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SPECTRAL PROBLEMS IN LIPSCHITZ DOMAINS IN SOBOLEV-TYPE BANACH SPACES

Dedicated to Roland Duduchava
with best wishes in connection with his jubilee

Abstract. This paper contains a short presentation of author's results on spectral properties of main boundary value problems for strongly elliptic second-order systems in bounded Lipschitz domains. We consider the questions on the completeness of root functions, on the summability of Fourier series with respect to them and on their basis property in spaces H_p^s with indices s, p close to $\pm 1, 2$. The complete presentation will be published elsewhere.

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რეზიუმე. ამ სტატიაში მოკლედ არის გადმოცემული ავტორის შედეგები, რომლებიც ეხება ძლიერ ელიფსური მეორე რიგის სისტემებისთვის შემოსაზღვრულ ლიფშიცის არეებში დასმული მთავარი სასაზღვრო ამოცანების სპექტრალურ თვისებებს. ჩვენ განვიხილავთ საკითხებს რომლებიც ეხება ფესვი ფუნქციების სისრულეს, მათ მიმართ ფურიეს მწკრივების კრებადობასა და მათ ძირითადი თვისებებს H_p^s

1. Let Ω be a bounded domain in $\mathbb{R}^n$, $n \geq 2$, with Lipschitz boundary Γ. Assume that we have a matrix strongly elliptic [16] second-order operator

$$Lu := -\sum_{j,k=1}^{n} \partial_j a_{j,k} \partial_k u + \sum_{j=1}^{n} b_j \partial_j u + cu$$

in Ω with complex-valued coefficients of small smoothness (in particular, with Lipschitz higher-order coefficients). The form

$$\Phi(u, v) = \int_\Omega \Big[\sum a_{j,k} \partial_k u \cdot \partial_j \overline{v} + \sum b_j \partial_j u \cdot \overline{v} + cu \cdot \overline{v} \Big] \, dx$$

is associated with L. We first consider the Dirichlet and Neumann problems in a weak sense for the equation $Lu = f$ with homogeneous boundary conditions. Solutions are defined by the Green formula

$$(Lu, v)_\Omega = \Phi(u, v). \tag{1}$$

In the simplest setting, in the Dirichlet problem

$$u, v \in \overset{\circ}{H}{}^1(\Omega) = \widetilde{H}^1(\Omega), \quad Lu = f \in H^{-1}(\Omega),$$

and in the Neumann problem

$$u, v \in H^1(\Omega) = W_2^1(\Omega), \quad Lu = f \in \widetilde{H}^{-1}(\Omega).$$

(The definitions of more general spaces can be seen in Section 2 below.) In such a generality, the Green formula is postulated. The functions f and u, v belong to spaces dual with respect to a continuation of the standard inner product in $L_2(\Omega)$

$$(u, v)_\Omega = \int_\Omega u \cdot \overline{v} \, dx.$$

The bounded operators

$$L_D : \widetilde{H}^1(\Omega) \longrightarrow H^{-1}(\Omega) \ \text{ and } \ L_N : H^1(\Omega) \longrightarrow \widetilde{H}^{-1}(\Omega)$$

correspond to these problems. The domains of these operators are compactly and densely embedded in the right-hand spaces. We wish to consider spectral properties of these operators. We assume that the form Φ is *coercive*:

$$\|u\|^2_{H^1(\Omega)} \leq C_1 \operatorname{Re} \Phi(u, u) + C_2 \|u\|^2_{L_2(\Omega)}. \tag{2}$$

In the Dirichlet problem, the coerciveness is needed only on $\overset{\circ}{H}{}^1(\Omega)$ and follows from the strong ellipticity, For the Neumann problem, the simple sufficient conditions are known, fulfilled, in particular, for elasticity systems (see e.g. [2, Section 11]).

The last term in (2) can be removed by using a shift of the spectral parameter. After this, we have the *strong coercivity of* Φ. Below *it is assumed*. From it, the *invertibility* of the operators L_D and L_N follows by *the Lax–Milgram theorem* (see e.g. [2, Section 18]). The same is true for the adjoint operators L_D^* and L_N^* defined by the operator L^* formally adjoint to L (in Ω or $\overline{\Omega}$, respectively, see [2, Section 11]) and the Green formula

$$\Phi(u, v) = (u, L^* v)$$

with the same Φ.

The inverse operators are compact. Hence L_D and L_N are the operators with a discrete spectrum in their ranges. Our main question is: when their root functions are *complete*, i.e. their finite linear combinations are dense (in the ranges and hence in the domains), or are "better".

For the problems in the simplest setting indicated above, there are simple tools for the investigation of the completeness since only Hilbert spaces are used in this setting. In particular, L can be a formally self-adjoint operator in Ω or $\overline{\Omega}$:

$$\Phi(u, v) = \overline{\Phi(v, u)}$$

for u, v in $\widetilde{H}^1(\Omega)$ or $H^1(\Omega)$, respectively. Then we take the form $\Phi(u, v)$ for the inner product in the domain of L_D or L_N, respectively. In the ranges, we introduce the corresponding inner product e.g. $\Phi(L_D^{-1} f, L_D^{-1} g)$ in the case of the Dirichlet problem. The operators become self-adjoint, and a unique orthogonal basis of eigenfunctions exists in the both spaces.

Here, elementary, but very important remark consists in the fact that we need *the inner product defined by the operator*.

The asymptotics of the eigenvalues λ_k of self-adjoint operators L_D and L_N in a Lipschitz domain is known [12]. Namely, if λ_k are enumerated in the non-decreasing order taking multiplicities into account, then, as for the smooth problems,

$$\lambda_k \sim c k^{\frac{n}{2}}$$

(even with a fairly good remainder estimate). For non-self-adjoint compact operators L_D^{-1} and L_N^{-1}, this implies the estimate of "s-numbers" (see [7, Chapter 2])

$$s_k \leq C k^{-\frac{n}{2}}. \tag{3}$$

We have also the completeness if L is a *weak perturbation of a formally self-adjoint operator* (i.e. a perturbation in terms of order not greater than 1).

A more general condition, sufficient for the completeness, gives the *Dunford–Schwartz theorem* which is formulated in terms of angles between rays on the complex plane from the origin with power estimate for the norm of the resolvent (see [9, Chapter XI]). We only formulate a corollary for our problems in the simplest spaces.

Denote by Λ_θ the closed sector on the complex plane of opening 2θ with bisector $\mathbb{R}_+$. By M_θ we denote the closure of the complement to Λ_θ. Let θ_0 be such that the values of $\Phi(u, u)$ (with zero boundary values for u in the case of the Dirichlet problem) are contained in Λ_{θ_0}. Obviously, it contains all eigenvalues of L_D or L_N.

Note that $\theta_0 < \frac{\pi}{2}$ and that $e^{i\alpha}\Phi$ is strongly coercive if $0 < \alpha < \frac{\pi}{2} - \theta_0$.

Proposition 1. *The root functions of the operators L_D and L_N are complete in their ranges and domains if*

$$\theta_0 < \frac{\pi}{n}\,. \tag{4}$$

The proof uses (3) and the optimal resolvent estimate in M_θ with θ a little greater than θ_0 (see (8) below), it is easily obtained in our simplest spaces, see [2, Section 11].

2. However, our problems can be considered in more general spaces H_p^s of *Bessel potentials*. (For $p = 2$, they are H^s.) We remind definitions and some facts from their theory (cf. [2, Sections 14]).

1. $H_p^s(\mathbb{R}^n) = \Lambda^{-s}L_p(\mathbb{R}^n)$ for $1 < p < \infty$, $s \in \mathbb{R}$, where $\Lambda^{-s} = F^{-1}(1+|\xi|^2)^{-s/2}F$ and F is the Fourier transform in the sense of distributions.
2. $H_p^s(\Omega)$ is the space of restrictions of elements in $H_p^s(\mathbb{R}^n)$ to Ω with inf-norm. For integers $s > 0$, they are the Sobolev spaces $W_p^s(\Omega)$.
3. $\widetilde{H}^s(\Omega)$ is the subspace in $H^s(\mathbb{R}^n)$ of elements supported in $\overline{\Omega}$.

We need to mention the following facts.

These spaces are separable and reflexive Banach spaces.

There is a universal bounded operator of continuation from $H_p^s(\Omega)$ to $H_p^s(\mathbb{R}^n)$ [13].

There is an operator of passage to the trace on Γ acting boundedly from $H_p^{s+\frac{1}{p}}(\Omega)$ to the Besov–Slobodetskii space $B_p^s(\Gamma) = W_p^s(\Omega)$ for $0 < s < 1$ (only) with a bounded right inverse.

The spaces $\widetilde{H}_p^s(\Omega)$ can be identified with $H_p^s(\Omega)$ for small $|s|$.

The spaces $H_p^s(\Omega)$ and $\widetilde{H}_{p'}^{-s}(\Omega)$ are dual. Here and below $\frac{1}{p} + \frac{1}{p'} = 1$.

We agree not to write Ω.

Now, in the Dirichlet problem

$$u \in \widetilde{H}_p^{\frac{1}{2}+s+\frac{1}{p}},\;\; f \in H_p^{-\frac{1}{2}+s-\frac{1}{p'}},\;\; v \in \widetilde{H}_{p'}^{\frac{1}{2}-s+\frac{1}{p'}},$$

and in the Neumann problem

$$u \in H_p^{\frac{1}{2}+s+\frac{1}{p}},\;\; f \in \widetilde{H}_p^{-\frac{1}{2}+s-\frac{1}{p'}},\;\; v \in H_{p'}^{\frac{1}{2}-s+\frac{1}{p'}}.$$

The solutions are defined by the same Green formula (1). The domains of the operators L_D and L_N: $u \longmapsto f$ are again compactly and densely embedded in their ranges. The functions u and f belong to the spaces with difference of superscripts equal 2. The functions f and v belong to the dual spaces. But $|s| < \frac{1}{2}$ in view of the trace theorem, and the functions f and u are generally not in dual spaces; because of this fact, the Lax–Milgram theorem cannot be applied.

Instead, the remarkable *Shneiberg's theorem* from the interpolation theory of operators is applicable. See [14] or [2, Section 13]. This is a theorem on the extrapolation of the invertibility of operators. According to it, there exist some numbers $\varepsilon \in (0, \frac{1}{2}]$ and (small) $\delta > 0$ such that our problem (Dirichlet or Neumann) is uniquely solvable for $|s| < \varepsilon$, $|r - \frac{1}{2}| < \delta$, where $r = \frac{1}{p}$. Simultaneously, this is a statement on the smoothness of solutions. If L has a formally self-adjoint principal part, then, under an easy additional condition at the points near Γ, $\varepsilon = \frac{1}{2}$.

Let $Q_{\varepsilon,\delta}$ be the rectangle of corresponding points $(s, \frac{1}{p})$. For convenience, we assume that it is common for the Dirichlet and Neumann problem and that $\varepsilon > \delta$. Below, we will consider only $(s,t) \in Q_{\varepsilon,\delta}$.

What can be said about spectral properties of our operators in these Banach spaces? Spectral properties of problems in abstract Banach spaces were investigated by many mathematicians (Grothendieck, Pietsch, König, Edmunds, Evans, Triebel, Markus, Matsaev, and many others). In particular, there are extensions of Dunford–Schwartz theorem ([6], [1]). But to apply them, one needs to have an extension of the resolvent estimate.

However, it turned out that *for our problems special theorems on the completeness in Banach spaces are non-necessary at all.* Let us explain this.

For a fixed p with $|\frac{1}{p} - \frac{1}{2}| < \delta$, denote by I_p the interval

$$\Big(-\frac{3}{2} - \varepsilon + \frac{1}{p}, \ \frac{1}{2} + \varepsilon + \frac{1}{p}\Big).$$

This is the union of superscripts of "the most right" domain of our operator, "the most left" range of it and intermediate points. These spaces form a unique scale. When the superscript decreases, the space is expanded. The embedding is dense since smooth functions are dense in all spaces. Since L_D and L_N are invertible, their root functions belong to the domain and to the range simultaneously. If we have the completeness in one of these spaces, then this is true in the other one as well.

We obtain the following

Proposition 2. *The root functions of the operator L_D belong to all spaces corresponding to points of I_p, and if they are complete in one of them, they are complete in all other. The same is true for the operator L_N.*

This is useful in obtaining the following result.

Theorem 3. *The root functions belong to all spaces corresponding to points of the union of intervals I_p with $|\frac{1}{p} - \frac{1}{2}| < \delta$, and if they are complete for $p = 2$, then the same is true for all p.*

The proof uses, besides isomorphisms defined by our operator, the known embeddings for our spaces. For $p < 2$, the obvious embeddings are used for $s = \frac{1}{2} - \frac{1}{p}$. For $p > 2$, we use a less simple result (see [15, Section 4.6.1]):

Let

$$1 < p \leq q < \infty, \quad \sigma - \tau \geq n\Big(\frac{1}{p} - \frac{1}{q}\Big).$$

Then there is a continuous and dense embedding $H_p^\sigma \subset H_q^\tau$. A similar statement is true for the spaces $\widetilde{H}_p^\sigma$.

It follows that for our operators the domain with the subscript p and superscript $\frac{1}{2}+\frac{1}{p}$ is embedded into the range with the subscript $q > p$ and superscript $-\frac{1}{2}-\frac{1}{q'}$ if

$$\frac{2}{n-1} \geq \frac{1}{p} - \frac{1}{q}\,.$$

We increase p by small steps and obtain the result in a finite number of steps. □

In a simpler case of smooth elliptic problems in Sobolev spaces, such approach was used by Agmon in his classical paper [4].

Remark. In the case of a formally self-adjoint L, in the spaces corresponding to the points of the interval I_2, it is possible to introduce inner products by using powers of the operator L_D or L_N, and then we have the same orthogonal basis of eigenfunctions in these spaces.

3. For our spectral problems, there exists *a second realization.* The corresponding operators can be considered as acting in $L_p(\Omega)$ (in particular, in $L_2(\Omega)$, which is especially popular in the literature, see e.g. [12]) instead of spaces with negative superscripts. We consider the Neumann problem for definiteness.

Let p be fixed with $|\frac{1}{p}-\frac{1}{2}| < \delta$. Denote by $\widehat{H}_p(\Omega)$ the space of such u that the form $\Phi(u,v)$ defines *a continuous anti-linear functional on $L_{p'}(\Omega)$.* Of course, it is continuous on $H_{p'}^{\frac{1}{2}-s+\frac{1}{p'}}(\Omega)$ for $|s| < \varepsilon$ (since the superscript is positive here). Hence formula

$$(L_N u, v) = \Phi(u,v) \tag{5}$$

defines a solution u of the equation $L_N u = f$ belonging to all $H_p^{\frac{1}{2}+s+\frac{1}{p}}(\Omega)$ with $|s| < \varepsilon$. In $\widehat{H}_p(\Omega)$, we introduce the graph norm by the equality

$$\|u\|^p_{\widehat{H}_p}(\Omega) = \|u\|^p_{L_p(\Omega)} + \|f\|^p_{L_p(\Omega)}.$$

For $p = 2$, it corresponds to the natural inner product in $\widehat{H}_2(\Omega)$. The first term in the right-hand side can be omitted.

Theorem 4. *The $\widehat{H}_p(\Omega)$ is a Banach space continuously embedded into the spaces $H_p^{\frac{1}{2}+s+\frac{1}{p}}(\Omega)$ for $|s| < \varepsilon$. The operator L_N defined by* (5) *maps the space $\widehat{H}_p(\Omega)$ onto $L_p(\Omega)$ isomorphically. Its spectrum and root functions remain the same, and the root functions are complete in $\widehat{H}_p(\Omega)$ if they are complete in $\widetilde{H}^{-1}(\Omega)$. In $L_2(\Omega)$, this operator is self-adjoint if it is self-adjoint in $\widetilde{H}^{-1}(\Omega)$, and then the orthonormal basis of eigenfunctions in $\widetilde{H}^{-1}(\Omega)$ remains an orthogonal basis in $\widehat{H}_2(\Omega)$.*

Remark. If the boundary Γ and the coefficients in L are smooth, then $\widehat{H}_p(\Omega)$ coincides with the subspace in $W_p^2(\Omega)$ of functions satisfying the homogeneous Neumann boundary conditions in the usual sense. Otherwise, $\widehat{H}_p(\Omega)$ can contain less smooth functions. The exact description of $\widehat{H}_p(\Omega)$ in a general Lipschitz domain is unavailable including $p = 2$.

The situation with the Dirichlet problem is similar.

4. Now we discuss *the summability of Fourier series with respect to root functions by the Abel–Lidskii method.* This is an intermediate property between the completeness and the basis property.

First, we define *the formal Fourier series with respect to the root vectors.* Let X and Y be separable Banach spaces with a compact and dense embedding $Y \subset X$, and let A be a bounded and invertible operator $Y \to X$. Assume that A has a complete minimal system $\{x_j\}_1^\infty$ of root vectors in X. Then the biorthogonal to it system $\{z_j\}_1^\infty$ is uniquely constructed from the root vectors of A^*, and to each vector $x \in X$ its formal Fourier series with respect to $\{x_j\}_1^\infty$ is associated:

$$x \sim \sum_1^\infty c_k x_k, \quad \text{where} \quad c_k = (x, z_k), \tag{6}$$

$(\,\cdot\,,\,\cdot\,)$ is the duality between X and X^*. We enumerate the corresponding eigenvalues λ_k of A in order of increasing moduli taking multiplicities into account.

Let now A be one of our operators L_D and L_N, X and Y be its range and domain. Under some conditions (discussed below), it is possible to represent each vector $x \in X$ in the form

$$x = \frac{1}{2\pi i} \lim_{t \to 0} \int\limits_{\partial\Lambda^\theta} e^{-t\lambda^\gamma} R_A(\lambda)\, d\lambda\, x. \tag{7}$$

Here, the number γ and the parameter t are positive, the contour $\partial\Lambda_\theta$ is the boundary of Λ_θ with negative direction, and $R_A(\lambda)$ is the resolvent of A:

$$R_A(\lambda) = (A - \lambda I)^{-1}.$$

Moreover, assume that the domain Λ_θ can be divided into subdomains by arcs of radii $R_l \uparrow \infty$ not containing eigenvalues and that the integral (7) can be represented as the sum of integrals along the boundaries of these subdomains. Each integral is calculated via the residues of the integrand at the eigenvalues λ_k lying in the subdomain.

This is a summability method of order γ of the series (6) to the original vector x. This method was proposed by Lidskii in the case of a Hilbert space under the name Abel's method. Lidskii has found the conditions sufficient for the realization of this method [11]; see also [3, Chapter 5].

For our problems, it suffices to have (4). The key tool is the optimal resolvent estimate

$$\|R_A(\lambda)\| \le C(1 + |\lambda|)^{-1} \tag{8}$$

in M_θ for $\theta > \theta_0$. For our operators in the simplest spaces, it is easily verified, and thus a deep strengthening of Proposition 1 is obtained.

To generalize this result to the spaces H^s_p, first, it is necessary to generalize the Lidskii theorem for the operators in Banach spaces. This was done in [1]. Here the abstract theorem is required. Secondly, it is necessary to generalize estimate (8) to these spaces. It turned out that this is not easy.

How to obtain the estimate, the paper by Gröger–Rehberg [8] suggested to the author. In this and some subsequent papers, the aim was to estimate the resolvent of the mixed problem in a very general statement, with domain of the corresponding operator contained in $W^1_p(\Omega)$, which is the diagonal direction $s + \frac{1}{p} = \frac{1}{2}$ in our notation. To obtain the estimate, they used Agmon's idea from the same paper [4].

Following this idea, we introduce the additional variable t and consider the Lipschitz cylinder $\Omega' = \Omega \times [-1, 1]$. In Ω', we consider the operator

$$L - \eta\partial_t^2$$

with the form

$$\int_0^1 \Phi(U, V)\, dt + \eta \int_\Omega \int_{-1}^1 \partial_t U \cdot \partial_t \overline{V}\, dt\, dx,$$

where $|\eta| = 1$, $|\arg \eta| < \frac{\pi}{2}$. *This form is strongly coercive on functions from* $H^1(\Omega')$*, equal to zero at* $t = \pm 1$. We apply the estimate that follows from Shneiberg's theorem to functions depending on the parameter μ:

$$U(x,t) = u(x)v(t), \quad \text{where } v(t) = \varphi(t)e^{i\mu t}, \quad \mu = |\lambda|, \quad \lambda = \eta\mu,$$

and $\varphi(t)$ is a function from $C_0^\infty[-1, 1]$ equal to 1 on $[-\frac{1}{2}, \frac{1}{2}]$.

Theorem 5. *Let $\theta > \theta_0$. Then for the resolvents of the operators L_D and L_N in the spaces corresponding to the points of some neighborhood of the centrum of the rectangle $Q_{\varepsilon,\delta}$ the uniform estimate* (8) *is valid for $\lambda \in M_\theta$.*

The proof is carried out first in two convenient directions $s + \frac{1}{p} = \frac{1}{2}$ (of Gröger–Rehberg) and $\frac{1}{p} = \frac{1}{2}$, on which the usual Sobolev–Slobodetskii norms can be used, and then the interpolation is applied.

Theorem 6. *Let condition* (4) *be fulfilled. Then the Fourier series with respect to the root functions of the operators L_D and L_N in the spaces corresponding to the points of some neighborhood of the centrum of the rectangle $Q_{\varepsilon,\delta}$, are summed to the corresponding vectors by the Abel–Lidskii method of order $\gamma \in (\frac{n}{\pi}, \theta_0^{-1})$.*

Remark. The estimate in Theorem 5 allows one to construct analytic semigroups e^{-tL_D} and e^{-tL_N} to solve "parabolic" problems in a Lipschitz cylinder in our Banach spaces. See [2, Section 17]. An essential additional remark: *the strong coerciveness of the form Φ is sufficient for this aim*, no additional assumptions on the coerciveness are needed.

5. A similar approach can be applied to other spectral problems. We indicate some of them. Cf. [2].

The mixed problem (with homogeneous Dirichlet and Neumann boundary conditions on two parts of Γ with common Lipschitz boundary of dimension $n-2$).

The Robin problem with boundary condition $T^+u + \beta u^+ = 0$, where u^+ is the boundary value of a solution and T^+u is its conormal derivative, $\operatorname{Re}\beta(x) \geq 0$.

The Dirichlet and Neumann problems for high-order strongly elliptic systems.

Of special interest is the Poincaré-Steklov spectral problem

$$Lu = 0 \ \text{ in } \ \Omega, \quad T^+u = \lambda u^+.$$

To it, *the Dirichlet-to-Neumann operator* is associated:

$$D : u^+ \longrightarrow T^+u.$$

Originally, it is considered as a bounded operator from $H^{\frac{1}{2}}(\Gamma) = B_2^{\frac{1}{2}}(\Gamma)$ to $H^{-\frac{1}{2}}(\Gamma) = B_2^{-\frac{1}{2}}(\Gamma)$. Its form (Du^+, u^+) coincides with $\Phi(u,u)$, which implies its strong coerciveness and the invertibility of the operator. By Shneiberg's theorem, for small $|s|$ and $|p-\frac{1}{2}|$ it has a bounded and invertible extension

$$B_p^{\frac{1}{2}+s}(\Gamma) \longrightarrow B_p^{-\frac{1}{2}+s}(\Gamma)$$

in Besov spaces on Γ, and we can investigate its spectral properties in these spaces. Cf. [5].

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References

1. M. S. Agranovich, Spectral problems in Lipschitz domains for strongly elliptic systems in the Banach spaces H_p^σ and B_p^σ. (Russian) *Funktsional. Anal. i Prilozhen.* **42** (2008), No. 4, 2–23; translation in *Funct. Anal. Appl.* **42** (2008), No. 4, 249–267.
2. M. S. Agranovich, Sobolev spaces, their generalizations, and elliptic problems in smooth and Lipschitz domains. *Springer, Berlin etc.*, 2015.
3. M. S. Agranovich, B. Z. Katsenelenbaum, A. N. Sivov, and N. N. Voitovich, Generalized method of eigenoscillations in diffraction theory. Translated from the Russian manuscript by Vladimir Nazaikinskii. *WILEY-VCH Verlag Berlin GmbH, Berlin*, 1999.
4. Sh. Agmon, On the eigenfunctions and on the eigenvalues of general elliptic boundary value problems. *Comm. Pure Appl. Math.* **15** (1962), 119–147.
5. W. Arendt and R. Mazzeo, Spectral properties of the Dirichlet-to-Neumann operator on Lipschitz domains. *Ulmer Seminare* **12** (2007), 23–37.

6. J. Burgoyne, Denseness of the generalized eigenvectors of a discrete operator in a Banach space. *J. Operator Theory* **33** (1995), No. 2, 279–297.
7. I. C. Gohberg and M. G. Kreĭn, Introduction to the theory of linear nonselfadjoint operators. Translated from the Russian by A. Feinstein. Translations of Mathematical Monographs, Vol. 18 *American Mathematical Society, Providence, R.I.*, 1969.
8. K. Gröger and J. Rehberg, Resolvent estimates in $W^{-1,p}$ for second order elliptic differential operators in case of mixed boundary conditions. *Math. Ann.* **285** (1989), No. 1, 105–113.
9. N. Dunford and J. T. Schwartz, Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space. With the assistance of William G. Bade and Robert G. Bartle, *Interscience Publishers John Wiley & Sons New York–London*, 1963.
10. P. D. Lax and A. N. Milgram, Parabolic equations. *Contributions to the theory of partial differential equations*, pp. 167–190. Annals of Mathematics Studies, No. 33. *Princeton University Press, Princeton, N. J.*, 1954.
11. V. B. Lidskii, On summability of series in principal vectors of nonselfadjoint operators. *Amer. Math. Soc. Transl., Ser. 2*, **40** (1964), 193–228.
12. G. Métivier, Valeurs propres de problèmes aux limites elliptiques irrégulières. (French) *Bull. Soc. Math. France Suppl. Mém.*, No. 51–52 (1977), 125–219.
13. V. S. Rychkov, On restrictions and extensions of the Besov and Triebel-Lizorkin spaces with respect to Lipschitz domains. *J. London Math. Soc. (2)* **60** (1999), No. 1, 237–257.
14. I. Ya. Shneĭberg, Spectral properties of linear operators in interpolation families of Banach spaces. (Russian) *Mat. Issled.* **9** (1974), No. 2(32), 214–229, 254–255.
15. H. Triebel, Interpolation theory, function spaces, differential operators. North-Holland Mathematical Library, 18. *North-Holland Publishing Co., Amsterdam–New York*, 1978.
16. M. I. Vishik, On strongly elliptic systems of differential equations. (Russian) *Mat. Sbornik N.S.* **29(71)** (1951), 615–676.

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