

ABSTRACT. The map  $\phi(x, y) = (\sqrt{1+x^2} - y, x)$  of the plane is area preserving and has the remarkable property that in numerical studies it shows exact integrability: The plane is a union of smooth, disjoint, invariant curves of the map  $\phi$ . However, the integral has not explicitly been known. In the current paper we will show that the map  $\phi$  does not have an algebraic integral, i.e., there is no non-constant function  $F(x, y)$  such that

1.  $F \circ \phi = F$ ;
2. There exists a polynomial  $G(x, y, z)$  of three variables with

$$G(x, y, F(x, y)) = 0.$$

Thus, the integral of  $\phi$ , if it does exist, will have complicated singularities. We also argue that if there is an analytic integral  $F$ , then there would be a dense set of its level curves which are algebraic, and an uncountable and dense set of its level curves which are not algebraic.