

BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL EQUATIONS WITH FRACTIONAL ORDER

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Abstract. In this paper, we shall establish sufficient conditions for the existence of solutions for a first order boundary value problem for fractional differential equations.

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