

ABSTRACT. Let  $\Gamma$  be a discrete group which is a split extension of a group  $\Delta$  by a Coxeter group  $W$ , with  $\Delta$  acting on  $W$  by Coxeter graph automorphisms with kernel  $\Delta_0$ . Let  $M_i$ ,  $i = 1, 2$ , be two  $\Gamma$ -manifolds (possibly with boundary) such that the isotropy groups are finite and the fixed point sets are contractible and  $W$  acts by reflections. Let  $f$  be a  $\Gamma$ -homotopy equivalence between them that it is a homeomorphism outside the orbit of a compact subset. Then  $f$  is  $\Gamma$ -homotopic to a  $\Gamma$ -homeomorphism, provided that certain finite extensions of  $\Delta_0$  that fix the faces of the fundamental domains are topologically rigid groups.