New York Journal of Mathematics

New York J. Math. **18** (2012) 551–554.

Errata and corrigenda: Ergodic and chaotic properties of Lipschitz maps on smooth surfaces

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Abstract. Errata and corrigenda are given for *Ergodic and chaotic properties of Lipschitz maps on smooth surfaces*, New York J. Math. 18 (2012), 95–120.

Contents

1. Erratum

There is an error in the remarks that occur between Equations (4.2) and (4.3) in [1]. Checking Equation (3.6) for the map g^* is necessary but not sufficient for (4.2) to hold as is implied by the remarks. The sentence

> Therefore $G(x) = \varphi_{i+1} \circ g^* \circ \varphi_i^{-1}(x) = \varphi_i \circ g^* \circ \varphi_{i-1}^{-1}(x)$ is well-defined for every point $x \in X$.

does not hold. In fact G is not always well-defined when (3.6) holds. We remove the specific form of f from (3.6), and then we add condition (3.7); all equalities hold (mod1):

$$g^*(\{\theta\}) = -g^*(\{-\theta\}), \tag{3.6}$$

$$\theta \in [0,1/2] \Rightarrow g^*(\{\theta\}) \in [0,1/2]. \tag{3.7}$$

We note that (3.6) and (3.7) in turn imply (3.8) and (3.9):

$$\theta \in (1/2,1) \Rightarrow g^*(\{\theta\}) \in (1/2,1), \tag{3.8}$$

$$g^*(\{1/2\}) = 0 \quad \text{or} \quad g^*(\{1/2\}) = 1/2. \tag{3.9}$$

Received July 17, 2012.

2010 *Mathematics Subject Classification.* 37A, 37E, 57M.

Key words and phrases. Dynamics on surfaces, one-sided Bernoulli, measure preserving systems, chaotic dynamics.

ISSN 1076-9803/2012

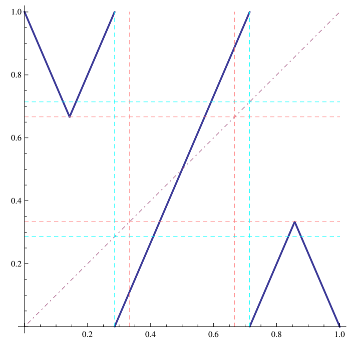

FIGURE 1. The graph of g_s from Remark 5.2

Conditions (3.6) and (3.7) are sufficient for (4.2) to hold. Only (3.6) appears in [1]; this omission led to errors in the statements of Theorem 4.1 and 6.3, for which we offer corrections in the next section. The particular map g^* defined just before (4.1) does not satisfy (3.7) and in fact G is not well-defined in Theorem 4.1; Theorem 6.3 contains a similar error. While Theorem 6.3 has a simple correction using an alternative map discussed in [1], we can only prove a modified version of Theorem 4.1. A similar modification to Theorem 4.2 is also required.

2. Corrigenda

We start with the simple correction needed in Section 6.

2.1. A correction for Theorem 6.3. If we use the map described in Remark 5.2 instead of the map f_s used (i.e., use the degree one circle map whose graph is the reflection about $x = 1/2$ of the map f_s used), then Theorem 5.3 remains unchanged, and in Theorem 6.3, statement (2) is replaced by "(2) The points A and B are repelling fixed points of G_s." The graph of the degree one map is shown in Figure 1.

2.2. Section 4 corrections. We cannot recover the statements of Theorems 4.1 and 4.2 as given in [1]. However we state revised versions here. Instead of $f_d(x) = dx$ (mod1) given in [1] (also written as $f(z) = z^d$), we use closely related maps denoted F_d, each one a d-to-one map with the property $|F_d'(x)| = d$ and which maps $[0, 1/2]$ onto $[0, 1/2]$. The formula for F_2 is:

$$F_2(x) = \begin{cases} 2x & \text{if } x \in \left[0, \frac{1}{4}\right), \\ 1 - 2x & \text{if } x \in \left[\frac{1}{4}, \frac{1}{2}\right), \\ 2 - 2x & \text{if } x \in \left[\frac{1}{2}, \frac{3}{4}\right), \\ 2x - 1 & \text{if } x \in \left[\frac{3}{4}, 1\right). \end{cases}$$

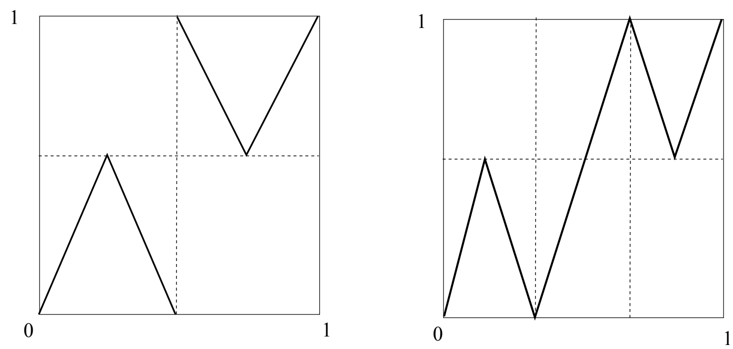

FIGURE 2. The graphs of F_2 and F_3

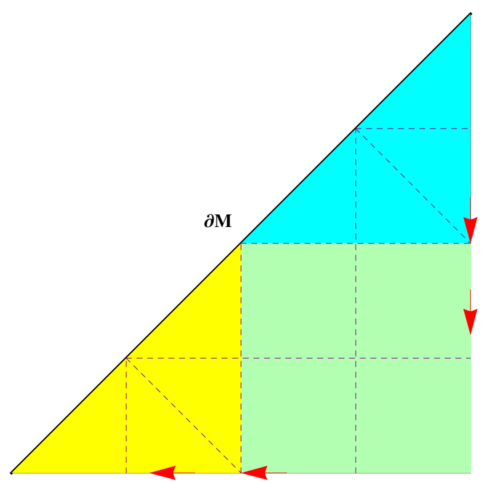

FIGURE 3. The ergodic decomposition and Bernoulli partitions of F_2^{*2} on $\mathbb{M}$

There is an analogous formula for each $d \geq 3$, and we describe the graph here. For $x \in [0, \frac{1}{2}]$, reflect any segment of the graph of $f_d(x) = dx \pmod 1$ which lies above the $y = 1/2$ line across that line, and for $x \in \left(\frac{1}{2}, 1\right]$, reflect any segment below the line $y = 1/2$ across that line. The graphs for $d = 2$ and $d = 3$ are illustrated in Figure 2. Evidently F_d satisfies the properties in Equations (3.6)–(3.9), making G well-defined on nP.

The map F_d has two ergodic components (the intervals $\left[0, \frac{1}{2}\right)$ and $\left[\frac{1}{2}, 1\right)$), and the map F_d^{*2} on the symmetric product $I^{*2} \cong \mathbb{M}$ (the Mobius band) has three ergodic components as shown in Figure 3; each color represents an ergodic component and a generating Bernoulli partition is shown within each ergodic component.

The revised Theorems 4.1 and 4.2 are as follows; the proofs are as in [1], but instead we use the maps F_d given here and make obvious minor modifications.

Theorem 4.1 (Revised). *Given any nonorientable compact surface X of genus $n \geq 2$, there exists a map $G : X \to X$ which is locally Lipschitz on X (Lipschitz in each coordinate chart), continuous, and smooth except on a finite number of curves, and satisfying:*

(i) *G preserves a smooth probability measure m_n on X.*
(ii) *G has three ergodic components with respect to m_n.*
(iii) *The restriction of G to each ergodic component is isomorphic to an n-point extension of a one-sided Bernoulli shift.*
(iv) *On each ergodic component G is transitive and chaotic, but not topologically exact.*
(v) $h_{top}(G) = 2 \log d$.

Theorem 4.2 (Revised). *Suppose $(\mathbb{S}^1, \mathcal{B}, m, f)$ is any nonsingular d-to-one dynamical system satisfying the following conditions:*

(1) *f is continuous on $\mathbb{S}^1$ and differentiable except at finitely many points.*
(2) *f is topologically exact.*
(3) *f is weak mixing.*
(4) *In additive coordinates, $f(1-x) = 1 - f(x)$ for all $x \in [0,1]$ and $f([0,1/2]) = [0,1/2]$.*

Then for any nonorientable compact surface X of genus > 1, f defines a d^2-to-one nonsingular map G on X with respect to a smooth measure μ, has at most three ergodic and chaotic components and G is continuous and differentiable μ-a.e.

At the end of Section 4 in [1] we show how the measure theoretic entropy of the examples we construct can be reduced; this still holds with a slight variation on the examples T_p given. By reflecting the inner two line segments of the graph shown in Figure 8 of [1] about the line $y = 1/2$, we can still obtain G_p of arbitrarily small entropy as claimed.

References

[1] Goodman, Sue; Hawkins, Jane. Ergodic and chaotic properties of Lipschitz maps on smooth surfaces. *New York J. Math.* **18** (2012), 95–120. Zbl 06032704.

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This paper is available via http://nyjm.albany.edu/j/2012/18-30.html.