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Parametrization of Internal-Gravity Waves in the Ionosphere with Nonuniform Shear Flow

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The linear generation and intensification of internal gravity waves (IGW) in the ionosphere with non-uniform zonal wind (shear flow) is studied. On the basis of non-modal approach, the equations of dynamics and the energy transfer of IGW disturbances in the ionosphere with a shear flow is obtained. The effectiveness of the linear amplification mechanism of IGW at interaction with non-uniform zonal wind is analyzed. It is shown that at initial linear stage of evolution IGW effectively temporarily draws energy from the shear flow significantly increasing (by order of magnitude) own amplitude and energy.

Keywords: Gravity waves; Shear flow; IGW transient amplification.

AMS Subject Classification: 35Q35, 37K10, 37K40.

1. Introduction

Internal gravity waves (IGWs) play an important role in the formation of the general circulation, thermal regime, and composition of the middle and upper atmosphere. According to present knowledge, the main portion of IGW energy reaches the middle and upper atmosphere from tropospheric sources. In the middle and upper atmosphere the amplitudes of waves increase, they break and produce substantial amounts of heat and momentum [25]. Several parametrizations have been developed for the turbulent viscosity, mean flow drag and heating rates produced by dissipating IGWs [30], 1981; [34]; [25]; [15]; [33]; [36]; [24]. One of the main problems for the development of such parametrizations is that of the inclusion of atmospheric IGW sources. The most developed recent theories are those of IGW generation by mountains [41]; [45]; [8]; [38]; [32]; [35]. In the case of other tropospheric sources of IGWs, a good correlation is noticed between IGW intensity in the atmosphere and the passage of atmospheric fronts [9]; [29]. IGWs may appear at the upper edge of thunderstorms and heavy cumulus clouds [28]; [47]. Among the IGW sources are also convection [49]), industrial explosions [51], and moving

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disturbances in the atmosphere [31], etc. Many of these mentioned sources are only intermittently present in the atmosphere.

One of the important properties of IGW is their significant influence on the distribution of the electromagnetic waves in atmospheric-ionosphere layers [43]; [16]. Consequently, ionosphere electric currents and electromagnetic fields may re-influence the wave properties of IGW at ionosphere altitudes. In the ionosphere, contrary to the lower layers of the atmosphere, investigating the dynamics of wave processes non-uniform and non-stationary properties of the wind process, the turbulent state of the lower ionosphere and the influence of non-uniform electromagnetic forces should be taken into account. These factors, which are due to the low density medium in the ionosphere and the relatively high conductivity of the ionosphere gas, are strongly pronounced and they can sufficiently affect the propagation characteristics of wave patterns. Consequently, the general circulation in the ionosphere must have specific features that are absent in the troposphere.

The stationary problem of the existence of ionosphere wave disturbances in case of rectilinear uniform medium flow (for large-scale Rossby type waves) has been discussed for the first time in [13]. It has been revealed that in the theoretical study and interpretation of the dynamics of the winds above 100 km it is necessary to consider the possible deviations from the geostrophic winds associated with the action of electromagnetic forces. Further, a number of other works have appeared [26]; [6]; [5]; [4]; [1]; [12] and others, which studied the non-stationary evolution of wind structure in the conducting ionosphere medium under the influence of the spatially non-uniform geomagnetic field.

The action of the geomagnetic field, on the one hand, leads to the inductive damping of the waves associated with Pedersen or transverse (with respect to the geomagnetic field) conductivity, on the other hand–to the gyroscopic effect due to the Hall conductivity of the ionosphere acting on the perturbation like the Coriolis force. As a result of the joint action of spatially non-uniform Coriolis and electrodynamic (related to the geomagnetic field) forces the new type waves with different characteristics from the usual waves in the neutral medium may exist in the ionosphere. These waves can be called magnetized. In this paper we study the linear evolution of IGW in shear zonal flows (winds) in the ionosphere. At the linear stage in the dynamic equations the perturbed hydrodynamic quantities are given by SFH, which corresponds to non-modal analysis in a moving coordinate system along the background wind. Non-modal mathematical analysis allows replacement of the spatial non-uniform nature of the perturbed quantities, associated with the basic zonal flow, by temporal one in the basic equations and trace the evolution of SFH disturbances according to time.

The aim of this paper is theoretical investigation of the peculiarities of generation; intensification and further evolution of IGW structures due to the presence of local inhomogeneous zonal wind (shear flow). In section 2 we briefly outline the main principles of non-modal mathematical analysis of the generation and intensification of magnetized IGW in the linear stage based on the model of the medium and basic hydrodynamic equations for the lower ionosphere, given in [1] and the stability of the waves in shear flow and derive a necessary condition for instability. In Section 3 we study the characteristics of energy transfer by the IGW structures in the dissipative ionosphere with the shear flow. Discussion of the results is carried out in Section 4.

2. Generation and intensification of IGW at linear stage of evolution

To study the linear stage of interaction of internal gravity waves with the local non-uniform zonal wind and geomagnetic field, based on the model of the medium and basic hydrodynamic equations for the lower ionosphere, given in [1], we will have the following system of equations:

$$\rho_0\Big(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\Big)V_x=-\frac{\partial P}{\partial x}-\rho_0 v_0'(z)V_z-\sigma_p B_0^2 V_x+\rho_0\nu\Delta_\perp V_x, \tag{1}$$

$$\rho_0\Big(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\Big)V_z=-\frac{\partial P}{\partial z}-\rho_0 g-\sigma_p B_y^2 V_z+\rho_0\nu\Delta_\perp V_z, \tag{2}$$

$$\Big(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\Big)\rho=-\frac{d\rho_0}{dz}V_z, \tag{3}$$

$$\Big(\frac{\partial}{\partial t}+v_0(z)\frac{\partial}{\partial x}\Big)P=-\frac{dP_0}{dz}V_z, \tag{4}$$

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_z}{\partial z}=0. \tag{5}$$

Here, $\mathbf{V}_0(z)$ is a background of a plane zonal shear flow (wind) velocity, which is non-uniform along the vertical,$\mathbf{v}=\mathbf{v}_0(\mathbf{z})+\mathbf{v}(\mathbf{x},\mathbf{z},\mathbf{t})$, $\rho=\rho_0(z)+\rho(x,z,t)$, $P=P_0(z)+P(x,z,t)$, $v_0'(z)=dv_0(z)/dz$. In this system of five equations (1)–(5) any four of them creates a closed system. To facilitate further research, we choose equation (1), (2), (3) and (4) as a closed system.

2.1. *The local dispersion equation*

The system of equations (1)–(5) presents partial differential equations with variable coefficients, depending on the spatial coordinate z. For the existence of nontrivial solutions a local approximation should be applied, due to which the coefficients of equations (1)–(5) become locally homogeneous (constant). Then, for analyzes of the spectral characteristics, expressed by these equations of the disturbances, the Fourier expansion should be performed according to the spatial and temporal variables [21]. In Consequence a solution of equations (1)–(5) can be sought in the plane wave form [27]; [18]; [19]:

$$V_{x,z}(x,z,t)=\int V_{x,z}(k_x,k_z)\exp\big\{i\big[k_x x+(k_z-i/2H)z-\omega t\big]\big\}dk_x dk_z,$$

$$(P,\rho)(x,z,t)=\int(P,p)(k_x,k_z)\exp\big\{i\big[k_x x+(k_z+i/2H)z-\omega t\big]\big\}dk_x dk_z, \tag{6}$$

where the spatial Fourier expansion of the wave disturbances is carried out; $\mathbf{k}(k_x,0,k_z)$ is the wave vector and $\omega(k_x,k_z)$ is the frequency of the waves. Inserting (6) into equations (1)–(3) and (5), the following dispersion equation:

$$(\omega - k_x v_0)^2 - \frac{k_x^2}{K^2}\omega_g^2 + i\frac{(\omega - k_x v_0)}{K^2}$$
$$\times\Big[k_x^2\Big(\frac{\sigma_p B_{0y}^2}{\rho_0} + \nu K_1^2\Big) - k_x\Big(k_z + \frac{i}{2H}\Big)v_0' + \Big(k_z^2 + \frac{1}{4H^2}\Big)\Big(\frac{\sigma_P B_0^2}{\rho_0} + \nu K_1^2\Big)\Big] = 0. \quad (7)$$

Here, $\omega_g = (g/H)^{1/2} > 0$ is frequency of Brunt-Vaisala for stably stratified incompressible isothermal atmosphere; $K^2 = k_x^2 + k_z^2 + 1/(4H^2)$, $K_1^2 = K_2^2 - ik_z/H$, $K_2^2 = k_x^2 + k_z^2 - 1/(4H^2)$. Assuming the wave number K to be real and frequency $\omega = \omega_0 + i\gamma$, $|\gamma| \ll \omega_0$-complex, from (7), the expressions for the spectrum of linear fluctuations will be obtained:

$$\frac{\omega_0}{k_x} = v_0 - \frac{v_0'}{4K^2H^2} \pm \frac{\omega_g}{K}\sqrt{1 + \frac{v_0'^2}{16K^2H^2\omega_g^2}}, \quad (8)$$

and decrement (increment) of the perturbations

$$\gamma = -\frac{k_x^2\Big(\frac{\sigma_P B_{0y}^2}{\rho_0} + \nu K_2^2\Big) + \Big(k_z^2 + \frac{1}{4H^2}\Big)\Big(\frac{\sigma_P B_0^2}{\rho_0} + \nu K_2^2\Big) - k_x k_z v_0'}{2K^2\Big[1 + \frac{k_x v_0'}{4K^2H(\omega_0 - k_x v_0)}\Big]}. \quad (9)$$

In the absence of shear flow the formula (8) transforms into the expression for the frequency of ordinary internal gravity waves [18]:

$$\omega_0 = \pm\frac{k_x\omega_g}{(k_x^2 + k_z^2 + 1/(4H^2))^{1/2}}. \quad (10)$$

Formula (9) expresses the damping decrement of IGW due to induction (Pedersen) and viscous damping in the ionosphere medium: (11)

$$\gamma = -\frac{k_x^2\Big(\frac{\sigma_P B_{0y}^2}{\rho_0} + \nu K_2^2\Big) + \Big(k_z^2 + \frac{1}{4H^2}\Big)\Big(\frac{\sigma_P B_0^2}{\rho_0} + \nu K_2^2\Big)}{2K^2}. \quad (11)$$

According to (10), phase velocity of linear IGW is in the range:

$$-V_{\max} \le V_p \le V_{\max}, \quad (12)$$

where $V_{\max} = 2H\omega_g = 2(gH)^{1/2}$ in incompressible atmosphere. IGW is a low-frequency branch of acoustic-gravity waves (AGW), occupying an intermediate position between the frequency of inertial oscillations $\omega_i = 2\Omega_0$ and that of the Brunt-Vaisala for stably stratified incompressible isothermal atmosphere ω_g, $\omega_i < \omega_0 < \omega_g$ [16]; [19]). For the height of the uniform atmosphere $H \approx 4.5 \div 6km$, we can estimate the value of maximal phase velocity of linear IGW m/s, $V_{\max} \approx 440m/s$, the frequency $\omega_g \approx 1.7 \times 10^{-2}$ and $\Omega_0 \approx 10^{-4}$. So, IGW disturbances cover the following range of low-frequency oscillations $10^{-4}c^{-1} < \omega < 1.7\times10^{-2}c^{-1}$–and can be supersonic $V_p \geq c_s \approx 330m/s$.

At the different levels of the ionosphere the values of the viscous and induction damping of IG structures are different, and it should be considered in dynamic problems involving IGW structures.

It should be noted, that according to (8), the non-uniform zonal wind greatly expands the range of IGW in the ionosphere. Moreover, the shear flow feeds the medium with energy (see formula (9)), which is responsible for the generation-swing of IGW and development of linear shear instability with a characteristic growth rate:

$$\gamma_A \sim \frac{k_x k_z}{K^2} A. \tag{13}$$

From (11) it is obvious, that the considered ionospheric shear flow can become the source of the instability at the condition $\gamma_A \geq \gamma_\nu, \gamma_\sigma$. According to (13), for generation of the IGW structures it is necessary for the shear flow velocity to have at least the first derivative according to the vertical coordinate, different from zero ($v_0'(z) = A \neq 0$). As it was mentioned in [22]; [20], the typical value of the dimensional parameter of the shear flow $(A)s^{-1}$ for the ionosphere levels equals $A = v_0' \approx (0.015 \div 0.15)s^{-1}$ as well. Taking it into account from (13) it follows $\gamma_A \geq 10^{-1}s^{-1}$. Thus, the condition of the generation and amplification of IGW perturbations (inequality $\gamma_A \geq \gamma_\nu, \gamma_\sigma$) at different levels of the ionosphere (especially, in D and E-regions) can be satisfied and the shear instability can be developed. This conclusion can be made by virtue of above used modal (local–spectral) approach, which cannot give more information about the features of the shear flow instability.

3. Non-modal analysis of shear instability of the waves in the ionosphere

Instabilities, discussed in the previous section do not always arise and remain in a form, considered in the previous section. In shear flows the modal approach can detect only possibility of instability. But for the investigation of instability generation conditions and its temporal development in the ionosphere an alternative approach, namely, non-modal mathematical analysis is more appropriate. On the basis of non-modal approximation, shear flows can become unstable transiently till the condition of the strong relationship between the shear flows and wave perturbations is satisfied [10]; [4], e. i. the perturbation falls into amplification region in the wave number space. Leaving this region, e. i. when the perturbation passes to the damping region in the wave vector space, it returns an energy to the shear flow [2]. The experimental and observation data show the same [19]; [40]; [17].

Thus, non-uniform zonal wind or shear flow can generate and/or intensify the internal gravity waves in the ionosphere and provoke transient growth of amplitude, i.e. transient transport the medium into an unstable state. In the next subsection we confirm this view by using a different, more self-consistent method for the shear flow.

According to the above discussions, further analysis of the features of magnetized IGW wave at the linear stage in the ionosphere should be conducted in accordance with a non-modal approach. For this purpose, the moving coordinate system is more convenient with origin and the axis , which coincides with the same characteristics of the equilibrium local system, the axis flowing along the unperturbed (background) wind. In our problem, this transformation of the coordinate system is equivalent to the following replacement of the variables:

$$x_1 = x - azt, \quad y_1 = y, \quad t_1 = t, \tag{14}$$

or

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_1} - az\frac{\partial}{\partial x_1}, \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial x_1}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial z_1} - at_1\frac{\partial}{\partial x_1}. \tag{15}$$

With these new variables equation (1), (2), (3) and (5) take the form

$$\rho_0\frac{\partial V_x}{\partial t_1} = -\frac{\partial P}{\partial x_1} - \rho_0 v_0' V_z - \sigma_P B_0^2 V_x + \nu\rho_0\Big\{\frac{\partial^2}{\partial x^2} + \Big(\frac{\partial}{\partial z_1} - at_1\frac{\partial}{\partial x_1}\Big)^2\Big\}V_x, \tag{16}$$

$$\rho_0\frac{\partial V_z}{\partial t_1} = -\Big(\frac{\partial}{\partial z_1} - At_1\frac{\partial}{\partial x_1}\Big)P - \rho_0 g - \sigma_P B_{0y}^2 V_z$$

$$+\nu\rho_0\Big\{\frac{\partial^2}{\partial x^2} + \Big(\frac{\partial}{\partial z_1} - at_1\frac{\partial}{\partial x_1}\Big)^2\Big\}V_z, \tag{17}$$

$$\frac{\partial\rho}{\partial t_1} = \frac{\rho_0}{H}V_z, \tag{18}$$

$$\frac{\partial V_x}{\partial x} + \Big(\frac{\partial}{\partial z_1} - At_1\frac{\partial}{\partial x_1}\Big)V_z = 0. \tag{19}$$

Coefficients of the initial system of linear equations (1)–(4) depend on the spatial coordinate. Such mathematical transformations replace this spatial non-uniform property into temporal one. Thus, the initial-boundary problem is reduced to the initial problem of Cauchy type. Since now the coefficients of (16)–(19) are independent of spatial variables, the Fourier transformation of these equations with respect to spatial variables is already possible without any local approximation, the temporal evolution of these spatial Fourier harmonics (SFH) we consider independently:

$$\left\{\begin{matrix} V_x, z(x_1, z_1, t_1) \\ \rho, P(x_1, z_1, t_1) \end{matrix}\right\} = \int\int_{-\infty}^{\infty}\int dk_{x_1}dk_{z_1}\left\{\begin{matrix} \tilde{V}_{x,z}(k_{x_1}, k_{z_1}, t_1) \\ \tilde{\rho}, \tilde{P}(k_{x_1}, k_{z_1}, t_1) \end{matrix}\right\} \tag{20}$$

$$\times \exp(ik_{x_1}x_1 + ik_{z_1}z_1).$$

Here the factors with a tilde (for example $\tilde{V_x}$) indicate spatial Fourier harmonics (SFH) of the relevant physical quantities. Inserting (20) into equations (16)–(19), and passing to dimensionless variables,

$$\tau \Rightarrow \omega_g t_1; \quad V_{x,z} \Rightarrow \frac{\tilde{V}_{x,z}}{\omega_g H}; \quad \rho \Rightarrow \frac{\tilde{\rho}}{\rho_0}; \quad P \Rightarrow \frac{-i\tilde{P}}{\rho_0 \omega_g^2 H^2};$$

$$(x, z) \Rightarrow \frac{(x_1, z_1)}{H}; \quad S \Rightarrow \frac{A}{\omega_g}; \quad k_{x,z} \Rightarrow k_{x_1,z_1} H; \quad k_z = k_z(0) - k_x S\tau;$$

$$k^2(\tau) = (k_x^2 + k_z^2(\tau)); \quad \nu \Rightarrow \frac{\nu}{\omega_g H^2}; \quad b_0 \Rightarrow \frac{\sigma_P B_0^2}{\rho_0 \omega_g}; \quad b_y \Rightarrow \frac{\sigma_P B_y^2}{\rho_0 \omega_g}; \tag{21}$$

for each SFH perturbed quantities, we obtain

$$\frac{\partial V_x}{\partial \tau} = -SV_z + k_x P - \big[b_0 + \nu k^2(\tau)\big] V_x, \tag{22}$$

$$\frac{\partial V_z}{\partial \tau} = k_z(\tau) P - \rho - \big[b_y + \nu k^2(\tau)\big] V_z, \tag{23}$$

$$\frac{\partial \rho}{\partial \tau} = V_z, \tag{24}$$

$$k_x V_x + k_z(\tau) V_z = 0. \tag{25}$$

The closed system of equations (22)–(25) describes the linear interaction of IGW with a shear flow and the evolution of the generated disturbances in the dissipative ionosphere medium. We note once again that after these transformations the wave vector $k(k_x, k_z(\tau))$ of the perturbation became dependent on time: $k_z(\tau) = k_z(0) - k_x S \cdot \tau$; $k^2(\tau) = (k_x^2 + k_z^2(\tau))$. Variation of the wave vector according to time (i.e. splitting of the disturbances scales in the linear stage) leads to significant interaction in the medium even of such perturbations, the characteristic scale of which are very different from each other at the initial time [4].

On the basis of (22)–(25) an equation of energy transfer of the considered wave structures can be obtained, which gives possibility to identify the pattern of energy density variation with time:

$$\frac{dE(\tau)}{d\tau} = -\frac{S}{2}\big(V_x^*(\tau) \cdot V_z(\tau) + V_x(\tau) \cdot V_z^*(\tau)\big) - b_1(\tau)|V_x|^2 - b_2(\tau)|V_z|^2, \tag{26}$$

Here the asterisk denotes the complex conjugate values of the perturbation, $b_1(\tau) = b_0 + \nu k^2(\tau)$, $b_2(\tau) = b_y + \nu k^2(\tau)$ and the density of the total dimensionless energy of the wave perturbations $E(\tau)$ in the wave number space is given by:

$$E[k(\tau)] = \frac{1}{2}\big(|V_x|^2 + |V_z|^2 + |\rho|^2\big). \tag{27}$$

It's obvious that the transient evolution of wave energy structures in the ionosphere is due to the shear flow ($S \neq 0, A \neq 0$), dissipative processes–induction decay ($b_0 \neq 0, b_y \neq 0$) and viscosity ($\nu \neq 0$). In the absence of shear flow ($S = 0, A = 0$), and dissipative processes ($\nu = 0, \sigma_p = 0$), the energy of the considered wave disturbances in the ionosphere conserves $dE(\tau)/d\tau = 0$. The total energy density of the perturbations (27) consists of two parts: $E[k] = E_k + E_t$, where the first term is the kinetic energy of perturbation $E_k = \left(\left||v_x|^2 + |V_z|^2\right|\right)/2$, and the second–thermobaric energy $E_t = |\rho|^2/2$, stipulated due to the elasticity of perturbations.

To emphasize the pure effect of shear flow on the evolution of IGW, for simplicity, we consider non-dissipative ionosphere, i.e. we suppose that $(\nu = 0, \sigma_p = 0)$. Further, we determine on the basis of equation (26) what actually leads the evolution of the energy of the wave disturbance to does their energy increase or decrease? To answer we must calculate the right-hand side part of equation (26). For this purpose we must find the solutions of equations (22)–(25) at $b_1 = b_2 = 0$. Differentiating (23) with respect to time and using (22) (24) and (25), we obtain the second-order equation for the vertical velocity components:

$$\frac{d^2V_z}{d\tau^2} + R_1(\tau)\frac{dV_z}{d\tau} + R_2(\tau)V_z = 0, \tag{28}$$

where

$$R_1(\tau) = -4Sk_x\frac{k_z(\tau)}{k^2(\tau)}, \quad R_2(\tau) = (2S^2+1)\frac{k_x^2(\tau)}{k^2(\tau)}. \tag{29}$$

Equation (28) can be simplified by introducing a new variable (Magnus, 1976). Assuming

$$V_z = V\exp[(-1/2)\int R_1(\tau')d\tau']. \tag{30}$$

Let's transform (28) to the equation of a linear oscillator with time dependent parameters:

$$\ddot{V} + \Omega^2(\tau)V = 0, \tag{31}$$

where

$$\ddot{V} = \frac{d^2V}{d\tau^2}; \quad \Omega^2(\tau) = R_2(\tau) - \frac{1}{2}\dot{R}_1(\tau) - \frac{1}{4}R_1^2(\tau) = \frac{k_x^2}{k^2(\tau)}. \tag{32}$$

The equation (31) is well known in mathematical physics. This is an equation of linear oscillations of a mathematical pendulum, length of which changes. The value $\Omega(\tau)$ determines the angular velocity of the pendulum.

We solve equation (31) in the adiabatic approximation [52], i.e. when dependence of $\Omega(\tau)$ on time is adiabatically slow:

$$|\dot{\Omega}(\tau)| \ll \Omega^2(\tau). \tag{33}$$

Taking into account the definition of the parameter equation (33) can be rewritten as

$$S \cdot |k_z(\tau)| \ll \left[k_x^2 + k_z^2(\tau)\right]^{1/2}. \tag{34}$$

For the real ionospheric shear flow $S \ll 1$ (see definition (21)), it can be said that condition (34) holds for a wide range of variations of wave numbers $|k_z(\tau) =$

$k_z(0) - k_x S\tau|$. In other words, when the temporary variation of $|k_z(\tau)$ is due to the linear drift of the wave vector in the space of wave numbers, condition (33) or (34) is valid at all stages of the evolution of IGW. In this case, an approximate solution of homogeneous equation (31) can be represented as:

$$V = \frac{C}{\sqrt{\Omega(\tau)}} \exp[i\varphi(\tau)], \tag{35}$$

where $C = \text{const}$ and

$$\varphi(\tau) = \int_0^\tau \Omega(\tau')d\tau' = \frac{1}{S} \ln \left| \frac{k_z(0) + k(0)}{k_z(\tau) + k(\tau)} \right|.$$

Substituting (35) in (30), and then–into equations (22)–(25), we can finally construct the solutions for physical quantities:

$$V_z(\tau) = \frac{V_z(0) \cdot k^2(0)}{k_x^{1/2} \cdot k^{3/2}(\tau)} \exp[i\varphi(\tau)], \tag{36}$$

$$V_x(\tau) = -\frac{V_x(0) \cdot k_z(\tau) \cdot k^2(0)}{k_x^{3/2} \cdot k^{3/2}(\tau)} \exp[i\varphi(\tau)], \tag{37}$$

$$\rho(\tau) = -i\frac{\rho(0) \cdot k^2(0)}{k_x^{3/2} \cdot k^{1/2}(\tau)} \exp[i\varphi(\tau)], \tag{38}$$

$$P(\tau) = \frac{P(0) \cdot k^2(0)}{k_x^{3/2} \cdot k^{5/2}(\tau)} [2Sk(\tau) - ik_z(\tau)] \exp[i\varphi(\tau)], \tag{39}$$

$$k_x V_x(0) + k_z(0)V_z(0) = 0 \tag{40}$$

Here, in expressions (36)–(40) for the values of physical quantities are considered the real parts. Substituting (49)–(51) into equations (39) and (40), we obtain an expression for the normalized energy density of the Fourier harmonics:

$$\bar{E}(\tau) = \frac{E(\tau)}{E(0)} = \frac{(1 + k_0^2)^2}{[1 + (k_0 - S\tau)^2]^{1/2}}, \tag{41}$$

and for the IGW energy transport equation (at $b_1 = b_2 = 0$)

$$\frac{d\bar{E}(\tau)}{d\tau} = \frac{(1 + k_0^2)^2 \cdot (k_0 - S\tau)}{[1 + (k_0 - S\tau)^2]^{3/2}}, \tag{42}$$

Here for the convenience of numerical analysis a new parameter $k_0 = k_z(0)/k_x$ is introduced.

Using equations (54) and (55) we can determine an expression for the increment (decrement) of the shear instability $\Gamma(\tau) = (1/\bar{E}(\tau)) \cdot d\bar{E}(\tau)/d\tau$ in the nondissipative ionosphere:

$$\Gamma(\tau) = \frac{k_0 - S\tau}{1 + (k_0 - S\tau)^2}. \tag{43}$$

At the initial stage of evolution when $k_0 = k_z(0)/k_x > 0$ (when $k_z(\tau) > 0$) over time τ, $0 < \tau < \tau^* = k_z(0)/(Sk_x)$, the denominator (54) decreases and, accordingly, the energy density of IGW increases monotonically and reaches its maximum value (exceeding its initial value by an order) at the time $\tau = \tau^*$. Further, at $\tau^* < \tau < \infty$ the energy density begins to decrease (when $k_z(\tau) < 0$), and monotonically returns to its initial approximately constant value. In other words, at the early stages of evolution, temporarily, when $k_z(\tau) > 0$ and IGW perturbations are in the intensification region in wave-number space, the disturbances draw energy from the shear flow and increase own amplitude and energy by an order during the period of time $0 < \tau < \tau^* = k_z(0)/(Sk_x) = 100$. Then (if the nonlinear processes and the self-organization of the wave structures are not turned on), when $k_z(\tau) < 0$, IGW perturbation enters the damping region in wave number space and the perturbation returns energy back to the shear flow over time $\tau^* < \tau < \infty$ (Fig. 1, 2) and so on. Such transient redistribution of energy in the medium with the shear flow is due to the fact that the wave vector of the perturbation becomes a function of time $k = k(\tau)$, i.e. disturbances' scale splitting takes place. The structures of comparable scales effectively interact and redistribute free energy between them. Taking into account the induction and viscous damping (see equation (39)) the perturbation's energy reduction in the time interval $\tau^* < \tau < \infty$ is more intensive than that shown on fig. 1, the decay curve in the region $\tau^* < \tau < \infty$ becomes more asymmetric (right-hand side curve becomes steeper), and part of the energy of the shear flow passes to the medium in the form of heat.

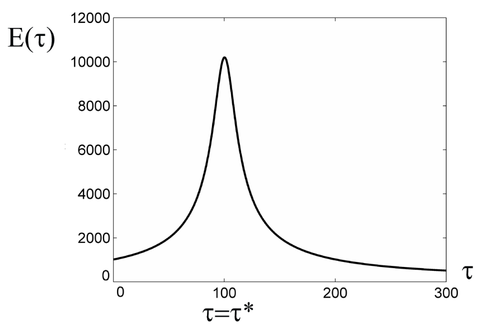

Figure 1. Evolution of the non-dimensional energy density $E(\tau)$

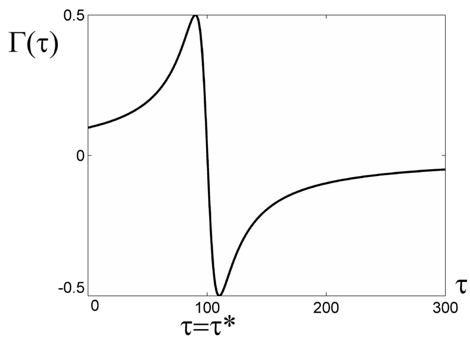

Figure 2. Increment of shear instability $\Gamma(\tau)$

Thus, even in a stable stratified ionosphere ($\omega_g^2 > 0$), temporarily, during the time interval $0 < t^* \approx 100/(\omega_g) \sim 5 \cdot 10^3 s \sim 1.5$ hour IGW-intensively draws energy from the shear flow and increases own energy and amplitude by an order. Accordingly, the wave activity will intensify in the given region of the ionosphere due to the shear flow (inhomogeneous wind) energy.

4. Discussion and conclusion

In this article the linear stage of generation and further nonlinear evolution of IGW structures in the dissipative stably stratified ($\omega_g^2 > 0$) ionosphere in the presence of shear flow (non-uniform zonal wind) is studied. A model system of dynamic nonlinear equations describing the interaction of internal gravity structures with viscous ionosphere, non-uniform local zonal wind, and the geomagnetic field is obtained. On the basis of analytical solutions and theoretical analysis of the corresponding system of dynamic equations a new mechanism of linear transient pumping of shear flow energy into that of the wave perturbation, wave amplification (multiple times), self-organization of nonlinear wave perturbations into the solitary vortex structures and the transformation of the perturbation energy into heat is revealed. A necessary condition for shear instability of IGW at their interaction with local non-uniform zonal wind, which is a generalization of the Rayleigh condition, is obtained.

The equation of energy transfer by nonlinear wave structure in the dissipative ionosphere is established. Based on the analysis of this equation it is revealed that the IGW structure effectively interacts with the local background non-uniform zonal wind and self-sustained by the shear flow energy in the ionosphere.

Linear amplification of IGW perturbation is not exponential as in the case of the AGW in the inverse-unstably stratified ($\omega_g^2 < 0$, when IGW cannot be generated) atmosphere [2], but in algebraic-power law manner. Intensification of IGW is possible temporarily, for certain values of environmental parameters, shear and waves, which form an unusual way of heating of the shear flow in the ionosphere: the waves draw their energy from the shear flow through a linear drift of SFH in the wave number space (fragmentation of disturbances due to scale) and pump energy into the region of small-scale perturbations, i.e. in the damping region. Finally, the dissipative processes convert this energy into heat. The process is permanent and can lead to strong heating of the medium. Intensity of heating depends on the level of the initial disturbance and the parameters of the shear flow.

A remarkable feature of the shear flow is the dependence of the frequency and wave number of perturbations on time $k_z = k_z(0) - k_x S\tau$, $k(\tau) = (k_x^2 + k_z^2(\tau))^{1/2}$. In particular, frequency and wave number transient growth leads to a reduction of scales of the wave disturbances due to time in the linear regime and, accordingly, to energy transfer into a short scale region–the dissipation region. On the other hand a significant change in the frequency range of the generated disturbances stipulates in the environment the formation of a broad range of spectral lines of the perturbations, which is linked to the linear interactions and not to the strong turbulent effects. Moreover, amplification of the SFH perturbation and broadening of wave modes' spectra occur in a limited period of time (transient interval), yet satisfied the relevant conditions of amplification and a strong enough interaction between the modes.

It should be emphasized that the detection of the mechanism of the intensification and broadening of the spectrum of perturbations became possible within the non-modal mathematical analysis (these processes are overlooked by more traditional modal approach). Thus, non-modal approach, taking into account the nonorthogonality of the eigenfunctions of the linear wave dynamics, proved to be more appropriate mathematical language to study the linear stage of the wave processes in shear flows.

The frequency of considered linear IGW perturbations varies in the interval of $10^{-4}c^{-1} < \omega_0 < 1.7 \times 10^{-2}c^{-1}$ and includes low-frequency range of AGW. Wavelength lies in the interval $\lambda \sim 100m \div 10km$, the period from 5 minutes to–3 hours. Considering intermediate values of the IGW wavelengths ($k \sim 1/H, H \sim 10km; \omega \sim \omega_g \sim 10^{-2}s^{-1}$) we find that the group and phase velocities are of the same order $V_g \sim V_p \sim \omega_g H \sim 10^{-2}s^{-1} \times 10^4 m \sim 10^2 m/s$. This estimation agrees with existing observations and they move with velocity $(0,1 \div 200)m/s$ in a random direction along the horizontal lines, depending on daytime and nighttime conditions. IGW is characterized by an exponential growth of the amplitude of the perturbed velocity at the vertical propagation in an environment with exponentially decaying vertical equilibrium density and pressure [27]; [19]). According to observational data, IGW disturbances manifest themselves in a wide range of heights–from the troposphere to the upper ionosphere heights $z \leq 600km$ [19]; [14]; [44]; [23]. At ionospheric altitudes (above 90 km) the conductive medium strongly impacts on the IGW, causing its remarkable damping due to local Pedersen currents.

IGW structures are eigen degrees of freedom of the ionospheric resonator. Therefore, influence of external sources on the ionosphere above or below (magnetic storms, earthquakes, artificial explosions, etc.) will excite these modes (or intensified) in the first, [7]. For a certain type of pulsed energy source the nonlinear solitary vortical structures will be generated [4]; [4], which is confirmed by experimental observations [42]; [11]; [39]; [46]; [48]). Thus, these wave structures can also be the ionospheric response to natural and artificial activity.

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