NON-CLASSICAL BOUNDARY VALUE PROBLEMS AND MAXWELL'S EQUATIONS

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We consider boundary value problem (BVP) for an elliptic system of partial differential equations $\mathbf{A}(x, D)\mathbf{U} = \mathbf{F}$ of arbitrary order $2m$ on a domain $\Omega \subset \mathbb{R}^n$ with the smooth boundary $\mathscr{S} := \partial$ and a normal system of boundary conditions $\gamma_{\mathscr{S}}(\mathbf{B}_j(x, D)\mathbf{U}) = \mathbf{G}_j$, $j = 0, \dots, m-1$ on $\mathscr{S}$ in the scale of Bessel potential $\mathbb{H}^s_p(\Omega)$ and Besov $\mathbb{B}^s_{p,p}(\mathscr{S})$ spaces, including the case of negative $s < 0$ (spaces of distributions). We define rigorously the traces $\gamma_{\mathscr{S}}\mathbf{U}$ of solutions on the boundary $\mathscr{S}$, obtain the representation formulae for $\mathbf{U}$ and write conditions for the unique solvability in terms of boundary pseudodifferential equations on the boundary.

The obtained results are applied to an extension of distributions $\mathbf{G} \in \mathbb{B}^s_{p,p}(\mathscr{C})$ from a subsurface $\mathscr{C} \subset \mathscr{S}$ into the space $\mathbb{R}^n_{\mathscr{C}} := \mathbb{R}^n \setminus \mathscr{C}$, slit by the hypersurface $\mathscr{C}$.

Another, more important, application is the regularization of the non-elliptic BVP for the Maxwell's system, which is reduced to an equivalent pair of elliptic BVPs.

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