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ERRATUM TO "TOWARDS A HOMOTOPY THEORY OF HIGHER DIMENSIONAL TRANSITION SYSTEMS"

PHILIPPE GAUCHER

ABSTRACT. Counterexamples for Proposition 8.1 and Proposition 8.2 are given. They are used in the paper only to prove Corollary 8.3. A proof of this corollary is given without them. The proof of the fibrancy of some cubical transition systems is fixed.

1. PROPOSITION. [Counterexample for Proposition 8.1] *The canonical map*

$$\psi_X : \mathrm{CSA}_1(\underline{\mathrm{Cub}}(X)) \to \underline{\mathrm{Cub}}(\mathrm{CSA}_1(X))$$

is bijective on states, one-to-one on actions and one-to-one on transitions for all cubical transition systems X. There exists a cubical transition system Z such that ψ_Z is not surjective on actions and on transitions.

PROOF. The map $\psi_X : \mathrm{CSA}_1(\underline{\mathrm{Cub}}(X)) \to \underline{\mathrm{Cub}}(\mathrm{CSA}_1(X))$ is bijective on states and one-to-one on actions: see the proof of [Gau11, Proposition 8.1]. Therefore, it is one-to-one on transitions by a standard argument already used several times in this series of papers (see also [Gau14, Proposition 4.4]): if $(\alpha, u_1, \dots, v_n, \beta)$ and $(\alpha, u'_1, \dots, v'_{n'}, \beta)$ are two transitions of $\mathrm{CSA}_1(\underline{\mathrm{Cub}}(X))$ such that

$$(\psi_X(\alpha), \psi_X(u_1), \dots, \psi_X(v_n), \psi_X(\beta)) = (\psi_X(\alpha), \psi_X(u'_1), \dots, \psi_X(v'_{n'}), \psi_X(\beta)),$$

then $n = n'$ and since ψ_X is one-to-one on states and actions, one has

$$(\alpha, u_1, \dots, v_n, \beta) = (\alpha, u'_1, \dots, v'_{n'}, \beta).$$

We are now going to find a cubical transition system Z such that ψ_Z is not surjective on actions and on transitions. Consider the quotient set

$$S = \big(\{\alpha, \beta, \alpha_1, \beta_1, \alpha_2, \beta_2, \alpha_3, \beta_3\} \cup \{0,1\}^3 \times \{-,+\}\big) \\ /((0,0,0,\pm) = \alpha \text{ and } (1,1,1,\pm) = \beta),$$

i.e. with the identifications $(0,0,0,-) = (0,0,0,+) = \alpha$ and $(1,1,1,-) = (1,1,1,+) = \beta$. Let $L = \{u_1, u_2, u_3, u'_1, u'_2, u'_3\}$ be a set of actions with the labelling map defined by $\mu(u_i) = \mu(u'_i) = x_i$ for $i = 1,2,3$. The cubical transition system Z is intuitively the smallest one having the set of states S and the set of actions L such that there are the following maps of cubical transition systems:

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1. The map $C_3[x_1,x_2,x_3]\to Z$ taking the states (i,j) to $(i,j,-)$ for all $(i,j)\in\{0,1\}^3$ and the actions x_i to u_i for $i=1,2,3$ (the actions of $C_3[x_1,x_2,x_3]$ are not denoted by $(x_1,1),\dots,(x_3,3)$ because it is already understood that x_1,x_2,x_3 are distinct).

2. The map $C_3[x_1,x_2,x_3]\to Z$ taking the states (i,j) to $(i,j,+)$ for all $(i,j)\in\{0,1\}^3$ and the actions x_i to u'_i for $i=1,2,3$.

3. The maps $\mathrm{Cyl}(C_1[x_i])\to Z$ for $i=1,2,3$ taking the initial state to α_i, the final state to β_i, the action $(x_i,0)$ to u_i and the action $(x_i,1)$ to u'_i.

It is defined rigorously as the final lift of a cone of maps like as follows, which always exists since the functor $\omega:\mathbf{WHDTS}\to\mathbf{Set}^{\{s\}\cup\Sigma}$ forgetting the transitions is topological:

$$\begin{aligned}
\omega(C_3[x_1,x_2,x_3])&\to(\{0,1\}^3\times\{-\},\{u_1\},\{u_2\},\{u_3\})\subset W\\
\omega(C_3[x_1,x_2,x_3])&\to(\{0,1\}^3\times\{+\},\{u'_1\},\{u'_2\},\{u'_3\})\subset W\\
\omega(\mathrm{Cyl}(C_1[x_1]))&\to(\{\alpha_1,\beta_1\},\{u_1,u'_1\},\varnothing,\varnothing)\subset W\\
\omega(\mathrm{Cyl}(C_1[x_2]))&\to(\{\alpha_2,\beta_2\},\varnothing,\{u_2,u'_2\},\varnothing)\subset W\\
\omega(\mathrm{Cyl}(C_1[x_3]))&\to(\{\alpha_3,\beta_3\},\varnothing,\varnothing,\{u_3,u'_3\})\subset W
\end{aligned}$$

with $W=(S,\{u_1,u'_1\},\{u_2,u'_2\},\{u_3,u'_3\})$. One has $\omega(Z)=W$. The weak HDTS Z is cubical since $\mathbf{CTS}$ is a coreflective subcategory of the category of weak HDTS. The key fact is that Z contains the transitions (α_i,u_i,β_i) and (α_i,u'_i,β_i) for $i=1,2,3$. Therefore the canonical map $\phi_Z:Z\to\mathrm{CSA}_1(Z)$ identifies the actions u_i and u'_i for $i=1,2,3$: $\phi_Z(u_i)=\phi_Z(u'_i)=x_i$ for $i=1,2,3$. So $\mathrm{CSA}_1(Z)$ contains the five transitions (remember that ϕ_Z is bijective on states)

$$(\alpha,x_1,x_2,x_3,\beta),(\alpha,x_1,(1,0,-)),((1,0,-),x_2,x_3,\beta),(\alpha,x_1,x_2,(1,1,+)),((1,1,+),x_3,\beta).$$

By the composition axiom, $\mathrm{CSA}_1(Z)$ contains the transition $((1,0,-),x_2,(1,1,+))$ which corresponds to a unique map $C_1[x_2]\to\mathrm{CSA}_1(Z)$. Hence the cubical transition system $\underline{\mathrm{Cub}}(\mathrm{CSA}_1(Z))$ contains a transition from $(1,0,-)$ to $(1,1,+)$ indexed by an action u''_2 labelled by x_2 which is distinct from u_2 and u'_2. The point is that in $\underline{\mathrm{Cub}}(Z)$, the transition (α_i,u_i,β_i) becomes a transition (α_i,v_i,β_i) and the transition (α_i,u'_i,β_i) becomes a transition (α_i,v'_i,β_i) with $\mu(u_i)=\mu(v_i)=\mu(u'_i)=\mu(v'_i)=x_i$ for $i=1,2,3$. So the canonical map $\phi_{\underline{\mathrm{Cub}}(Z)}:\underline{\mathrm{Cub}}(Z)\to\mathrm{CSA}_1(\underline{\mathrm{Cub}}(Z))$ does not identify the actions u_i and u'_i for $i=1,2,3$. Therefore the composition axiom cannot be applied in $\mathrm{CSA}_1(\underline{\mathrm{Cub}}(Z))$ to create a transition from $(1,0,-)$ to $(1,1,+)$. Hence the map $\psi_Z:\mathrm{CSA}_1(\underline{\mathrm{Cub}}(Z))\to\underline{\mathrm{Cub}}(\mathrm{CSA}_1(Z))$ is not surjective on actions and on transitions. ■

2. Proposition. [Counterexample for Proposition 8.2] *There exists a weak equivalence of* **CTS** *(the left determined model structure) such that* $\underline{\mathrm{Cub}}(f)$ *is not a weak equivalence of* **CTS**.

Proof. Let $f = \phi_Z : Z \to \mathrm{CSA}_1(Z)$ with Z as above. Then f is a weak equivalence of **CTS** by [Gau11, Theorem 7.10] since $\mathrm{CSA}_1(f) : \mathrm{CSA}_1(Z) \to \mathrm{CSA}_1(\mathrm{CSA}_1(Z))$ is an isomorphism. The source of $\mathrm{CSA}_1(\underline{\mathrm{Cub}}(f))$ is $\mathrm{CSA}_1(\underline{\mathrm{Cub}}(Z))$. The target of $\mathrm{CSA}_1(\underline{\mathrm{Cub}}(f))$ is $\mathrm{CSA}_1(\underline{\mathrm{Cub}}(\mathrm{CSA}_1(Z)))$ which is equal to $\underline{\mathrm{Cub}}(\mathrm{CSA}_1(Z))$ because the latter satisfies CSA1 (see the beginning of the proof of [Gau11, Proposition 8.1]). Therefore $\mathrm{CSA}_1(\underline{\mathrm{Cub}}(f))$ cannot be an isomorphism and $\underline{\mathrm{Cub}}(f)$ is not a weak equivalence of **CTS** by [Gau11, Theorem 7.10]. ∎

3. Proposition. [Corollary 8.3 fixed] *Every weak equivalence of* **CTS** *belongs to* $\mathcal{W}_{\underline{\mathrm{Cub}}}$.

Proof. The class of maps $\mathcal{W}_{\underline{\mathrm{Cub}}}$ is, by definition, the localizer generated by the maps of cubical transition systems f such that $\underline{\mathrm{Cub}}(f)$ is a weak equivalence of **CTS**. This localizer contains the smallest one, which is precisely the class of weak equivalences of the left determined model structure **CTS**. ∎

Let $\mathcal{I}$ be the set of generating cofibrations of **CTS**. Let S be an arbitrary set of maps in **CTS**. It is claimed in [Gau11] that the class of fibrant objects of the Bousfield localization by the set of maps S of the left determined model structure **CTS** is the class of $\Lambda_{\mathbf{CTS}}(V, S, \mathcal{I})$-injective objects. Using Olschok's theorems, it is only possible to say that the class of fibrant objects is the class of $\Lambda_{\mathbf{CTS}}(V, S^{cof}, \mathcal{I})$-injective objects where S^{cof} is a set of cofibrant replacements for the maps of S. Since $\varnothing = \varnothing^{cof}$, it is correct to say that the class of fibrant objects of the left determined model structure of **CTS** is the class of $\Lambda_{\mathbf{CTS}}(V, \varnothing, \mathcal{I})$-injective objects. So the proof of Proposition 7.8 is correct. However, "$\Lambda_{\mathbf{CTS}}(V, S, \mathcal{I})$-injective" must be replaced by "$\Lambda_{\mathbf{CTS}}(V, S^{cof}, \mathcal{I})$-injective" page 318 before and in the proof of Theorem 6.3. And the proofs of Theorem 8.10 and Theorem 8.11 must be modified. More precisely, the proof of the following fact must be modified, and without using Theorem 8.10 (the characterization of the weak equivalences of $\underline{\mathbf{L}}_{\mathcal{S}}(\mathbf{CTS})$) and Theorem 8.11 to avoid any vicious circle:

4. Proposition. [Proof of fibrancy fixed] *Let* $\mathcal{S} = \{p_x : C_1[x] \sqcup C_1[x] \to \uparrow x \uparrow \mid x \in \Sigma\}$. *Then any* $\mathcal{S}$*-injective cubical transition system is fibrant in the Bousfield localization* $\underline{\mathbf{L}}_{\mathcal{S}}(\mathbf{CTS})$ *of the left determined model structure of* **CTS** *by the set of maps* $\mathcal{S}$.

Proof. This is [Gau14, Proposition 8.4]. ∎

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Laboratoire PPS (CNRS UMR 7126)
Université Paris 7–Denis Diderot
Site Chevaleret
Case 7014
75205 PARIS Cedex 13
France

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