



Professor Michael Drakhlin

(1936–2007)

It is with great sadness that we report the passing of our friend, an excellent mathematician, Professor Michael Drakhlin. He died on July 11, 2007, at the age of 70.

Michael Drakhlin was born on November 22, 1936, in Kiev, USSR. During the Second World War his family was evacuated to Perm, Ural. After receiving his graduate degree from the Perm State University in 1960, Michael worked for one year as a school teacher in a small town Gremyachinsk near Perm. Then he started his academic career at the Perm Polytechnical Institute as an Assistant Professor at the Chair of Theoretical Mechanics. In 1962 he began to work on his doctoral thesis and received the Ph.D. degree from the Perm State University in 1965. The topic of the thesis was “Qualitative study of solution’s behavior for a second order differential equation and its relation to solvability of some boundary value problems”. The same year Michael Drakhlin returned to the Chair of Theoretical Mechanics at the Perm Polytechnical Institute where he worked till 1986 as a Senior Lecturer and then as an Associate Professor. In 1986 Michael Drakhlin obtained the Doctor of Science degree and got the professorship at the same Chair. The topic of his doctoral dissertation was “Equations with internal superposition operators”. In 1988 Michael Drakhlin became the Head of the Chair of Higher Mathematics.

Starting from 1977 till 1991 he was Managing Editor of the annual journal “Boundary Value Problems” and from 1985 till 1991 the Managing Editor of annual journal “Functional Differential Equations”.

In February 1991 Drakhlin moved to Israel. Three months later he started working at the Research Institute of the College of Judea and Samaria.

He founded there a bi-weekly seminar “Functional Differential Equations”. Hundreds of prominent mathematicians from all over the world came to lecture at this seminar. In 1993 Michael Drakhlin together with Elena Litsyn started the journal “Functional Differential Equations” devoted to differential, integral, functional and functional differential equations. This journal was immediately recognized by specialists all over the world. Michael Drakhlin also served as an Associate Editor on the boards of the following journals: “Nonlinear Oscillation” (Ukrainian National Academy of Science), “Mathematica” (Czech Republic) and “Memoirs on Differential Equations and Mathematical Physics” (Tbilisi, Georgia).

The impact of Professor Drakhlin on the development of the basic theory of functional differential equations is hard to over-estimate. He has written more than 200 scientific papers published in leading international profile journals.

His early papers (see, e.g., [1, 2]) were devoted to Riccati equations. He investigated zeros of these equations, obtained some comparison principles for Riccati differential equations. Later he concentrated mostly on properties of differential equations with deviation of the argument. In [5] the existence, uniqueness and convergence of consequent approximations for neutral type differential equations were studied, in [6] he investigated a boundary value problem for neutral type nonlinear differential equations. Of special importance are the contributions of Michael Drakhlin to the investigation of the internal superposition (shift) operator. The importance of this operator in the theory of abstract functional differential equations is great. Let us point out some topics in this direction covered by Drakhlin’s research: internal superposition operator (ISO) in Orlich spaces, nonlinear integral equations with ISO, action of ISO in Lebesgue spaces, convergence of ISOs and optimal control problems (see, e.g., [7, 8, 10, 14, 16, 15, 17, 19, 20, 30, 32, 43, 55, 65]), etc.

In 1975 Professor Nikolaï Azbelev moved to Perm. Michael Drakhlin immediately joined his seminar on functional differential equations (FDE). In cooperation with Azbelev, Rakhmatullina, Maksimov and other participants of the seminar, he developed a deep theoretical background for the general theory of FDE. In particular, in [27, 28] Drakhlin investigated oscillation and asymptotic properties of some functional differential equations. In [35, 37, 40, 44, 51, 47] he showed that the theory of abstract FDE is a very useful tool for solving some variational problems, especially in cases where the problem of minimization of a functional is unsolvable within the framework of the classical calculus of variations.

During last years Prof. Drakhlin became interested in some topics of the classical operator theory. The results in this direction are worth to be mentioned. In [64, 67, 69], notions of memory and comemory of an operator were introduced. Properties of the memory of operator were considered. Roughly speaking, the memory is an information about the preimages the operator remembers given some information about images. It was shown that some

properties of the memory allow to single out two classes of nonlinear operators generalizing the notion of local operator between ideal function spaces. The first class, named atomic, contains in particular all linear shifts (inner superpositions), while the second one, called coatomic, contains all adjoints to atomic operators, and, in particular, the conditional expectations. Both classes include local (in particular, the Nemytsky) operators and are closed with respect to compositions of operators. Results about representation of operators of both classes were provided. In [68] a new definition of Volterra operator was introduced. The new approach stems mainly from the initial considerations of Volterra–Tonelli–Tikhonov, i.e. uses mainly the evolutionary nature of the Volterra operator. Basing on the notions of operator’s memory and chain, a class of operators (called Volterra) possessing the evolutionary property was singled out. Properties of nilpotentness, quasi-nilpotentness and compactness of linear Volterra operators in some functional spaces were defined. Also conditions for solvability of some functional equations with linear and non-linear Volterra operators in certain complete metric spaces were derived.

Michael Drakhlin was a person of extraordinary features in all areas (not only in mathematics). As for mathematics, he had a striking ability of feeling and foreseeing the potential and beauty of new rough ideas long before they were converted into rigorous mathematical statements. This very rare ability attracted to him other unordinary people, who may always count on his intuition and judgement. His enthusiasm and professional qualities made possible establishment of the community of mathematicians working on functional differential equations, united by common mathematical ideas, the seminar, the journal and personal communications.

Professor Drakhlin influenced many young mathematicians. He was an advisor for more than 30 Ph.D. and Post-Doctoral students.

Michael’s interests were not restricted only to mathematics. He always was very interested in what was happening in the world and around him, and had very original and deep opinions about politics and culture. He had many close friends who always found in him a person able to understand and help. Tee personal kindness he radiated attracted people to him at the very first acquaintance.

Persons like Michael Drakhlin are unbelievably rare, and unfortunately when they leave us the world becomes worse since there is nobody to substitute them.

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A. LOMTATIDZE, V. MAKSIMOV, N. PARTSVANIA, N. ROZOV, M. TVRDY

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