ABSTRACT. Let M be a compact manifold and D a Dirac type differential operator on M. Let A be a C^*-algebra. Given a bundle W (with connection) of A-modules over M, the operator D can be twisted with this bundle. One can then use a trace on A to define numerical indices of this twisted operator. We prove an explicit formula for these indices. Our result does complement the Mishchenko–Fomenko index theorem valid in the same situation. We establish generalizations of these explicit index formulas if the trace is only defined on a dense and holomorphically closed subalgebra $\mathcal{B}$.

As a corollary, we prove a generalized Atiyah L^2-index theorem if the twisting bundle is flat.

There are actually many different ways to define these numerical indices. From their construction, it is not clear at all that they coincide. A substantial part of the paper is a complete proof of their equality. In particular, we establish the (well-known but not well-documented) equality of Atiyah's definition of the L^2-index with a K-theoretic definition.

In case A is a von Neumann algebra of type 2, we put special emphasis on the calculation and interpretation of the center valued index. This completely contains all the K-theoretic information about the index of the twisted operator.

Some of our calculations are done in the framework of bivariant KK-theory.