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FUNCTION VALUED METRIC SPACES

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Abstract. In this paper we introduce the notion of an $\mathcal{F}$-metric, as a function valued distance mapping, on a set X and we investigate the theory of $\mathcal{F}$-metric spaces. We show that every metric space may be viewed as an $\mathcal{F}$-metric space and every $\mathcal{F}$-metric space (X, δ) can be regarded as a topological space (X, τ_δ). In addition, we prove that the category of the so-called extended $\mathcal{F}$-metric spaces properly contains the category of metric spaces. We also introduce the concept of an $\bar{\mathcal{F}}$-metric space as a completion of an $\mathcal{F}$-metric space and, as an application to topology, we prove that each normal topological space is $\bar{\mathcal{F}}$-metrizable.

Full text

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