

ABSTRACT. The concept of the index of a vector field is one of the oldest in Algebraic Topology. First stated by Poincare and then perfected by Heinz Hopf and S. Lefschetz and Marston Morse, it is developed as the sum of local indices of the zeros of the vector field, using the idea of degree of a map and initially isolated zeros. The vector field must be defined everywhere and be continuous. A key property of the index is that it is invariant under proper homotopies.

In this paper we extend this classical index to vector fields which are not required to be continuous and are not necessarily defined everywhere. In this more general situation, proper homotopy corresponds to a new concept which we call proper otopy. Not only is the index invariant under proper otopy, but the index classifies the proper otopy classes. Thus two vector fields are properly otopic if and only if they have the same index. This allows us to go back to the continuous case and classify globally defined continuous vector fields up to proper homotopy classes. The concept of otopy and the classification theorems allow us to define the index for space-like vector fields on Lorentzian space-time where it becomes an invariant of general relativity.