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Zero Order Approximation of Hierarchical Models for Elastic Prismatic Shells with Microtemperatures

George Jaiani

I. Javakhishvili Tbilisi State University
I. Vekua Institute of Applied Mathematics & Faculty of Exact and Natural Sciences
e-mail: george.jaiani@gmail.com
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In the present paper on the basis of the linear theory of thermoelasticity of homogeneous isotropic bodies with microtemperatures the zero order approximation of hierarchical models of elastic prismatic shells with microtemperatures is constructed.

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1. Introduction

The linear theory for elastic materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures, was constructed by Iesan and Quintanilla [1] in 2000. The fundamental solutions of the equations of the three-dimensional (3D) theory of thermoelasticity with microtemperatures were constructed by Svanadze [2] in 2004. The representations of the Galiorkin type and general solutions of the system of statics of the above theory were obtained by Scalia, Svanadze, and Tracina [3] in 2010. The linear theory for microstretch elastic materials with microtemperatures was constructed by Iesan [4] in 2001, where the uniqueness and existence theorems in the dynamical case for isotropic materials are proved. The fundamental solutions of the equations of the two-dimensional (2D) theory of thermoelasticity with microtemperatures were constructed by Basheleishvili, Bitsadze, and Jaiani [5] in 2011. Some basic boundary value problems of 2D version of statics of the linear theory of thermoelasticity with microtemperatures that cannot be considered as a particular case of the 3D version because of some peculiarities intrinsic only for the 2D version are studied by Bitsadze and Jaiani [6] in 2012.

In the present paper on the basis of the linear theory of thermoelasticity of homogeneous isotropic bodies with microtemperatures the zero order approximation of hierarchical models of elastic prismatic shells with microtemperatures is constructed.

Let $Ox_1x_2x_3$ be an anticlockwise-oriented rectangular Cartesian frame of origin O. We conditionally assume the x_3-axis to be vertical. The elastic body is called a prismatic shell if it is bounded above and below by, respectively, the surfaces (so called face surfaces)

$$x_3 = \overset{(+)}{h}(x_1,x_2) \text{ and } x_3 = \overset{(-)}{h}(x_1,x_2),\ \ (x_1,x_2) \in \omega \subset \mathbb{R}^2,$$

laterally by a cylindrical surface Γ of generatrix parallel to the x_3-axis and its vertical dimension is sufficiently small compared with other dimensions of the body. In other words, the 3D elastic prismatic shell-like body occupies a bounded region $\overline{\Omega}$ with boundary $\partial\Omega$, which is defined as:

$$\Omega := \left\{(x_1,x_2,x_3) \in \mathbb{R}^3 : \ (x_1,x_2) \in \omega, \ \overset{(-)}{h}(x_1,x_2) < x_3 < \overset{(+)}{h}(x_1,x_2)\right\}, \tag{1}$$

where $\overline{\omega} := \omega \cup \partial\omega$ is the so-called projection of the prismatic shell $\overline{\Omega} := \Omega \cup \partial\Omega$ $\gamma := \partial\omega$ and $\partial\Omega$ denote boundaries of ω and Ω, respectively; $\mathbb{R}^n$ is an n-dimensional Euclidian space.

In what follows we assume that

$$\overset{(\pm)}{h}(x_1,x_2) \in C^2(\omega) \cap C(\overline{\omega}),^1$$

and

$$2h(x_1,x_2) := \overset{(+)}{h}(x_1,x_2) - \overset{(-)}{h}(x_1,x_2) \ \begin{cases} > 0 & \text{for} \quad (x_1,x_2) \in \omega, \\ \geq 0 & \text{for} \quad (x_1,x_2) \in \partial\omega \end{cases}$$

is the thickness of the prismatic shell $\overline{\Omega}$ at the points $(x_1,x_2) \in \overline{\omega}$; $\max\{2h\}$ is essentially less than the characteristic dimensions of ω;

$$a := \frac{1}{h}, \quad b := \frac{\overset{(+)}{h}(x_1,x_2) + \overset{(-)}{h}(x_1,x_2)}{2h}.$$

2. Hierarchical Models

In order to construct hierarchical models we use Vekua's dimension reduction method [7], [8]. In what follows X_{ij} and e_{ij} are the stress and strain tensors, respectively, ρ is the reference mass density; Φ_i is the volume force; u_i is the displacement vector; T is the temperature measured from the constant absolute temperature $T_* > 0$; η is the entropy per unit mass; q_i is the heat flux vector; S is the heat supply; ε_i is the first moment of energy vector; $\tilde{q}_{ij}$ is the first heat flux moment vector; Q_i is the mean heat flux vector; M_i is the first heat source moment vector; λ, μ, β, $\tilde{a}$, $\tilde{b}$, k, κ_s, $(s = 1,2,...,5)$ are constitutive coefficients; δ_{ij} is the Kronecker delta, and w_i is the microtemperature vector. Throughout this article we use a superposed dot to denote partial differentiation with respect to time. Moreover, repeated indices imply summation (Greek letters run from 1 to 2, and Latin letters run from 1 to 3, unless otherwise stated), bar under one of the repeated indices means that we do not sum.

By u_{ir}, X_{ijr}, e_{ijr}, Φ_{ir}, T_r, w_{ir}, S_r, M_{ir}, Q_{ir}, q_{ir}, η_r, $\tilde{q}_{ijr}$, ε_{ir} we denote the r-th order mathematical moments (with respect to the Legendre polynomicals P_r) of the corresponding

[1] $C(\overline{\omega})$ denotes a class of functions continuous on $\overline{\omega}$; $C^2(\omega)$ denotes a class of twice continuously differentiable functions with respect to the variables x_1 and x_2 with $(x_1,x_2) \in \omega$.

quantities u_i, X_{ij}, e_{ij}, Φ_i, T, w_i, S, M_i, Q_i, q_i, η, $\tilde{q}_{ij}$, ε_i as defined below:

$$\Big(u_{ir}, X_{ijr}, e_{ijr}, \Phi_{ir}, T_r, w_{ir}, S_r, M_{ir}, Q_{ir}, q_{ir}, \eta_r, \tilde{q}_{ijr}, \varepsilon_{ir}\Big)(x_1,x_2,t)$$

$$:= \int\limits_{\overset{(-)}{h}(x_1,x_2)}^{\overset{(+)}{h}(x_1,x_2)} \Big(u_i, X_{ij}, e_{ij}, \Phi_i, T, w_i, S, M_i, Q_i, q_i, \eta, \tilde{q}_{ij}, \varepsilon_i\Big)(x_1,x_2,x_3,t)$$

$$\times P_r(ax_3 - b)\, dx_3, \quad (x_1,x_2) \in \omega \subset \mathbb{R}^2, \quad i,j = 1,2,3. \tag{2}$$

Hierarchical models for elastic prismatic shells with microtemperatures are the mathematical models. Their constructing is based on the multiplication of the basic equations of the linear theory of thermoelasticity with microtemperatures (see [1]):

The Motion Equations

$$X_{ij,i} + \Phi_j = \rho \ddot{u}_j(x_1,x_2,x_3,t), \quad (x_1,x_2,x_3) \in \Omega \subset \mathbb{R}^3, \quad t > t_0, \quad j = 1,2,3; \tag{3}$$

The Balance of Energy

$$\rho T_* \dot{\eta} = q_{i,i} + \rho S; \tag{4}$$

The First Moment of Energy

$$\rho \dot{\varepsilon}_j = q_{ij,i} + q_j - Q_j + \rho M_j, \quad j = 1,2,3; \tag{5}$$

Generalized Hooke's law (isotropic case)

$$X_{ij} = \lambda e_{ll} \delta_{ij} + 2\mu e_{ij} - \beta T \delta_{ij}, \quad i,j = 1,2,3, \tag{6}$$

$$\rho \eta = \beta e_{ll} + \tilde{a} T, \qquad \rho \varepsilon_j = -\tilde{b} w_j, \quad j = 1,2,3, \tag{7}$$

$$q_j = k T_{,j} + \kappa_1 w_j, \quad j = 1,2,3, \tag{8}$$

$$Q_j = (\kappa_1 - \kappa_2) w_j + (k - \kappa_3) T_{,j}, \quad j = 1,2,3, \tag{9}$$

$$\tilde{q}_{ij} = -\kappa_4 w_{l,l} \delta_{ij} - \kappa_5 w_{i,j} - \kappa_6 w_{j,i}, \quad i,j = 1,2,3; \tag{10}$$

The Kinematic Relations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad i,j = 1,2,3, \tag{11}$$

by Legendre polynomials $P_r(ax_3 - b)$ and then integration with respect to x_3 within the limits $\overset{(-)}{h}(x_1,x_2)$ and $\overset{(+)}{h}(x_1,x_2)$. Roughly speaking, assuming all the mathematical moments for $r > N$ to be equal to zero in the obtained relations, we get the Nth order hierarchical model. We suppose that:

(i) Φ_i, S, and M_i are continuous on $\overline{\Omega} \times I$, where $I = [0, \infty[$;
(ii) ρ is strictly positive;
(iii) the constitutive coefficients are constants.

The components of surface the traction X_n, the heat flux q, and the first heat flux moment Λ_i at a regular point of $\partial\Omega \times I$ are defined by

$$X_{ni} = X_{ji}n_j, \quad q = q_i n_i, \quad \Lambda_i = \tilde{q}_{ji}n_j, \tag{12}$$

respectively.

In the context of the linear theory, the Clausius-Duhem inequality has the form

$$q_i T_{,i} - T_* \tilde{q}_{ji} w_{i,j} - T_0(Q_i - q_i)w_i \geq 0. \tag{13}$$

Inequality (13) implies that

$$3\kappa_4 + \kappa_5 + \kappa_6 \geq 0, \quad \kappa_6 + \kappa_5 \geq 0, \tag{14}$$

$$\kappa_6 - \kappa_5 \geq 0, \quad k \geq 0, \quad (\kappa_1 + T_0\kappa_3)^2 - 4T_0 k \kappa_2 \leq 0.$$

We must adjoin boundary conditions and initial conditions to equations (3)-(11). Let us consider the subsets Σ_r $(r = 1, 2, ..., 6)$ of $\partial\Omega$ such that $\overline{\Sigma}_1 \cup \Sigma_2 = \overline{\Sigma}_3 \cup \Sigma_4 = \overline{\Sigma}_5 \cup \Sigma_6 = \partial\Omega$, and $\Sigma_1 \cap \Sigma_2 = \Sigma_3 \cap \Sigma_4 = \Sigma_5 \cap \Sigma_6 = \varnothing$.

In the case of the mixed boundary value problem the boundary conditions are

$$u_i = \tilde{u}_i \quad on \quad \overline{\Sigma}_1 \times I, \quad i = 1, 2, 3, \tag{15}$$

$$X_{ji}n_j = \tilde{X}_{ni} \quad on \quad \Sigma_2 \times I, \quad i = 1, 2, 3, \tag{16}$$

$$w_i = \tilde{w}_i \quad on \quad \overline{\Sigma}_5 \times I, \quad i = 1, 2, 3, \tag{17}$$

$$T = \widetilde{T} \quad on \quad \overline{\Sigma}_3 \times I, \quad i = 1, 2, 3, \tag{18}$$

$$\tilde{q}_{ji}n_j = \widetilde{\Lambda}_i \quad on \quad \Sigma_6 \times I, \quad i = 1, 2, 3, \tag{19}$$

$$q_i n_i = \tilde{q} \quad on \quad \Sigma_4 \times I, \quad i = 1, 2, 3, \tag{20}$$

where $\tilde{u}_i$, $\tilde{T}$, $\tilde{w}_i$, $\tilde{X}_{ni}$, $\tilde{q}$, and $\tilde{\Lambda}_i$ are prescribed functions. The initial conditions are

$$u(x,0) = u^0(x), \quad \dot{u}(x,0) = \underset{\sim}{u}^0(x), \tag{21}$$

$$T(x,0) = T^0(x), \quad w(x,0) = w^0(x), \quad x \in \bar{B}, \tag{22}$$

where u^0, v^0, T^0, and w^0 are given. We assume that: (i) $\tilde{u}_i$, $\tilde{T}$, and $\tilde{w}_i$ are continuous functions; (ii) $\tilde{X}_{ni}$, $\tilde{q}$, and $\tilde{\Lambda}_i$ are continuous in time and piecewise regular on $\Sigma_2 \times I$, $\Sigma_4 \times I$, and $\Sigma_6 \times I$, respectively; (iii) u_i^0, v_i^0, T^0, and w_i^0 are continuous on $\overline{\Omega}$.

The mixed problem consists of finding the functions $u_i \in C^{2.1}(\Omega \times I)$, $T \in C^{2.1}(\Omega \times I)$, and $w_i \in C^{2.1}(\Omega \times I)$ on $\Omega \times I$ that satisfy equations (3)-(11) on $\Omega \times I$, the boundary conditions (15)-(20), and the initial conditions (21)-(22).

Substitution of equations (11) into (6) and then the obtained and (7)-(10) into equations (3)-(5) yields the following system of linear partial differential equations for the fields u_i, T, and w_i:

$$\mu \triangle u + (\lambda + \mu) grad\ div\ u - \beta grad\ T + \Phi = \rho \ddot{u}, \tag{23}$$

$$k \triangle T - \beta T_* div\ \dot{u} + \kappa_1 div\ w - c\dot{T} = -\rho S, \tag{24}$$

$$\kappa_6 \triangle w + (\kappa_4 + \kappa_5) grad\ div\ w - \kappa_3 grad\ T - \kappa_2 w - \tilde{b}\dot{w} = \rho M, \tag{25}$$

where $u := (u_1, u_2, u_3)$, $w := (w_1, w_2, w_3)$, $c := \tilde{a} T_*$.

By constructing hierarchical models, on the upper and lower face surfaces temperatures, stress-vectors $X_{ji} \overset{(\pm)}{n}_j$ and first heat flux moment vectors $\tilde{q}_{ji} \overset{(\pm)}{n}_j$ are assumed to be known, while on the upper and lower face surfaces the values of the displacement, microtemperature and heat flux vectors are calculated from their Fourier-Legendre–series expansions on the segment

$$x_3 \in \left[\overset{(-)}{h}(x_1, x_2), \overset{(+)}{h}(x_1, x_2) \right].$$

3. The $N = 0$ Approximation (Hierarchical Model)

Let

$$v_{i0} := h^{-1} u_{i0}, \quad W_{i0} := h^{-1} w_{i0}, \quad i = 1,2,3.$$

Then the basic relations (constructed in the way pointed out in Section 2) of the zero approximation for elastic isotropic prismatic shells with microtemperatures have the following form:

The Motion Equations

$$X_{\alpha j0,\alpha} + \overset{\circ}{X}_j = \rho \ddot{u}_{j0}, \quad j = 1,2,3; \tag{26}$$

The Balance of Energy

$$\rho T_* \dot{\eta}_0 = q_{\alpha 0,\alpha} - \overset{(+)}{q}_\alpha \overset{(+)}{h}_{,\alpha} + \overset{(-)}{q}_\alpha \overset{(-)}{h}_{,\alpha} + \overset{(+)}{q_3} - \overset{(-)}{q_3} + \rho S_0; \tag{27}$$

The First Moment of Energy

$$\rho \dot{\varepsilon}_{j0} = -\tilde{b}\dot{w}_{j0} = \tilde{q}_{\alpha j0,\alpha} + \overset{0}{\tilde{q}}_j + q_{j0} - Q_{j0} + \rho M_{jo}, \quad j = 1,2,3; \tag{28}$$

Generalized Hooke's Law (isotropic case)

$$X_{ij0} = \lambda \delta_{ij} e_{ll0} + 2\mu e_{ij0} - \beta T_0 \delta_{ij}, \quad i,j = 1,2,3; \tag{29}$$

$$\begin{aligned}
e_{\alpha\beta 0} &= \frac{1}{2}(u_{\alpha 0,\beta} + u_{\beta 0,\alpha} + \overset{0}{b}_{\alpha 0} u_{\beta 0} + \overset{0}{b}_{\beta 0} u_{\alpha 0}) \\
&= \frac{1}{2}(u_{\alpha 0,\beta} + u_{\beta 0,\alpha} - \frac{h_{,\alpha}}{h} u_{\beta 0} - \frac{h_{,\beta}}{h} u_{\alpha 0}) = \frac{h}{2}(v_{\alpha 0,\beta} + v_{\beta 0,\alpha}), \quad \alpha,\beta = 1,2, \\
e_{3\alpha} &= e_{\alpha 3} = \frac{1}{2}(u_{30,\alpha} + \overset{0}{b}_{\alpha 0} u_{30}) = \frac{1}{2}(u_{30,\alpha} - \frac{h_{,\alpha}}{h} u_{30}) = \frac{h}{2} v_{30,\alpha}, \quad \alpha = 1,2, \\
e_{330} &= 0,
\end{aligned} \tag{30}$$

i.e.,

$$e_{ij0} = \frac{h}{2}(v_{i0,j} + v_{j0,i}), \quad i,j = 1,2,3;$$

$$\begin{aligned}
\rho \eta_0 &= \beta(u_{\gamma 0,\gamma} + \overset{0}{b}_{i0} u_{i0}) + \tilde{a} T_0 = \beta(u_{\gamma 0,\gamma} - \frac{h_{,\gamma}}{h} u_{\gamma 0}) + \tilde{a} T_0 \\
&= \beta h v_{\gamma 0,\gamma} + \tilde{a} T_0;
\end{aligned} \tag{31}$$

$$q_{i0} = \kappa_1 w_{i0} + k(T_{0,i} + \overset{0}{a}_{i0} T_0 + \overset{i}{T}) = \kappa_1 w_{i0} + k(T_{o,i} + \overset{i}{T}), \quad i = 1,2,3; \tag{32}$$

$$Q_{i0} = (\kappa_1 - \kappa_2) w_{i0} + (k - \kappa_3)(T_{0,i} + \overset{i}{T}), \quad i = 1,2,3; \tag{33}$$

$$\begin{aligned}
\tilde{q}_{\alpha\beta 0} &= -\kappa_4 (w_{\gamma 0,\gamma} + \overset{0}{b}_{l0} w_{l0}) \delta_{\alpha\beta} - \kappa_5 (w_{\alpha 0,\beta} + \overset{0}{b}_{\beta 0} w_{\alpha 0}) - \kappa_6 (w_{\beta 0,\alpha} + \overset{0}{b}_{\alpha 0} w_{\beta 0}) \\
&= -\kappa_4 (w_{\gamma 0,\gamma} - \frac{h_{,\gamma}}{h} w_{\gamma 0}) \delta_{\alpha\beta} - \kappa_5 (w_{\alpha 0,\beta} - \frac{h_{,\beta}}{h} w_{\alpha 0}) - \kappa_6 (w_{\beta 0,\alpha} - \frac{h_{,\alpha}}{h} w_{\beta 0}) \\
&= -\kappa_4 h W_{\gamma 0,\gamma} \delta_{\alpha\beta} - \kappa_5 h W_{\alpha 0,\beta} - \kappa_6 h W_{\beta 0,\alpha}, \quad \alpha,\beta = 1,2, \\
\tilde{q}_{\alpha 30} &= -\kappa_6 (w_{30,\alpha} + \overset{0}{b}_{\alpha 0} w_{30}) = -\kappa_6 h W_{30,\alpha}, \quad \alpha = 1,2, \\
q_{3\beta 0} &= -\kappa_5 (w_{30,\beta} + \overset{0}{b}_{\beta 0} w_{30}) = -\kappa_5 h W_{30,\beta}, \quad \beta = 1,2, \\
\tilde{q}_{330} &= -\kappa_4 (w_{\gamma 0,\gamma} + \overset{0}{b}_{l0} w_{l0}) = -\kappa_4 h W_{\gamma 0,\gamma},
\end{aligned}$$

i.e.,

$$\begin{aligned}
&\tilde{q}_{ij0} = -\kappa_4 hW_{\gamma 0,\gamma}\delta_{ij} - \kappa_5 hW_{i0,j} - \kappa_6 hW_{j0,i}, \quad i,j = 1,2,3;\\
&\overset{0}{q}_j = \overset{(+)}{\widetilde{q}}_j \sqrt{1+\big(\overset{(+)}{h}_{,1}\big)^2+\big(\overset{(+)}{h}_{,2}\big)^2} + \overset{(-)}{\widetilde{q}}_j \sqrt{1+\big(\overset{(-)}{h}_{,1}\big)^2+\big(\overset{(-)}{h}_{,2}\big)^2},\\
&\overset{(\pm)}{\widetilde{q}}_j = \overset{(\pm)}{\widetilde{q}}_{ij} n_i, \quad j = 1,2,3,\\
&\overset{i}{T} = \begin{cases} -\overset{(+)}{T}\overset{(+)}{h}_{,\alpha} + \overset{(-)}{T}\overset{(-)}{h}_{,\alpha}, & i=\alpha=1,2;\\ \overset{(+)}{T} - \overset{(-)}{T}, & i=3, \end{cases} \qquad (34)\\
&\overset{0}{X}_j = X_{\overset{(+)}{n}j}\sqrt{1+\big(\overset{(+)}{h}_{,1}\big)^2+\big(\overset{(+)}{h}_{,2}\big)^2} + X_{\overset{(-)}{n}j}\sqrt{1+\big(\overset{(-)}{h}_{,1}\big)^2+\big(\overset{(-)}{h}_{,2}\big)^2} + \Phi_{j0}, \quad j=1,2,3,
\end{aligned}$$

where $X_{\overset{(+)}{n}j}$ and $X_{\overset{(-)}{n}j}$ are components of the stress vectors, acting on upper and lower face surfaces.

By virtue of (26)-(34) and

$$b^0_{\alpha 0} := -h^{-1}h_{,\alpha}, \quad \alpha = 1,2, \quad b^0_{30} = 0,$$

we have

$$\begin{aligned}
&\mu\left[(hv_{\alpha 0,j})_{,\alpha} + (hv_{jo,\alpha})_{,\alpha}\right] + \lambda\delta_{\alpha j}(hv_{\gamma 0,\gamma})_{,\alpha}\\
&-\beta T_{0,j} + \overset{\circ}{X}_j = \rho h\ddot{v}_{j0}, \quad (x_1,x_2)\in\omega\subset\mathbb{R}^2, \quad j=1,2,3, \qquad (35)
\end{aligned}$$

$$\begin{aligned}
&-\tilde{b}h\dot{W}_{j0} = -\kappa_4(hW_{\gamma 0,\gamma})_{,\alpha}\delta_{\alpha j} - \kappa_5(hW_{\alpha 0,j})_{,\alpha} - \kappa_6(hW_{j0,\alpha})_{,\alpha}\\
&+\overset{\circ}{q}_j + \kappa_2 hW_{j0} + \kappa_3(T_{0,j} + \overset{j}{T}) + \rho M_{j0}, \quad (x_1,x_2)\in\omega\subset\mathbb{R}^2, \quad j=1,2,3, \qquad (36)
\end{aligned}$$

$$\begin{aligned}
&\beta T_*\left[(h\dot{v}_{\gamma 0})_{,\gamma} - h_{,\gamma}\dot{v}_{\gamma 0}\right] + \tilde{a}T_*\dot{T}_0 = \kappa_1(hW_{\alpha 0})_{,\alpha}\\
&+kT_{0,\alpha\alpha} + \overset{\alpha}{T}_{,\alpha} - \frac{h_{,\alpha}}{h}\left[\kappa_1 hW_{\alpha 0} + k(T_{0,\alpha} + \overset{\alpha}{T})\right] + \rho S_0, \quad (x_1,x_2)\in\omega\subset\mathbb{R}^2. \qquad (37)
\end{aligned}$$

To the system (35)-(37) should be added the initial and boundary conditions reformulated in mathematical zero moments:

$$\begin{aligned}
&u_{i0}(x_1,x_2,0) = u^0_{i0}(x_1,x_2), \;\; \dot{u}_{i0}(x_1,x_2,0) = \underset{\sim}{u}^0_{i0}(x_1,x_2),\\
&w_{i0}(x_1,x_2,0) = w^0_{i0}(x_1,x_2), \;\; T_0(x_1,x_2,0) = T^0_0(x_1,x_2),
\end{aligned}$$

$$(x_1,x_2)\in\overline{\omega}, \;\; i=1,2,3,$$

and e.g.

$$u_{i0} = \widetilde{u}_{i0}, \;\; w_{i0} = \widetilde{w}_{i0}, \;\; T_0 = \widetilde{T}_0 \;\; on \;\; \partial\omega \;\; for \;\; t > 0, \;\; i = 1,2,3.$$

If we consider the static case of the prismatic shell of constant thickness $2h(x_1,x_2) = const$ and take $j = 1,2$ in (35), (36), the obtained system along with (37) will give the system considered in [5], [6], while in the case of the variable thickness (in particular for cusped prismatic shells) we can use correspondingly modified methods presented in [9].

From the specificity of system (35)-(37) it follows that setting boundary conditions on the cusped edge for v_{j0} and W_{j0}, $j = 1,2,3$, differs from classical setting and depends on the character of sharpening prismatic shells but it is not so for T_0, while setting initial conditions for T_0, v_{j0} and W_{j0}, $j = 1,2,3$, does not depend on the character of sharpening and does not differ from classical setting.

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