

ABSTRACT. We consider multiple tilings of \mathbb{Z} by translates of a finite multiset A of integers (called a *tile*). We say that a set of integers T is an *A -tiling of level d* if each integer can be written in exactly d ways as the sum of an element of T and an element of A . We find new exponential lower bounds on the longest period of A -tiling as a function of the diameter of A , which rejoin the exponential upper bounds given by Ruzsa (preprint, 2002) and Kolountzakis (2003). We also show the existence of tiles whose level semigroups have arbitrarily many generators (where the *level semigroup* of a tile A is the set of integers d such that A admits a tiling of level d).