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THE LOCAL-GLOBAL PRINCIPLE IN LEAVITT PATH ALGEBRAS

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Abstract. This is a short note on how a particular graph construction on a subset of edges that lead to a subalgebra construction, provided a tool in proving some ring theoretical properties of Leavitt path algebras.

1 Introduction

This paper is an expository note publicizing how a particular subalgebra construction which first appeared in the paper [5] by G.Abrams and K.M.Rangaswamy was used in proving many theorems on Leavitt path algebras. The power of the subalgebra construction relies on extending a particular property on a Leavitt path algebra over a "smaller" graph to the Leavitt path algebra of the whole graph. This can be visualised as from a local view to a global setting, "local-to-global jump".

We start by recalling the definitions of a path algebra and a Leavitt path algebra, (see [2] for a more extended study on Leavitt path algebras). A *directed graph* $E = (E^0, E^1, r, s)$ consists of two countable sets E^0, E^1 and functions $r, s : E^1 \to E^0$. The elements E^0 and E^1 are called *vertices* and *edges*, respectively. For each $e \in E^0$, $s(e)$ is the source of e and $r(e)$ is the range of e. If $s(e) = v$ and $r(e) = w$, then we say that v emits e and that w receives e. A vertex which does not receive any edges is called a *source,* and a vertex which emits no edges is called a *sink.* A graph is called *row- finite* if $s^{-1}(v)$ is a finite set for each vertex v. For a row-finite graph the edge set E^1 of E is finite if its set of vertices E^0 is finite. Thus, a row-finite graph is finite if E^0 is a finite set.

A path in a graph E is a sequence of edges $\mu = e_1 \dots e_n$ such that $r(e_i) = s(e_{i+1})$ for $i = 1, \dots, n-1$. In such a case, $s(\mu) := s(e_1)$ is the *source* of μ and $r(\mu) := r(e_n)$ is the *range* of μ, and n is the *length* of μ, i.e., $l(\mu) = n$.

If $s(\mu) = r(\mu)$ and $s(e_i) \neq s(e_j)$ for every $i \neq j$, then μ is called a *cycle.* If E does not contain any cycles, E is called *acyclic.* For $n \geq 2$, define E^n to be the set

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of paths of length n, and $E^* = \bigcup_{n\geq 0} E^n$ the set of all finite paths. Denote by E^∞ the set of all infinite paths of E, and by $E^{\leq\infty}$ the set E^∞ together with the set of finite paths in E whose range vertex is a sink. We say that a vertex $v \in E^0$ is *cofinal* if for every $\gamma \in E^{\leq\infty}$ there is a vertex w in the path γ such that $v \geq w$. We say that a graph E is cofinal if every vertex in E is cofinal.

The path K-algebra over E is defined as the free K-algebra $K[E^0 \cup E^1]$ with the relations:

(1) $v_i v_j = \delta_{ij} v_i$ for every $v_i, v_j \in E^0$.

(2) $e_i = e_i r(e_i) = s(e_i) e_i$ for every $e_i \in E^1$.

This algebra is denoted by KE. Given a graph E, define the extended graph of E as the new graph $\widehat{E} = (E^0, E^1 \cup (E^1)^*, r', s')$ where $(E^1)^* = \{e_i^* \mid e_i \in E^1\}$ and the functions r' and s' are defined as

$$r'|_{E^1} = r, \quad s'|_{E^1} = s, \quad r'(e_i^*) = s(e_i) \quad \text{and} \quad s'(e_i^*) = r(e_i).$$

The Leavitt path algebra of E with coefficients in K is defined as the path algebra over the extended graph $\widehat{E}$, with relations:

(CK1) $e_i^* e_j = \delta_{ij} r(e_j)$ for every $e_j \in E^1$ and $e_i^* \in (E^1)^*$.

(CK2) $v_i = \sum_{\{e_j \in E^1 \mid s(e_j) = v_i\}} e_j e_j^*$ for every $v_i \in E^0$ which is not a sink.

This algebra is denoted by $L_K(E)$. The conditions (CK1) and (CK2) are called the Cuntz-Krieger relations. In particular condition (CK2) is the Cuntz-Krieger relation at v_i. If v_i is a sink, we do not have a (CK2) relation at v_i. Note that the condition of row-finiteness is needed in order to define the equation (CK2).

Given a graph, we define a new graph built upon the given one that will be necessary for the subalgebra construction. The construction is based on an idea presented by Raeburn and Szymański in [12, Definition 1.1]. Then, we construct several examples.

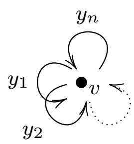
Definition 1. *[5, Definition 2] Let E be a graph, and F be a finite set of edges in E. We define $s(F)$ (resp. $r(F)$) to be the sets of those vertices in E which appear as the source (resp. range) vertex of at least one element of F. We define a graph E_F as follows:*

$$E_F^0 = F \cup (r(F) \cap s(F) \cap s(E^1 \backslash F)) \cup (r(F) \backslash s(F)),$$

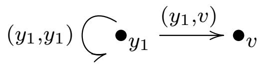
$$E_F^1 = \{(e, f) \in F \times E_F^0 \mid r(e) = s(f)\},$$

and where $s((x, y)) = x$, $r((x, y)) = y$ for any $(x, y) \in E_F^1$.

Example 2. *[5, Example 1] Let E be the rose with n-petals graph*

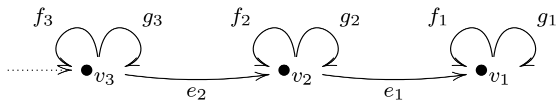


Let $F = \{y_1\}$. Then $E_F^0 = \{y_1\} \cup \{v\}$, and $E_F^1 = \{(y_1, y_1), (y_1, v)\}$. Pictorially, E_F is given by

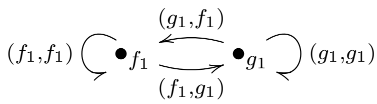


This example indicates that various properties of the graph E need not pass to the graph E_F. For instance, E is cofinal, while E_F is not. In particular, $L_K(E)$ is a simple algebra, while $L_K(E_F)$ is not.

Example 3. *Let E be the graph*

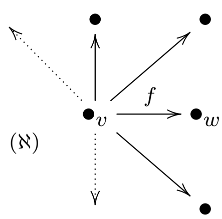


and $F = \{f_1, g_1\}$. Then, E_F is given by

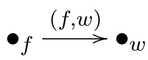


In this example E is not cofinal but E_F is cofinal. Also, $L_K(E)$ is not purely infinite simple while $L_K(E_F)$ is.

Example 4. *Consider the infinite clock graph E with one source which emits countably many edges as follows:*



Let $F = \{f\}$ and then E_F is



This is an example which shows that both E and E_F are acyclic graphs where F is any subset of vertices. Actually, if E is any acyclic graph and F any subset of vertices then E_F is acyclic is proved in [5, Lemma 1].

2 The Subalgebra Construction

Although in general E_F need not be a subgraph of E, the Leavitt path algebras $L_K(E_F)$ and $L_K(E)$ are related via a homomorphism which leads to a subalgebra construction of $L_K(E)$.

In [5, Proposition 1], for a finite set of edges F in a graph E, the algebra homomorphism $\theta : L_K(E_F) \to L_K(E)$ having the properties

(1) $F \cup F^* \subseteq \mathrm{Im}(\theta)$,

(2) If $w \in r(F)$, then $w \in \mathrm{Im}(\theta)$,

(3) If $w \in E^0$ has $s_E^{-1}(w) \subseteq F$, then $w \in \mathrm{Im}(\theta)$,

is defined by using the following subsets G^0 and G^1 of $L_K(E)$

$$G^0 = \{ee^* \mid e \in F\} \cup \{v - \sum_{f\in F, s(f)=v} ff^* \mid v \in r(F) \cap s(F) \cap s(E^1\backslash F)\}$$
$$\cup \{v \mid v \in r(F)\backslash s(F)\}$$

and

$$G^1 = \{eff^* \mid e,f \in F, s(f) = r(e)\} \cup \{e - \sum_{f\in F, s(f)=r(e)} eff^* \mid r(e) \in r(F) \cap s(F) \cap s(E^1\backslash F)\}$$
$$\cup \{e \in F \mid r(E) \in r(F)\backslash s(F)\}$$

In particular, $\theta(w) \in G^0$ for all vertices in E_F and $\theta(w) \in G^1$ for all edges in E_F.

Let E be any graph, K any field, and $\{a_1, a_2, \ldots, a_l\}$ any finite subset of nonzero elements of $L_K(E)$. For each $1 \leq r \leq l$ write

$$a_r = k_{c_1}v_{c_1} + k_{c_2}v_{c_2} + \ldots + k_{c_{j(r)}}v_{c_{j(r)}} + \sum_{i=1}^{t(r)} k_{r_i}p_{r_i}q_{r_i}^*$$

where each k_j is a nonzero element of K, and , for each $1 \leq i \leq t(r)$, at least one of p_{r_i} or q_{r_i} has length at least 1. Let F be denote the (necessarily finite) set of those edges in E which appear in the representation of some p_{r_i} or q_{r_i}, $1 \leq r_i \leq t(r)$, $1 \leq r \leq l$. Now consider the set

$$S = \{v_{c_1}, v_{c_2}, \ldots, v_{c_{j(r)}} \mid 1 \leq r \leq l\}$$

of vertices which appear in the displayed description of a_r for some $1 \leq r \leq l$. We partition S into subsets as follows:

$$S_1 = S \cap r(F),$$

and, for remaining vertices $T = S \backslash S_1$, we define

$$\begin{aligned} S_2 &= \{v \in T \mid s_E^{-1}(v) \subseteq F \text{ and } s_E^{-1}(v) \neq \emptyset\} \\ S_3 &= \{v \in T \mid s_E^{-1}(v) \cap F = \emptyset\} \\ S_4 &= \{v \in T \mid s_E^{-1}(v) \cap F \neq \emptyset \text{ and } s_E^{-1}(v) \cap (E^1 \backslash F) \neq \emptyset\}. \end{aligned}$$

Definition 5. *[5, Definition 3] Let E be any graph, K any field, and $\{a_1, a_2, \ldots, a_l\}$ any finite subset of nonzero elements of $L_K(E)$. Consider the notation presented in The Subalgebra Construction. We define $B(a_1, a_2, \ldots, a_l)$ to be the K-subalgebra of $L_K(E)$ generated by the set $Im(\theta) \cup S_3 \cup S_4$. That is,*

$$B(a_1, a_2, \ldots, a_l) =< Im(\theta), S_3, S_4 > .$$

Proposition 6. *[5, Proposition 1] Let E be any graph, K any field, and $\{a_1, a_2, \ldots, a_l\}$ any finite subset of nonzero elements of $L_K(E)$. Let F denote the subset of E^1 presented in The Subalgebra Construction. For $w \in S_4$ let u_w denote the element $w - \sum_{f \in F, s(f)=w} ff^*$ of $L_K(E)$. Then*

(1) $\{a_1, a_2, \ldots, a_l\} \subseteq B(a_1, a_2, \ldots, a_l)$.

(2) $B(a_1, a_2, \ldots, a_l) = Im(\theta) \oplus (\oplus_{v_i \in S_3} K v_i) \oplus (\oplus_{w_j \in S_4} K u_{w_j})$.

(3) The collection $\{B(S) \mid S \subseteq L_K(E),\ S \text{ finite}\}$ is an upward directed set of subalgebras of $L_K(E)$.

(4) $L_K(E) = \varinjlim_{\{S \subseteq L_K(E), S \quad finite\}} B(S)$.

Proposition 6, can be modified to include some more properties of the subalgebra construction in [5]. For instance, the morphism θ in the construction is actually a graded morphism whose image is a graded submodule of $L_K(E)$ and it also reveals some properties of cycles.

The stronger version of Proposition 6 is given in [10] as Theorem 4.1

Theorem 7. *[10, Theorem 4.1] For an arbitrary graph E, the Leavitt path algebra $L_K(E)$ is a directed union of graded subalgebras $B = A \oplus K\epsilon_1 \oplus \cdots \oplus K\epsilon_n$ where A is the image of a graded homomorphism θ from a Leavitt path algebra $L_K(F_B)$ to $L_K(E)$ where F_B a finite graph which depends on B, the elements ϵ_i are homogeneous mutually orthogonal idempotents and $\oplus$ is a ring direct sum. Moreover, if E is acyclic, so is each graph F_B and in this case θ is a graded monomorphism.*

Moreover, any cycle c in the graph F_B gives rise to a cycle c' in E such that if c has an exit in F_B then c' has an exit in E. In particular, a cycle in F_B is of the form $(f_1, f_2)(f_2, f_3) \dots (f_n, f_1)$ and this case $f_1 f_2 \dots f_n$ is a cycle in E.

Throughout recent literature this subalgebra construction has been a powerful tool. The first theorem that appears in the literature is the following:

Theorem 8. *[5, Theorem 1] $L_K(E)$ is von Neumann regular if and only if E is acyclic. If E is acyclic, then $L_K(E)$ is locally K-matricial; that is, $L_K(E)$ is the direct union of subrings, each of which is isomorphic to a finite matrix rings over K.*

Now, we give one implication of the statement to demonstrate how the subalgebra construction is used in the proof:

Proof. We assume E is acyclic. Let $\{B(S) \mid S \subseteq L_K(E), S \text{ finite}\}$ be the collection of subalgebras of $L_K(E)$ indicated in Proposition 6(3). By Proposition 6(4), it suffices to show that each such $B(S)$ is of the indicated form. But by Proposition 6 (2), $B(S) = B(a_1, a_2, \dots, a_l) = \mathrm{Im}(\theta) \oplus (\oplus_{v_i \in S_3} K v_i) \oplus (\oplus_{w_j \in S_4} K u_{w_j})$. Since terms appearing in the second and third summands are clearly isomorphic as algebras to $K \cong M_1(K)$, it suffices to show that $\mathrm{Im}(\theta)$ is isomorphic to a finite direct sum of finite matrix rings over K. Since E is acyclic, by Lemma 1 in [5] we have that E_F is acyclic. But E_F is always finite by definition, so we have by [3, Proposition 3.5], that $L_K(E_F) \cong \oplus_{i=1}^{l} M_{m_i}(K)$ for some $m_1, \dots, m_l$ in $\mathbb{N}$. Since each $M_{m_i}(K)$ is a simple ring, we have that any homomorphic image of $L_K(E_F)$ must have this same form. So we get that $\mathrm{Im}(\theta) \cong \oplus_{i=1}^{t} M_{m_i}(K)$ for some $m_1, \dots, m_t$ in $\mathbb{N}$, and we are done. (As remarked previously, since θ is in fact an isomorphism we have $t = l$.) □

We list the following theorems which are using the same Subalgebra Construction in their proofs. In particular, we only quote the parts that uses the Subalgebra Construction.

Theorem 9. *[10, Theorem 5.1] Let E be an arbitrary graph. Then for the Leavitt path algebra $L_K(E)$ the following are equivalent:*

(1) Every left/right ideal of $L_K(E)$ is graded;

(2) The class of all simple left/right $L_K(E)$-modules coincides with the class of all graded-simple left/right $L_K(E)$-modules;

(3) The graph E is acyclic.

Proof. (3) $\Rightarrow$ (1) For the sake of simplicity of the notation, let $L := L_K(E)$. Suppose E is acyclic. Now, by Theorem 7, L is a direct union of graded subalgebras B_λ where $\lambda \in I$, an index set and where each B_λ is a finite direct sum of copies of K and a graded homomorphic image of a Leavitt path algebra of a finite acyclic graph. By [8, Theorem 4.14], Leavitt path algebras of finite acyclic graphs are semi-simple algebras which have elementary gradings, that is, all the matrix units are homogeneous. Consequently, every ideal of each B_λ is graded. Let $L = \bigoplus_{n\in\mathbb{Z}} L_n$ be the $\mathbb{Z}$-graded decomposition of L. Since the B_λ are graded subalgebras, each $B_\lambda = \bigoplus_{n\in\mathbb{Z}} (B_\lambda \cap L_n)$. Let M be a left ideal of L. To show that M is graded, we need only to show that $M = \bigoplus_{n\in\mathbb{Z}} (M \cap L_n)$. Let $a \in M$. Then, for some λ, $a \in M \cap B_\lambda$. Note that $M \cap B_\lambda = B_\lambda$ or a left ideal of B_λ. Since every left ideal of B_λ and in particular $M \cap B_\lambda$ is graded, we can write $a = a_{n_1} + \cdots + a_{n_k}$ where

$$a_{n_i} \subset (M \cap B_\lambda) \cap (B_\lambda \cap L_{n_i}) \subset M \cap L_{n_i}$$

for $i = 1, \ldots, k$. This show that $M = \bigoplus_{n\in\mathbb{Z}} (M \cap L_n)$ and hence M is a graded left ideal of L. □

The next result is about graded von Neumann regular Leavitt path algebras. A ring R is von Neumann regular if for every $x \in R$ there exists $y \in R$ such that $x = xyx$. Moreover, a graded ring R is graded von Neumann regular if each homogeneous element is von Neumann regular.

Theorem 10. *[10, Theorem 4.2]; [9, Theorem 10] Every Leavitt path algebra $L_K(E)$ of an arbitrary graph E is a graded von Neumann regular ring.*

Proof. [10, Proof of Theorem 4.2] Suppose E is an arbitrary graph. By [10, Theorem 4.1], $L_K(E)$ is a directed union of graded subalgebras $B = A \oplus K\epsilon_1 \oplus \cdots \oplus K\epsilon_n$ where A is the image of a graded homomorphism θ from a Leavitt path algebra $L_K(F_B)$ to $L_K(E)$ with F_B a finite graph (depending on B), the elements ϵ_i are homogeneous mutually orthogonal idempotents and $\oplus$ is a ring direct sum. Since F_B is a finite graph, $L_K(F_B)$ and hence B is graded von Neumann regular by [9]. It is then clear from the definition that the direct union $L_K(E)$ is also graded von Neumann regular. □

Recall that a ring R is called left Bézout in case every finitely generated left ideal of R is principal. If the graph E is finite, then $L_K(E)$ is Bézout [4, Theorem 15]. The proof of this statement is given via a nice induction argument which we do not quote here. The generalization of this result to arbitrary graphs, which again appears in [4], uses the subalgebra construction.

Theorem 11. *[4, Corollary 16] Let E be an arbitrary graph and K any field. Then $L_K(E)$ is Bézout.*

Proof. By Theorem 7, $L_K(E)$ is the direct limit of unital subalgebras, each of which is isomorphic to the Leavitt path K-algebra of a finite graph. By [4, Theorem 15], each of these unital subalgebras is a Bézout subring of $L_K(E)$.

Now, we are going to prove that for any ring R, if every finite subset of R is contained in a unital Bézout subring of R, then R is Bézout. Let us consider a finitely generated left ideal of R with generators $x_1, x_2, \ldots, x_n \in R$. Then there is a unital Bézout subring S of R that contains $\{x_1, x_2, \ldots, x_n\}$. Hence, there exists $x \in S$ such that the left S-ideal $Sx_1 + Sx_2 + \cdots + Sx_n = Sx$.

Since $1_S x_i = x_i$ for all $1 \leq i \leq n$, and each x_i is in $Sx_1 + Sx_2 + \cdots + Sx_n = Sx$ which implies that for each i there exists $s_i \in S$ with $x_i = s_i x$.

Hence $Rx_1 + Rx_2 + \cdots + Rx_n = Rs_1x + Rs_2x + \cdots + Rs_nx \subseteq Rx$. Also, $x = 1_s x \in Sx$ implies $x \in Sx_1 + Sx_2 + \cdots + Sx_n \subseteq Rx_1 + Rx_2 + \cdots + Rx_n$. Therefore, $Rx_1 + Rx_2 + \cdots + Rx_n = Rx$ and R is a Bézout ring.

Hence, if R is taken to be $L_K(E)$, the result follows. □

Recall that a ring with local units R is said to be *directly finite* if for every $x, y \in R$ and an idempotent element $u \in R$ such that $xu = ux = x$ and $yu = uy = y$, we have that $xy = u$ implies $yx = u$.

Theorem 12. *[13, Proposition 4.3] $L_K(E)$ is directly finite if and only if no cycle in E has an exit.*

The converse of Theorem 12 for Leavitt path algebras of finite graphs has been proven in [7, Theorem 3.3]. To get the infinite graphs, Lia Vas proved the theorem by using Cohn-Leavitt approach. In particular, the localization of the graph is used by considering a finite subgraph generated by the vertices and edges of just those paths that appear in representations of x, y and u in $L_K(E)$ where $xy = u$ for some local unit u. However, the subgraph F defined in this way may not produce a subalgebra $L_K(F)$ of $L_K(E)$. This problem is avoided by considering an appropriate finite subgraph F such that the Cohn-Leavitt algebra of F is a subalgebra of $L_K(E)$ and then adapts [7, Theorem 3.3] to Cohn-Leavitt algebras of finite graphs.

An alternative proof using the subalgebra construction is pointed out in [11, Theorem 3.7] using the grading on matrices. We outline the proof below (without considering the grading to refer to Theorem 12).

Theorem 13. *([11, Theorem 3.7] rephrased) For an arbitrary graph E, the following properties are equivalent for $L_K(E)$:*

(a) *No cycle in E has an exit;*

(b) *$L_K(E)$ is a directed union of graded semisimple Leavitt path algebras; specifically, $L_K(E)$ is a directed union of direct sums of matrices of finite order over K or $K[x, x^{-1}]$.*

(c) *$L_K(E)$ is directly-finite.*

Proof. (a) implies (b) Assume (a). By Theorem 7, $L_K(E)$ is a directed union of graded subalgebras $B = A \oplus K\epsilon_1 \oplus \cdots \oplus K\epsilon_n$, where A is the image of a graded homomorphism θ from a Leavitt path algebra $L_K(F_B)$ to $L_K(E)$ with F_B a finite graph depending on B. Moreover, any cycle with an exit in F_B gives rise to a cycle with an exit in E. Since no cycle in E has an exit, no cycle in the finite graph F_B has an exit. So by using [2, Theorem 2.7.3],

$$L_K(F_B) \cong \bigoplus_{i\in I} M_{n_i}(K) \oplus \bigoplus_{j\in J} M_{m_j}(K[x, x^{-1}]),$$

where n_i and m_j are positive integers I, J are index sets. Since the matrix rings $M_{n_i}(K)$ and $M_{m_j}(K[x, x^{-1}])$ are simple rings, A and hence B is a direct sum of finitely many matrix rings of finite order over K and/or $K[x, x^{-1}]$. This proves (b).

(b) implies (c) follows from the known fact that matrix rings $M_{n_i}(K)$ and $M_{m_j}(K[x, x^{-1}])$ are directly-finite and finite ring direct sums of such matrix rings are directly-finite. Hence, by condition (b), $L_K(E)$ is directly-finite. □

We want to finish the survey with another application of the Subalgebra Construction. In [6], the authors do not use the exact results, however they carry the same techniques and proofs to another subgraph (dual graph) construction.

The authors present the notion of a dual of a subgraph in a graph, which is the generalization of the usual notion of dual graph found in the literature that we quote here:

Usual dual: Let E be an arbitrary graph. The *usual dual* of E, $D(E)$, is the graph formed from E by taking

$$\begin{aligned} D(E)^0 &= \{e \mid e \in E^1\} \\ D(E)^1 &= \{ef \mid ef \in E^2\} \\ s_{D(E)}(ef) &= e, \quad r_{D(E)}(ef) = f \quad \text{for all } ef \in E^2. \end{aligned}$$

The interest on the usual dual graph notion in the context of Leavitt path algebras lies on the fact that, if E is a row-finite graph without sinks, then there is an algebra isomorphism $L_K(E) \cong L_K(D(E))$ ([1, Proposition 2.11]). These statement is untrue for usual dual of a graph with sinks. The authors propose a new definition of dual graph which generalizes this important property to row-finite graphs with sinks.

Dual of F in E: Let E be a graph and let F be a subgraph of E. Denote $F_1^0 = \{v \in F^0 \mid s_F^{-1}(v) = \emptyset\}$, $F_1^1 = r_F^{-1}(F_1^0)$ and $F_2^0 = s(F^1) \cap s(E^1 \setminus F^1)$, $F_2^1 = r_F^{-1}(F_2^0)$. The graph $D_E(F)$, the *dual of* F *in* E is defined by

$$\begin{aligned} D_E(F)^0 &= D(F)^0 \cup F_1^0 \cup F_2^0 \\ D_E(F)^1 &= D(F)^1 \cup F_1^1 \cup F_2^1 \\ s_{D_E(F)}|_{D(F)} &= s_{D(F)},\ r_{D_E(F)}|_{D(F)} = r_{D(F)} \end{aligned}$$

For all $e \in F_i^1$ with $i \in \{1, 2\}, s_{D_E(F)} = e \in D(F)^0,\ r_{D_E(F)}(e) = r_F(e) \in F_i^0$.

Dual graph: Given a graph E, they define $d(E) = D_E(E)$ and call it the dual graph of E.

Then they prove the graded algebra isomorphism $L_K(d(E)) \cong L_K(E)$ when E is a row-finite graph ([6, Proposition 3.6]). In this paper the authors also prove that for a graph E and a row-finite subgraph of E there is a graded monomorphism $\theta : L_K(D_E(F)) \to L_K(E)$. In addition, $F^0 \cup F^1 \subseteq \theta(L_K(D_E(E)))$. This result is stated as [6, Proposition 3.8] and the proof is basically rephrasing [5, Proposition 1,2].

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References

[1] G. Abrams, P. N. Anh, A. Louly and E. Pardo, *The classification question for Leavitt path algebras*, J. Algebra **320** (2008), 1983-2026. MR2437640(2009f:16026). Zbl 1205.16013.

[2] G. Abrams , P. Ara and M. Siles Molina, *Leavitt Path Algebras. Lecture Notes in Mathematics*, London: Springer-Verlag, 2017. MR3729290. Zbl 06755637.

[3] G. Abrams, G. Aranda Pino and M. Siles Molina, *Finite-dimensional Leavitt path algebras*, J. Pure Appl. Algebra. **209** (3) (2007), 753-762. MR2298853(2007m:16038). Zbl 1128.16008.

[4] G. Abrams, F. Matese and A. Tonolo, *Leavitt path algebras are Bézout*, arXiv: 1605.08317v1[math RA] 26 May 2016. To appear Israel J. Math.

[5] G. Abrams and K.M. Rangaswamy, *Regularity conditions for arbitrary Leavitt path algebras*, Algebr. Represent. Theory **13** (2010), 319-334. MR2630124(2011b:16108). Zbl 1201.16016.

[6] G. Aranda Pino, J. Brox and M. Siles Molina, *Cycles in Leavitt path algebras by means of idempotents*, Forum Mathematicum, **27** (2015), 601–633. MR3334075. Zbl 1311.16004.

[7] G. Aranda Pino and L. Vas, *Noetherian Leavitt path algebras and their regular algebras*, Mediterr. J. Math., **10** (4) (2013), 1633 – 1656. MR3119324. Zbl 1308.16004.

[8] R. Hazrat, *The graded structure of Leavitt path algebras*, Israel J. Math **195** (2013), 833-895. MR3096577. Zbl 1308.16005.

[9] R. Hazrat, *Leavitt path algebras are graded von Neumann regular rings*, Journal of Algebra 401 (2014), 220-233. MR3151256. Zbl 1303.16005.

[10] R. Hazrat and K.M. Rangaswamy, *On graded irreducible representations of Leavitt path algebras*, Journal of Algebra **450** (2016), 458-486. MR3449700. Zbl 1350.16006.

[11] R. Hazrat, K.M. Rangaswamy and A.K. Srivastava, *Leavitt Path Algebras: Graded Direct-finiteness and Graded Sigma-injective Simple Modules*, Journal of Algebra **503** (2018) 229-328. MR3779997. Zbl 06890591.

[12] I. Raeburn and I. W. Szymański, *Cuntz-Krieger algebras of infinite graphs and matrices*, Trans. Amer. Math. Soc. **356** (1) (2003), 39 - 59. MR2020023(2004i:46087). Zbl 1030.46067.

[13] L. Vas, *Canonical traces and directly finite Leavitt path algebras*, Algebr. Represent. Theory **18** (2015), 711 - 738. MR3357945. Zbl 1329.16004.

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