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ON COUPLED RANDOM FIXED POINT RESULTS IN PARTIALLY ORDERED METRIC SPACES

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Abstract. We prove a coupled random coincidence and a coupled random fixed point theorem under a set of conditions. Our result is a generalization of the recent result of Ćirić and Lakshmikantham [Stochastic Analysis and Applications 27:6 (2009), 1246–1259].

1. Introduction

Random fixed point theorems are stochastic generalizations of classical fixed point theorems. Several authors studied many random fixed point results on separable complete metric spaces (see [6, 7, 9, 10, 15–17, 20]). The study of fixed point results in partially ordered metric spaces is at the center of activity research for the importance of this subject in differential equations. For research in partially ordered metric spaces, we refer the reader to [1–5, 11, 13, 14, 19]. V. Bhaskar and Lakshmikantham [5] introduced the concept of a coupled coincidence point of a mapping F from $X \times X$ into X and a mapping g from X into X and studied fixed point theorems in partially ordered metric spaces. Shatanawi [18] extended the results of Bhaskar and Lakshmikantham to partially ordered cone metric spaces. In [11] V. Lakshmikantham and L. Ćirić studied some fixed point theorems for nonlinear contractions in partially ordered metric spaces. Recently, L. Ćirić and Lakshmikantham [8] studied two coupled random coincidence and coupled random fixed point theorems for a pair of random mappings $F : \Omega \times (X \times X) \to X$ and $g : \Omega \times X \to X$ under some contractive conditions. The aim of this paper is to study coupled random fixed point results in a more general form of L. Ćirić and Lakshmikantham. Our result generalizes Theorem 2.2 of L. Ćirić and Lakshmikantham results [8].

2. Basic concepts

Let $(X, \preceq)$ be a partially ordered set. The concept of a mixed monotone property of the mapping $F : X \times X \to X$ has been introduced by Bhaskar and Lakshmikantham in [5].

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Definition 2.1. [5] Let $(X, \preceq)$ be a partially ordered set and $F : X \times X \to X$. Then the map F is said to have mixed monotone property if $F(x, y)$ is monotone non-decreasing in x and is monotone non-increasing in y; that is, for any $x, y \in X$,

$$x_1 \preceq x_2 \text{ implies } F(x_1, y) \preceq F(x_2, y) \text{ for all } y \in X$$

and

$$y_1 \preceq y_2 \text{ implies } F(x, y_2) \preceq F(x, y_1) \text{ for all } x \in X.$$

Inspired by Definition 2.1, Lakshmikantham and Ćirić in [11] introduced the concept of a g-mixed monotone mapping.

Definition 2.2. [11] Let $(X, \preceq)$ be a partially ordered set and $F : X \times X \to X$. Then the map F is said to have mixed g-monotone property if $F(x, y)$ is monotone g-non-decreasing in x and is monotone g-non-increasing in y; that is, for any $x, y \in X$,

$$gx_1 \preceq gx_2 \text{ implies } F(x_1, y) \preceq F(x_2, y) \text{ for all } y \in X$$

and

$$gy_1 \preceq gy_2 \text{ implies } F(x, y_2) \preceq F(x, y_1) \text{ for all } x \in X.$$

Definition 2.3. [5] An element $(x, y) \in X \times X$ is called a coupled fixed point of a mapping $F : X \times X \to X$ if

$$F(x, y) = x \text{ and } F(y, x) = y.$$

Definition 2.4. [11] An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings $F : X \times X \to X$ and $g : X \to X$ if

$$F(x, y) = gx \text{ and } F(y, x) = gy.$$

Definition 2.5. Let X be a nonempty set. Then we say that the mappings $F : X \times X \to X$ and $g : X \to X$ are commutative if

$$gF(x, y) = F(gx, gy).$$

Let (Ω, Σ) be a measurable space with Σ a sigma algebra of subsets of Ω and let (X, d) be a metric space. A mapping $T : \Omega \to X$ is called measurable if for any open subset U of X, $T^{-1}(U) = \{\omega : T(\omega) \in U\} \in \Sigma$. A mapping $T : \Omega \times X \to X$ is called a random operator if for any $x \in X$, $T(\cdot, x)$is measurable.

Definition 2.6. A measurable mapping $\zeta : \Omega \to X$ is called a random coincidence of $T : \Omega \times X \to X$ if $\zeta(\omega) = T(\omega, \zeta(\omega))$for every $\omega \in \Omega$. A measurable mapping $\zeta : \Omega \to X$ is called a random coincidence of $T : \Omega \times X \to X$ and $g : \Omega \to X$ if $g(\omega, \zeta(\omega)) = T(\omega, \zeta(\omega))$ for every $\omega \in \Omega$.

Ćirić and Lakshmikantham in [8] proved the following theorem.

Theorem 2.1. [8, Theorem 2.2] *Let* $(X, \preceq)$ *be a partially ordered set,* (X, d) *be a complete separable metric space, and* (Ω, Σ) *be a measurable space. Let* F :

$\Omega\times(X\times X)\to X$ and $g:\Omega\times X\to X$ be mappings such that there is a nonnegative real number k with

$$d(F(\omega,(x,y)),F(\omega,(u,v)))\preceq\frac{k}{2}(d(g(\omega,x),g(\omega,u))+d(g(\omega,y),g(\omega,v)))$$

for all $x,y,u,v\in X$ with $g(\omega,x)\preceq g(\omega,u)$ and $g(\omega,v)\preceq g(\omega,y)$ for all $\omega\in\Omega$. Assume that F and g satisfies the following conditions:

1. *$F(\omega,\cdot)$, $g(\omega,\cdot)$ are continuous for all $\omega\in\Omega$,*
2. *$F(\cdot,v)$, $g(\cdot,x)$ are measurable for all $v\in X\times X$ and $x\in X$, respectively,*
3. *$F(\omega\times X)\subseteq X$ for each $\omega\in\Omega$*
4. *g is continuous and commutes with F and also suppose either*
   (a) *F is continuous or*
   (b) *X has the following properties:*
      i. *if a non-decreasing sequence $x_n\to x$, then $x_n\preceq x$ for all n,*
      ii. *if a non-increasing sequence $x_n\to x$, then $x\preceq x_n$ for all n.*

If there exist measurable mappings $\eta_0,\theta_0\in X$ such that $g(\omega,\eta_0)\preceq F(\omega,(\eta_0,\theta_0))$ and $F(\omega,(\eta_0(\omega),\theta_0(\omega)))\preceq g(\omega,\theta_0(\omega))$ and if $k\in[0,1)$, then there are measurable mappings $\eta,\theta:\Omega\to X$ such that $F(\omega,(\eta(\omega),\theta(\omega)))=g(\omega,\eta(\omega))$ and $F(\omega,(\eta(\omega),\theta(\omega)))=g(\omega,\theta(\omega))$ for all $\omega\in\Psi$, that is, F and g have a coupled random coincidence.

3. Main results

In this section, we study random version of a coupled random fixed point theorem for a pair of random mappings $F:\Omega\times(X\times X)\to X$ and $g:\Omega\times X\to X$ under a set of conditions.

THEOREM 3.1. *Let $(X,\preceq)$ be a partially ordered set, (X,d) be a complete separable metric space, and (Ω,Σ) be a measurable space. Let $F:\Omega\times(X\times X)\to X$ and $g:\Omega\times X\to X$ be mappings such that there are two nonnegative real numbers a and b with*

$$d(F(\omega,(x,y)),F(\omega,(u,v)))\preceq ad(g(\omega,x),g(\omega,u))+bd(g(\omega,y),g(\omega,v)) \qquad (1)$$

for all $x,y,u,v\in X$ with $g(\omega,x)\preceq g(\omega,u)$ and $g(\omega,v)\preceq g(\omega,y)$ for all $\omega\in\Omega$. Assume that F and g satisfies the following conditions:

1. *$F(\omega,\cdot)$, $g(\omega,\cdot)$ are continuous for all $\omega\in\Omega$,*
2. *$F(\cdot,v)$, $g(\cdot,x)$ are measurable for all $v\in X\times X$ and $x\in X$, respectively,*
3. *$F(\omega\times X)\subseteq X$ for each $\omega\in\Omega$*
4. *g is continuous and commutes with F and also suppose either*
   (a) *F is continuous or*
   (b) *X has the following properties:*
      i. *if a non-decreasing sequence $x_n\to x$, then $x_n\preceq x$ for all n,*

ii. if a non-increasing sequence $x_n \to x$, then $x \preceq x_n$ for all n.

If there exist measurable mappings $\eta_0, \theta_0 \in X$ such that $g(\omega, \eta_0) \preceq F(\omega, (\eta_0, \theta_0))$ and $F(\omega, (\eta_0(\omega), \theta_0(\omega))) \preceq g(\omega, \theta_0(\omega))$ and if $a + b \in [0, 1)$, then there are measurable mappings $\eta, \theta : \Omega \to X$ such that $F(\omega, (\eta(\omega), \theta(\omega))) = g(\omega, \eta(\omega))$ and $F(\omega, (\eta(\omega), \theta(\omega))) = g(\omega, \theta(\omega))$ for all $\omega \in \Psi$, that is, F and g have a coupled random coincidence.

Proof. Let $\Theta = \{\eta : \Omega \to X\}$ be a family of measurable mappings. Define a function $h : \Omega \times X \to \mathbf{R}^+$ as follows $h(w, x) = d(x, g(w, x))$. Since $x \to g(\omega, x)$ is continuous for all $\omega \in \Omega$, we conclude that $h(\omega, \cdot)$ is continuous for all $\omega \in \Omega$. Also, since $\omega \to g(\omega, x)$ is measurable for all $x \in X$, we conclude that $h(\cdot, x)$ is measurable for all $\omega \in \Omega$ (see Wangner [20, p. 868]). Thus, $h(\omega, x)$ is the Caratheodory function. Thus, if $\eta : \Omega \to X$ is measurable mapping, then $\omega \to h(\omega, \eta(\omega))$ is also measurable (see [16]). Also, for each $\theta \in \Theta$ the function $\eta : \Omega \to X$ defined by $\eta(\omega) = g(w, \theta(\omega))$ is measurable; that is, $\eta \in \Theta$. Now we are going to construct two sequences of measurable mappings (ζ_n) and (η_n) in Θ and two sequences $(g(\omega, \zeta_n(\omega)))$ and $(g(\omega, \eta_n(\omega)))$ in X as follows. Let $\zeta_0, \eta_0 \in \Theta$ be such that

$$g(\omega, \zeta_0(\omega)) \preceq F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \quad \text{and} \quad g(\omega, \eta_0(\omega)) \succeq F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$$

for all $\omega \in \Omega$. Since $F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \in X = g(\omega \times X$, by a sort of Filippov measurable implicit function theorem (see [4, 9, 10, 12]), there is $\zeta_1 \in \Theta$ such that $g(\omega, \zeta_1(\omega)) = F(\omega, (\zeta_0(\omega), \eta_0(\omega)))$. Similarly, as $F(\omega, (\eta_0(\omega), \zeta_0(\omega))) \in g(\omega \times X$, there is $\eta_1 \in \Theta$ such that $g(\omega, \eta_1(\omega)) = F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$. Thus $F(\omega, (\zeta_0(\omega), \eta_0(\omega)))$ and $F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$ are well defined now. Again, since

$$F(\omega, (\zeta_1(\omega), \eta_1(\omega))), F(\omega, (\eta_1(\omega), \zeta_1(\omega))) \in g(\omega \times X,$$

there are $\zeta_2, \eta_2 \in \Theta$ such that

$$g(\omega, \zeta_2(\omega)) = F(\omega, (\zeta_1(\omega), \eta_1(\omega))) \quad \text{and} \quad g(\omega, \eta_2(\omega)) = F(\omega, (\eta_1(\omega), \zeta_1(\omega))).$$

Continuing this process we can construct sequences $(\zeta_n(\omega))$ and $(\eta_n(\omega))$ in X such that

$$g(\omega, \zeta_{n+1}(\omega)) = F(\omega, (\zeta_n(\omega), \eta_n(\omega))) \quad \text{and} \quad g(\omega, \eta_{n+1}(\omega)) = F(\omega, (\eta_n(\omega), \zeta_n(\omega))) \tag{2}$$

for all $n \in \mathbf{N} \cup \{0\}$. Now, we use mathematical induction to prove that

$$g(\omega, \zeta_n(\omega)) \preceq g(\omega, \zeta_{n+1}(\omega)) \quad \text{and} \quad g(\omega, \eta_n(\omega)) \succeq g(\omega, \eta_{n+1}(\omega)) \tag{3}$$

for all $n \in \mathbf{N} \cup \{0\}$.

Let $n = 0$. By assumption we have

$$g(\omega, \zeta_0(\omega)) \preceq F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \quad \text{and} \quad g(\omega, \eta_0(\omega)) \succeq F(\omega, (\eta_0(\omega), \zeta_0(\omega))).$$

Since

$$g(\omega, \zeta_1(\omega)) = F(\omega, (\zeta_0(\omega), \eta_0(\omega))) \quad \text{and} \quad g(\omega, \eta_1(\omega)) = F(\omega, (\eta_0(\omega), \zeta_0(\omega)))$$

we have

$$g(\omega,\zeta_0(\omega)) \preceq g(\omega,\zeta_1(\omega)) \quad \text{and} \quad g(\omega,\eta_0(\omega)) \succeq g(\omega,\eta_1(\omega)).$$

Therefore (3) holds for $n=0$. Suppose (3) holds for some fixed $n \geq 0$. Then, since

$$g(\omega,\zeta_n(\omega)) \preceq g(\omega,\zeta_{n+1}(\omega)), \quad g(\omega,\eta_n(\omega)) \succeq g(\omega,\eta_{n+1}(\omega))$$

and F is monotone g-non-decreasing in its first argument, we have

$$F(\omega,(\zeta_n(\omega),\eta_n(\omega))) \preceq F(\omega,(\zeta_{n+1}(\omega),\eta_n(\omega))), \tag{4}$$

and

$$F(\omega,(\eta_{n+1}(\omega),\zeta_n(\omega))) \preceq F(\omega,(\eta_n(\omega),\zeta_n(\omega))). \tag{5}$$

Also, since

$$g(\omega,\zeta_n(\omega)) \preceq g(\omega,\zeta_{n+1}(\omega)), \quad g(\omega,\eta_n(\omega)) \succeq g(\omega,\eta_{n+1}(\omega))$$

and F is monotone g-non-decreasing in its second argument, we have

$$F(\omega,(\zeta_{n+1}(\omega),\eta_{n+1}(\omega))) \succeq F(\omega,(\zeta_{n+1}(\omega),\eta_n(\omega))), \tag{6}$$

and

$$F(\omega,(\eta_{n+1}(\omega),\zeta_n(\omega))) \succeq F(\omega,(\eta_{n+1}(\omega),\zeta_{n+1}(\omega))). \tag{7}$$

Thus, from (4)–(7), we get

$$g(\omega,\zeta_{n+1}(\omega)) \preceq g(\omega,\zeta_{n+1}(\omega)) \quad \text{and} \quad g(\omega,\eta_{n+1}(\omega)) \succeq g(\omega,\eta_{n+2}(\omega)).$$

Thus by mathematical induction we conclude that (3) holds for all $n \in \mathbf{N} \cup \{0\}$.

Now, we prove that $(g(\omega,\zeta_n(\omega)))$ and $(g(\omega,\eta_n(\omega)))$ are Cauchy sequences. Let $n \in \mathbf{N} \cup \{0\}$. Then by (1)–(3), we have

$$\begin{aligned} d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega))) &= d(F(\omega,(\zeta_{n-1}(\omega),\eta_{n-1}(\omega))),F(\omega,(\zeta_n(\omega),\eta_n(\omega)))) \\ &\leq ad(g(w,\zeta_{n-1}(\omega)),g(\omega,\zeta_n(\omega))) + bd(g(\omega,\eta_{n-1}(\omega)),g(\omega,\eta_n(\omega))) \end{aligned}$$

and

$$\begin{aligned} d(g(\omega,\eta_{n+1}(\omega)),g(\omega,\eta_n(\omega))) &= d(F(\omega,(\eta_n(\omega),\zeta_n(\omega))),F(\omega,(\eta_{n-1}(\omega),\zeta_{n-1}(\omega)))) \\ &\leq ad(g(w,\eta_{n-1}(\omega)),g(\omega,\eta_n(\omega))) + bd(g(\omega,\zeta_{n-1}(\omega)),g(\omega,\zeta_n(\omega))). \end{aligned}$$

By combining the above inequalities, we have

$$\begin{aligned} &d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_{n+1}(\omega))) + d(g(\omega,\eta_{n+1}(\omega)),g(\omega,\eta_n(\omega)) \\ &\quad \leq (a+b)(d(g(w,\zeta_{n-1}(\omega)),g(\omega,\zeta_n(\omega))) + d(g(\omega,\eta_{n-1}(\omega)),g(\omega,\eta_n(\omega))) \\ &\quad \leq (a+b)^2(d(g(w,\zeta_{n-2}(\omega)),g(\omega,\zeta_{n-1}(\omega))) + d(g(\omega,\eta_{n-2}(\omega)),g(\omega,\eta_{n-1}(\omega))) \\ &\quad\quad \vdots \\ &\quad \leq (a+b)^n(d(g(w,\zeta_0(\omega)),g(\omega,\zeta_1(\omega))) + d(g(\omega,\eta_0(\omega)),g(\omega,\eta_1(\omega))). \end{aligned}$$

Let $m,n\in\mathbf{N}$ with $m>n$. Since

$$d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_m(\omega)))+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_m(\omega))\\ \leq\sum_{i=n}^{m-1}d(g(\omega,\zeta_i(\omega)),g(\omega,\zeta_{i+1}(\omega)))+d(g(\omega,\eta_i(\omega)),g(\omega,\eta_{i+1}(\omega))$$

and $a+b<1$, we have

$$d(g(\omega,\zeta_n(\omega)),g(\omega,\zeta_m(\omega)))+d(g(\omega,\eta_n(\omega)),g(\omega,\eta_m(\omega))\\ \leq\frac{(a+b)^n}{1-a-b}(d(g(\omega,\zeta_0(\omega)),g(\omega,\zeta_1(\omega)))+d(g(\omega,\eta_0(\omega)),g(\omega,\eta_1(\omega)).$$

Letting $n\to+\infty$, we conclude that $(g(\omega,\zeta_n(\omega)))$ and $(g(\omega,\eta_n(\omega)))$ are Cauchy sequences in X.

Since X is complete and $g(\omega\times X)=X$, there exist $\zeta_0,\theta_0\in\Theta$ such that

$$\lim_{n\to+\infty}g(\omega,\zeta_n(\omega))=g(\omega,\zeta_0(\omega))\quad\text{and}\quad\lim_{n\to+\infty}g(\omega,\eta_n(\omega))=g(\omega,\theta_0(\omega)).$$

Define $\zeta,\theta:\Omega\to X$ by $\zeta(\omega)=g(\omega,\zeta_0(\omega))$ and $\theta(\omega)=g(\omega,\theta_0(\omega))$. Since $g(\omega,\zeta_0(\omega))$ and $g(\omega,\theta_0(\omega))$ are measurable, then the functions $\zeta(\omega)$ and $\theta(\omega)$ are measurable. Thus, we have

$$\lim_{n\to+\infty}g(\omega,\zeta_n(\omega))=\zeta(\omega\quad\text{and}\quad\lim_{n\to+\infty}g(\omega,\eta_n(\omega))=\theta(\omega)).\tag{8}$$

From (8) and continuity of g, we have

$$\lim_{n\to+\infty}g(\omega,g(\omega,\zeta_n(\omega)))=g(\omega,\zeta(\omega))\ \text{and}\ \lim_{n\to+\infty}g(\omega,g(\omega,\eta_n(\omega)))=g(\omega,\theta(\omega))).$$

By using the fact that F and g are commutative, from (2) we have

$$F(\omega,(g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega)))=g(\omega,F(\omega,\zeta_n(\omega),\eta_n(\omega)))=g(\omega,g(\omega,\zeta_{n+1}(\omega)))$$

and

$$F(\omega,(g(\omega,\eta_n(\omega)),g(\omega,\zeta_n(\omega)))=g(\omega,F(\omega,\eta_n(\omega),\zeta_n(\omega)))=g(\omega,g(\omega,\eta_{n+1}(\omega)))$$

Suppose F is continuous. Then

$$\begin{aligned}g(\omega,\zeta(\omega))&=\lim_{n\to+\infty}g(\omega,g(\omega,\zeta_{n+1}(\omega)))\\&=\lim_{n\to+\infty}F(\omega,(g(\omega,\zeta_n(\omega)),g(\omega,\eta_n(\omega))))\\&=F(\omega,(\lim_{n\to+\infty}g(\omega,\zeta_n(\omega)),\lim_{n\to+\infty}g(\omega,\eta_n(\omega)))\\&=F(\omega,(\zeta(\omega),\theta(\omega)))\end{aligned}$$

and

$$\begin{aligned}g(\omega,\theta(\omega))&=\lim_{n\to+\infty}g(\omega,g(\omega,\eta_{n+1}(\omega)))\\&=\lim_{n\to+\infty}F(\omega,(g(\omega,\eta_n(\omega)),g(\omega,\zeta_n(\omega))))\\&=F(\omega,(\lim_{n\to+\infty}g(\omega,\eta_n(\omega)),\lim_{n\to+\infty}g(\omega,\zeta_n(\omega)))\\&=F(\omega,(\theta(\omega),\zeta(\omega)))\end{aligned}$$

From above equalities, we deduce that $(\zeta(\omega), \theta(\omega)) \in X \times X$ is a coupled random coincidence of F and g.

Suppose (b) holds. From (3), the sequence $(g(\omega, \zeta_n))$ is nondecreasing and the sequence $(g(\omega, \eta_n))$ is non-increasing. Since X satisfies (b), we have $g(\omega, \zeta_n(\omega)) \preceq g(\omega, \zeta(\omega))$ and $g(\omega, \eta_n(\omega)) \succeq g(\omega, \theta(\omega))$. Thus

$$\begin{aligned} d(g(\omega,\zeta(\omega)), F(\omega,(\zeta(\omega),\theta(\omega)))) \\ &\leq d(g(\omega,\zeta(\omega)), g(\omega,\zeta_n(\omega))) + d(g(\omega,\zeta_n(\omega)), F(\omega,(\zeta(\omega),\theta(\omega)))) \\ &= d(g(\omega,\zeta(\omega)), g(\omega,\zeta_n(\omega))) + d(F(\omega,(\zeta_{n-1}(\omega),\eta_{n-1}(\omega))), F(\omega,(\zeta(\omega),\theta(\omega)))) \\ &\leq d(g(\omega,\zeta(\omega)), g(\omega,\zeta_n(\omega))) + ad(g(\omega,\zeta_{n-1}(\omega)), g(\omega,\zeta(\omega))) \\ &\qquad + bd(g(\omega,\eta_{n-1}(\omega)), g(\omega,\theta(\omega))). \end{aligned}$$

By letting $n \to +\infty$, we conclude that $d(g(\omega,\zeta(\omega)), F(\omega,(\zeta(\omega),\theta(\omega)))) = 0$. Hence $g(\omega,\zeta(\omega)) = F(\omega,(\zeta(\omega),\theta(\omega)))$. Similarly, we can show that $g(\omega,\theta(\omega)) = F(\omega,(\theta(\omega),\zeta(\omega)))$. Thus we prove that $(\zeta(\omega),\theta(\omega)) \in X \times X$ is a coupled random coincidence of F and g. ■

Corollary 3.1. *Let $(X, \preceq)$ be a partially ordered set, (X, d) be a complete separable metric space, and (Ω, Σ) be a measurable space. Let $F : \Omega \times (X \times X) \to X$ and $g : \Omega \times X \to X$ be mappings such that there is a nonnegative real number k with*

$$d(F(\omega,(x,y)), F(\omega,(u,v))) \preceq \frac{k}{2}(d(g(\omega,x), g(\omega,u)) + d(g(\omega,y), g(\omega,v)))$$

for all $x, y, u, v \in X$ with $g(\omega, x) \preceq g(\omega, u)$ and $g(\omega, v) \preceq g(\omega, y)$ for all $\omega \in \Omega$. Assume that F and g satisfies the following conditions:

1. *$F(\omega, \cdot)$, $g(\omega, \cdot)$ are continuous for all $\omega \in \Omega$,*
2. *$F(\cdot, v)$, $g(\cdot, x)$ are measurable for all $v \in X \times X$ and $x \in X$, respectively,*
3. *$F(\omega \times X) \subseteq X$ for each $\omega \in \Omega$*
4. *g is continuous and commutes with F and also suppose either*
    (a) *F is continuous or*
    (b) *X has the following properties:*
        i. *if a non-decreasing sequence $x_n \to x$, then $x_n \preceq x$ for all n,*
        ii. *if a non-increasing sequence $x_n \to x$, then $x \preceq x_n$ for all n.*

If there exist measurable mappings $\eta_0, \theta_0 \in X$ such that $g(\omega, \eta_0) \preceq F(\omega, (\eta_0, \theta_0))$ and $F(\omega, (\eta_0(\omega), \theta_0(\omega))) \preceq g(\omega, \theta_0(\omega))$ and if $k \in [0, 1)$, then there are measurable mappings $\eta, \theta : \Omega \to X$ such that $F(\omega, (\eta(\omega), \theta(\omega))) = g(\omega, \eta(\omega))$ and $F(\omega, (\eta(\omega), \theta(\omega))) = g(\omega, \theta(\omega))$ for all $\omega \in \Psi$, that is, F and g have a coupled random coincidence.

Proof. Follows from Theorem 3.1 by taking $a = b = \frac{k}{2}$. ■

Remark. Our Theorem 3.1 is a generalization of Theorem 2.2 of [8].

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