VARIOUS NOTIONS OF AMENABILITY FOR NOT NECESSARILY LOCALLY COMPACT GROUPOIDS

Mădălina Roxana Buneci

Abstract. We start with a groupoid G endowed with a family $\mathcal{W}$ of subsets mimicking the properties of a neighborhood basis of the unit space (of a topological groupoid with paracompact unit space). Using the family $\mathcal{W}$ we endow each G-space with a uniform structure. The uniformities of the G-spaces allow us to define various notions of amenability for the G-equivariant maps. As in [1], the amenability of the groupoid G is defined as the amenability of its range map. If the groupoid G is a group, all notions of amenability that we introduce coincide with the classical notion of amenability for topological (not necessarily locally-compact) groups.

Full text

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Mădălina Buneci

University Constantin Brâncuşi,

Str. Geneva, Nr. 3, 210136 Târgu-Jiu, Romania.

e-mail: ada@utgjiu.ro

http://www.utgjiu.ro/math/mbuneci/