

ABSTRACT. We prove a version of Koszul duality and the induced derived equivalence for Adams connected A_∞ -algebras that generalizes the classical Beilinson–Ginzburg–Soergel Koszul duality. As an immediate consequence, we give a version of the Bernstein–Gel’fand–Gel’fand correspondence for Adams connected A_∞ -algebras.

We give various applications. For example, a connected graded algebra A is Artin–Schelter regular if and only if its Ext-algebra $\text{Ext}_A^*(k, k)$ is Frobenius. This generalizes a result of Smith in the Koszul case. If A is Koszul and if both A and its Koszul dual $A^!$ are noetherian satisfying a polynomial identity, then A is Gorenstein if and only if $A^!$ is. The last statement implies that a certain Calabi–Yau property is preserved under Koszul duality.