 10^5$	$0{,}2109 \cdot 10^5$ $0{,}2236 \cdot 10^5$	$0{,}3117 \cdot 10^5$ $0{,}2769 \cdot 10^5$	$0{,}5043 \cdot 10^5$ $0{,}4657 \cdot 10^5$
18	$-0{,}2288 \cdot 10^5$ $-0{,}2153 \cdot 10^5$	$0{,}2731 \cdot 10^5$ $0{,}2765 \cdot 10^5$	$0{,}4219 \cdot 10^5$ $0{,}3550 \cdot 10^5$	$0{,}6782 \cdot 10^5$ $0{,}5955 \cdot 10^5$
20	$-0{,}1617 \cdot 10^5$ $-0{,}1521 \cdot 10^5$	$0{,}1961 \cdot 10^5$ $0{,}2058 \cdot 10^5$	$0{,}4219 \cdot 10^5$ $0{,}2661 \cdot 10^5$	$0{,}6782 \cdot 10^5$ $0{,}4423 \cdot 10^5$
22	$-0{,}5913 \cdot 10^4$ $-0{,}4433 \cdot 10^4$	$0{,}8109 \cdot 10^4$ $0{,}8770 \cdot 10^4$	$0{,}8997 \cdot 10^4$ $0{,}9763 \cdot 10^4$	$0{,}1565 \cdot 10^5$ $0{,}1619 \cdot 10^5$
24	$-0{,}1850 \cdot 10^4$ $-0{,}9381 \cdot 10^4$	$0{,}3483 \cdot 10^4$ $0{,}2827 \cdot 10^4$	$-0{,}6301 \cdot 10^3$ $0{,}3176 \cdot 10^3$	$0{,}1185 \cdot 10^4$ $0{,}9626 \cdot 10^3$

Table 2 shows the distributions of normal σ_s, circumferential σ_θ stresses on the outer and inner surfaces of the shell under normal load $q_3 = 100$. The first line shows the values of the indicated functions obtained from the linear theory, and the second - from the non-linear one. The sign $(+)$ corresponds to the outer surface, and $(-)$ corresponds to the inner surface.

Concluding remark. One of the authors of the present article, Ukrainian scientist and great patriot of his country, Academician of National Academy of Sciences of Ukraine Y.M. Grigorenko (born October 12, 1927) passed away in January 18, 2022.

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