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A GENERAL UNIQUE COMMON FIXED POINT THEOREM FOR HYBRID PAIRS OF MAPPINGS IN METRIC SPACES

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Abstract. The purpose of this paper is to prove a general unique common fixed point theorem for two pairs of mappings using Hausdorff - Pompeiu metric, which generalizes, in a correct form, the results from [8] and extends Theorem 2.4 [9], for occasionally (f, F) - weakly commuting mappings.

1 Introduction

Let f, g be self mappings of a metric space (X, d). Jungck [12] defined f and g to be compatible if

$$\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some $t \in X$.

Definition 1. *A point $x \in X$ is said to be a point of coincidence of f and g if $fx = gx$.*

We denote by $\mathcal{C}(f, g)$ the set of all coincidence points of f and g.

In [16], Pant defined the notions of pairwise R - weakly commuting mappings in metric spaces which is equivalent with commutativity in coincidence points.

In [13], Jungck defined the notion of weakly compatible mappings.

Definition 2. *Let X be a nonempty set and f, g be self mappings of X. f and g are weakly compatible if $fgx = gfx$ for all $x \in \mathcal{C}(f, g)$.*

If (X, d) is a metric space, then f and g are weakly compatible if and only if f and g are pointwise R - weakly commuting.

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Definition 3 ([7]). *Let f, g be self mappings of a nonempty set X. f and g are occasionally weakly compatible (owc) if $fgu = gfu$ for some $u \in X$.*

Remark 4. *If $\mathcal{C}(f,g) \neq \emptyset$ and f and g are weakly compatible, then f and g are owc, but the converse if not true (Example [6]).*

Let (X,d) be a metric space and $CL(X)$ (respectively, $CB(X)$) be the set of all nonempty closed (respectively, closed and bounded) subsets of X. For

$$d(x,A) = \inf_{y \in A} \{d(x,y)\},$$

we denote

$$D(A,B) = \inf \{d(a,b) : a \in A, b \in B\}$$

and by

$$H(A,B) = \max \left\{ \sup_{x \in A} d(x,B), \sup_{y \in B} d(y,A) \right\},$$

where $A, B \in CL(X)$ (respectively, $CB(X)$), the Hausdorff - Pompeiu metric on X.

Definition 5. *Let $f : X \to X$ and $F : X \to 2^X$ be.*

1) A point $x \in X$ is said to be a coincidence point of f and F if $fx \in Fx$. The set of all coincidence points of f and F is denoted by $\mathcal{C}(f,F)$.

2) A point $x \in X$ is a fixed point of F if $x \in Fx$.

Definition 6 ([14]). *Let X be a nonempty set, $f : X \to X$ and $F : X \to 2^X$. The pair (f,F) is weakly compatible if $fFx \subset Ffx$, for $x \in \mathcal{C}(f,F)$.*

Definition 7. *The hybrid pair (f,F), where $f : X \to X$, $F : X \to 2^X$ and X is a nonempty set, is occasionally weakly compatible (owc) if there exists $u \in X$ such that $fFu \subset Ffu$.*

Remark 8. *If $\mathcal{C}(f,F) \neq \emptyset$, every weakly compatible hybrid mappings are owc. The converse in not true (Example 1.7 [2], Example 1.3 [4]).*

In general, in literature, in the fixed point theorems for hybrid pairs of mappings involving Hausdorff - Pompeiu metric, the fixed point is not unique (Example 1.12 [6]).

The following theorem is "proved" in [8].

Theorem 9. *Let (X,d) be a metric space. Let $f, g : X \to X$ and $F, G : X \to CB(X)$ be such that (f,F) and (g,G) are owc satisfying the inequality*

$$\begin{aligned} H^p(Fx,Gy) \leq \max\{&ad(fx,gy) \cdot D^{p-1}(fx,Fx), ad(fx,gy) \cdot D^{p-1}(gy,Gy), \\ &aD(fx,Ax) \cdot D^{p-1}(gy,Gy), cD^{p-1}(fx,Gy) \cdot D(gy,Fx)\}, \end{aligned}$$

for all $x, y \in X$, where $p \geq 2$ is an integer, $a \geq 0$, $ac < 1$.

Then f, g, F and G have a unique common fixed point.

Remark 10. *The proof of this theorem is not correct because by* $a \in A$ *and* $b \in B$, *the inequality*

$$d(a, B) \le H(A, B)$$

is not correct.

In 2000, Shrivastava et al. [27] defined the notion of compatible of type N for a single valued mapping and a multivalued mapping.

Under another names, this notion was introduced in [2], [15], [26], [28].

Definition 11. *Let* (X, d) *be a metric space,* $f : X \to X$ *and* $F : X \to 2^X$. f *is said to be* (f, F) *commuting at* $x \in X$ *if* $ffx \in Ffx$.

The notion of occasionally (f, F) commuting is introduces in [24] under the name "occasionally weakly semi - compatible" and in [25] under the name of "occasionally F weakly commuting".

Definition 12. *Let* (f, F) *be a hybrid pair. The mapping* f *is said to be occasionally* F *- weakly commuting if there exists* $x \in X$ *such that* $x \in \mathcal{C}(f, F)$ *and* $ffx \in Ffx$.

Remark 13. *If* (f, F) *is occasionally* F *- weakly compatible, then* f *is occasionally* F *- weakly commuting but the converse is not true (see Example 1.6 [24] and Example 8 [25]).*

2 Preliminaries

The study of common fixed points for noncompatible mappings is also interesting, the work along this lines being initiated by Park [17], [18].

Aamri and El Moutawakil [1] introduced a generalization of noncompatible mappings.

Definition 14 ([1])**.** *Let* S, T *be self mappings of a metric space* (X, d). *We say that* S *and* T *satisfy* $(E.A)$ *- property if there exists a sequence* $\{x_n\}$ *in* X *such that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$$

for some $t \in X$.

Remark 15. *It is clear that two self mappings of a metric space* (X, d) *will be noncompatible if there exists a sequence* $\{x_n\}$ *in* X *such that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t,$$

for some $t \in X$, *but* $\lim_{n\to\infty}(STx_n, TSx_n)$ *is nonzero of non existent. Therefore, two noncompatible mappings satisfy* $(E.A)$ *- property.*

In 2011, Sintunavarat and Kumam [29] introduced the idea of limit range property.

Definition 16 ([29])**.** *A pair* (A, S) *of self mappings of a metric space* (X, d) *is said to satisfy the limit range property with respect to* S*, denoted* $CLR_{(S)}$ *- property, if there exists a sequence* $\{x_n\}$ *in* X *such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t,$$

for some $t \in S$*.*

Thus we can infer that a pair (A, S) satisfying $(E.A)$ - property along with the closedness of the subspace $S(X)$ always have the $CLR_{(S)}$ - property.

In [10], Imdad et al. introduced the notion of common limit range property of hybrid mappings.

Definition 17 ([10])**.** *Let* (X, d) *be a metric space and* $f : X \to X$*,* $F : X \to CL(X)$*.* (f, F) *has a common limit range property if there exists a sequence* $\{x_n\}$ *in* X *such that*

$$\lim_{n\to\infty} fx_n = fu \in A = \lim_{n\to\infty} Fx_n,$$

for $u \in A(X)$ *and* $A \in CL(X)$*.*

Quite recently, Imdad et al. [11] introduced the notions of joint common limit range property in metric spaces.

Definition 18 ([11])**.** *Let* (X, d) *be a metric space,* $f, g : X \to X$ *and* $F, G : X \to CL(X)$*. The pairs* (f, F) *and* (g, G) *are said to have joint common limit range property, denoted* $(JCLR)$ *- property, if there exist two sequences* $\{x_n\}$*,* $\{y_n\}$ *in* X *and* $A, B \in CL(X)$ *such that*

$$\lim_{n\to\infty} Fx_n = A, \lim_{n\to\infty} Gy_n = B, \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = t$$

such that $t \in A \cap B \subset f(X) \cap g(X)$*, i.e., there exist* $u, v \in X$ *such that* $t = fu = gv \in A \cap B$*.*

Now we introduce a new type of common limit range property for pairs of mappings.

Definition 19. *Let* (X, d) *be a metric space,* $A : X \to CL(X)$ *and* $S, T : X \to C$*. The pair* (A, S) *satisfy a common limit range property in respect to* T*, denoted* $CLR_{(A,S)T}$ *- property, if there exists a convergent sequence* $\{x_n\}$ *in* X *such that*

$$\lim_{n\to\infty} Sx_n = t \in D = \lim_{n\to\infty} Ax_n,$$

$D \in CL(X)$ *and* $t \in S(X) \cap T(X)$*.*

Example 20. *Let* $X = [0, \infty)$ *be a metric space with the usual metric.* $Ax = \left[\frac{1}{4}, 1\right]$, $Sx = \frac{x^2+1}{2}$, $Tx = x + \frac{1}{4}$. *Then* $S(X) = \left[\frac{1}{2}, \infty\right), T(X) = \left[\frac{1}{4}, \infty\right), S(X) \cap T(X) = \left[\frac{1}{2}, \infty\right)$.

Let $\{x_n\}$ *be a sequence in* X *such that* $\lim_{n\to\infty} x_n = 0$. *Then,*

$$\lim_{n\to\infty} Sx_n = t = \frac{1}{2} \in \left[\frac{1}{4}, 1\right] = \lim_{n\to\infty} Ax_n.$$

Hence, $t \in S(X) \cap T(X)$.

Remark 21. *1) Let* (X, d) *be a metric space,* $A, B : X \to CL(X)$ *and* $S, T : X \to X$. *If* (A, S) *and* (B, T) *satisfy* $(JCLR)$ *- property, then* (A, S) *and* T *satisfy* $CLR_{(A,S)T}$ *- property.*

2) If $BX = \left[0, \frac{1}{4}\right]$, *then* $A \cap B = \left\{\frac{1}{4}\right\}$, $A \cap B \not\subset S(X) \cap T(X)$ *and* (A, S) *and* T *satisfy* $CLR_{(A,S)T}$ *- property and not satisfy* $(JCLR)$ *- property.*

3 Implicit relations

Several classical fixed point theorems and common fixed point theorems have been recently unified considering a general condition by an implicit relation [19], [21]. The study of fixed points for hybrid pairs of mappings satisfying implicit relations is initiated in [20], [22], [23] and in other papers.

Definition 22. *Let* Φ_u *be the set of all continuous functions* $\phi(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ *such that:*

(ϕ_1) : ϕ is nondecreasing in variable t_1 and non increasing in variables t_5 and t_6,
(ϕ_2) : $\phi(t, 0, 0, t, t, 0) > 0$, $\forall t > 0$,
(ϕ_3) : $\phi(t, 0, t, 0, 0, t) > 0$, $\forall t > 0$,
(ϕ_4) : For every $t' > 0$, $\phi(t', t, 0, 0, t, t) > 0$, $\forall t > 0$.

Example 23. $\phi(t_1, ..., t_6) = t_1^p + t_2^p - \max\{at_2t_3^{p-1}, at_2t_4^{p-1}, at_3t_4^{p-1}, ct_5^{p-1}t_6\}$, *where* $p \geq 2$, $a \geq 0$, $0 < c < 1$.

Example 24. $\phi(t_1, ..., t_6) = t_1 - at_2 - bt_3 - ct_4 - dt_5 - et_6$, *where* $a, b, c, d, e \geq 0$, $c + d < 1$, $b + e < 1$ *and* $a > d + e$.

Example 25. $\phi(t_1, ..., t_6) = t_1^2 + t_2^2 - a\max\{t_3^2, t_5^2\} - b\max\{t_3t_5, t_4t_6\} - ct_5t_6$, *where* $a, b, c \geq 0$ ... ...

Example 26. $\phi(t_1, ..., t_6) = t_1 + t_2 - \alpha\max\{t_2, t_3, t_4\} - (1-\alpha)(at_5 + bt_6)$, *where* $\alpha \in (0, 1)$, $a, b \geq 0$ *and* $a + b < 1$.

Example 27. $\phi(t_1,...,t_6) = t_1 + t_2 - a\sqrt{t_3^2 + t_4^2} - b\sqrt{t_5 t_6}$, *where* $a, b \geq 0$, $a < 1$ *and* $b < 1$.

Example 28. $\phi(t_1,...,t_6) = t_1 + t_2 - a\max\{t_3, t_4\} - b\max\{t_5, t_6\}$, *where* $a, b \geq 0$ *and* $a + b < 1$.

Example 29. $\phi(t_1,...,t_6) = t_1 + t_2 - h\max\{t_3, t_4, \frac{t_5 + t_6}{2}\}$, *where* $h \in (0,1)$.

Example 30. $\phi(t_1,...,t_6) = t_1 + t_2 - k\max\{\frac{t_3 + t_4}{2}, \frac{t_5 + t_6}{2}\}$, *where* $k \in (0,1)$.

Remark 31. *The implicit relations satisfying conditions* (ϕ_2) *and* (ϕ_3) *- types are used in [15] and of* (ϕ_4) *- type is used in [9].*

The purpose of this paper is to prove a general unique common fixed point theorem for two pairs of mappings using Hausdorff - Pompeiu metric, which generalizes, in a correct form, the results from [8] and extends Theorem 2.4 [9], for occasionally (f, F) - weakly commuting mappings.

4 Main results

Theorem 32. *Let* (X, d) *be a metric space,* $f, g : X \to X$ *and* $F, G : X \to CL(X)$ *such that*

$$\phi\begin{pmatrix} H(Fx, Gy), d(fx, gy), d(fx, Fx), \\ d(gy, Gy), d(fx, Gy), d(gy, Fx) \end{pmatrix} \leq 0 \tag{4.1}$$

all $x, y \in X$ *and some* $\phi \in \Phi_u$.

If (f, F) *and* g *satisfy* $CLR_{(F,f)g}$ *- property, then*

1) $\mathcal{C}(F, f) \neq \emptyset$,

2) $\mathcal{C}(G, g) \neq \emptyset$.

Moreover, if f *is occasionally* F *- weakly commuting and* g *is occasionally* G *- weakly commuting, then* f, g, F *and* G *have a unique common fixed point.*

Proof. Since (f, F) and g satisfy $CLR_{(F,f)g}$ - property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = t \in D = \lim_{n\to\infty} Ax_n$$

and $t \in f(X) \cap g(X)$.

Since $t \in g(X)$, there exists $u \in X$ such that $t = gu$.

By (4.1) we have

$$\phi\begin{pmatrix} H(Fx_n, Gu), d(fx_n, gu), d(fx_n, Fx_n), \\ d(gu, Gu), d(fx_n, Gu), d(gu, Fx_n) \end{pmatrix} \leq 0 \tag{4.2}$$

Letting n tends to infinity we obtain

$$\phi(H(D, Gu), 0, 0, d(t, Gu), d(t, Gu), 0) \leq 0. \tag{4.3}$$

Since $t \in D$, $d(t, Gu) \leq H(D, Gu)$.

By (ϕ_1) and (4.2) we obtain

$$\phi(d(t, Gu), 0, 0, d(t, Gu), d(t, Gu), 0) \leq 0,$$

a contradiction of (ϕ_2) if $d(t, Gu) > 0$. Hence, $d(t, Gu) = 0$ which implies $t = gu \in Gu$ and $\mathcal{C}(G, g) \neq \emptyset$.

On the other hand, $t \in f(X)$. Hence, there exists $v \in X$ such that $t = fv$. By (4.1) we obtain

$$\phi\left(\begin{array}{c} H(Fv, Gu), d(fv, gu), d(fv, Fv), \\ d(gu, Gu), d(fv, Gu), d(gu, Fv) \end{array}\right) \leq 0. \tag{4.4}$$

Since $t \in Gu$, $d(t, Fv) \leq H(Fv, Gu)$.

By (ϕ_1) and (4.4) we obtain

$$\phi(d(t, Fv), 0, d(t, Fv), 0, 0, d(t, Fv)) \leq 0,$$

a contradiction of (ϕ_3) if $d(t, Fv) > 0$. Hence, $d(t, Fv) = 0$ which implies $t = fv \in Fv$ and $\mathcal{C}(f, F) \neq \emptyset$.

Moreover, if f is occasionally F - weakly commuting and $\mathcal{C}(f, F) \neq \emptyset$ and $\mathcal{C}(g, G) \neq \emptyset$, then there exists $a \in \mathcal{C}(f, F)$ and $b \in \mathcal{C}(g, G)$ such that $fa \in Fa$, $gb \in Gb$ and $f^2a \in Ffa$, $g^2a \in Gga$.

By (4.1) we obtain

$$\phi\left(\begin{array}{c} H(Fa, Gb), d(fa, gb), d(fa, Fa), \\ d(gb, Gb), d(fa, Gb), d(gb, Fa) \end{array}\right) \leq 0. \tag{4.5}$$

By (4.5) and (ϕ_1) we obtain

$$\phi(H(Fa, Gb), d(fa, gb), 0, 0, d(fa, gb), d(fa, gb)) \leq 0,$$

a contradiction of (ϕ_4) if $d(fa, gb) > 0$. Hence, $d(fa, gb) = 0$ which implies $fa = gb$.

Next we prove that $fa = f^2a$. Suppose that $fa \neq f^2a$.

By (4.1) we have

$$\phi\left(\begin{array}{c} H(Ffa, Gb), d\left(f^2a, gb\right), d\left(f^2a, Ffa\right), \\ d(gb, Gb), d\left(f^2a, Gb\right), d(gb, Ffa) \end{array}\right) \leq 0.$$

Since $f^2a \in Ffa$, by (ϕ_1) we obtain

$$\phi\left(H(Ffa, Gb), d\left(f^2a, gb\right), 0, 0, d\left(f^2a, gb\right), d\left(f^2a, gb\right)\right) \leq 0,$$

$$\phi\left(H(Ffa, Gb), d\left(f^2a, fa\right), 0, 0, d\left(f^2a, fa\right), d\left(f^2a, fa\right)\right) \leq 0,$$

a contradiction of (ϕ_4) if $d\left(f^2a, fa\right) > 0$. Hence, $d\left(f^2a, fa\right) = 0$ which implies $fa = f^2a$ and fa is a fixed point of f. Similarly, $gb = g^2b$ and $gb = gfa$. Therefore, $fa = f^2a = gb = g^2b = gfa$ and fa is a fixed point of g.

On the other hand, $fa = f^2a \in Ffa$ and fa is a fixed point of F. Similarly, $fa = f^2a = gb = g^2b \in Ggb = Gfa$. Hence, $fa \in Gfa$ and fa is a fixed point of g.

So, fa is a common fixed point of f, F, g and G.

Put $w = fu$ and let w' be another common fixed point of f, F, g and G. Then by (4.1) we have

$$\phi\left(\begin{array}{c} H\left(Fw, Gw'\right), d\left(fw, gw'\right), d\left(fw, Fw\right), \\ d\left(gw', Gw'\right), d\left(fw, Gw'\right), d\left(gw', Fw\right) \end{array}\right) \leq 0.$$

By (ϕ_1) we have

$$\phi\left(H\left(Fw, Gw'\right), d\left(w, w'\right), 0, 0, d\left(w, w'\right), d\left(w, w'\right)\right) \leq 0,$$

a contradiction of (ϕ_4) if $d\left(w, w'\right) > 0$. Hence, $d\left(w, w'\right) = 0$ which implies$w = w'$ and $w = fu$ is the unique common fixed point of f, F, g and G. □

By Example 23 and Theorem 32 we obtain

Theorem 33. *Let (X, d) be a metric space, $f, g : X \to X$ and $F, G : X \to CL(X)$ such that (f, F) and g satisfy $CLR_{(F,f)g}$ - property. If for all $x, y \in X$ for which $fx \neq gy$,*

$$\begin{array}{c} H^p\left(Fx, Gy\right) + d^p\left(fx, gy\right) \leq \max\{ad\left(fx, gy\right) \cdot D^{p-1}\left(fx, Fx\right), \\ ad\left(fx, gy\right) \cdot D^{p-1}\left(gy, Gy\right), ad\left(fx, Fx\right) \cdot D^{p-1}\left(gy, Gy\right), \\ cD^{p-1}\left(fx, Gy\right) \cdot d\left(gy, Fx\right)\}, \end{array}$$

where $p \geq 2$, $a \geq 0$, $c \in (0, 1)$, then

1) $\mathcal{C}(F, f) \neq \emptyset$,

2) $\mathcal{C}(G, g) \neq \emptyset$.

Moreover, if f is occasionally F - weakly commuting and g is occasionally G - weakly commuting, then f, g, F and G have a unique common fixed point.

Remark 34. *1. Theorem 33 is a correct generalization of Theorem 9.*

2. By Examples 24 - 30 we obtain new particular results.

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