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**FLUID-SOLID INTERACTION:
ACOUSTIC SCATTERING BY AN ELASTIC
OBSTACLE WITH LIPSCHITZ BOUNDARY**

Abstract. The potential method is developed for the three-dimensional interface problems of the theory of acoustic scattering by an elastic obstacle which are also known as fluid-solid (fluid-structure) interaction problems. It is assumed that the obstacle has a Lipschitz boundary. The sought for field functions belong to spaces having L_2 integrable nontangential maximal functions on the interface and the transmission conditions are understood in the sense of nontangential convergence almost everywhere. The uniqueness and existence questions are investigated. The solutions are represented by potential type integrals. The solvability of the direct problem is shown for arbitrary wave numbers and for arbitrary incident wave functions. It is established that the scalar acoustic (pressure) field in the exterior domain is defined uniquely, while the elastic (displacement) vector field in the interior domain is defined modulo Jones modes, in general. On the basis of the results obtained it is proved that the inverse fluid-structure interaction problem admits at most one solution.

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რეზიუმე. ნაშრომში დამუშავებულია პოტენციალის მეთოდი დრეკადი წინაღობის მიერ ბგერის გაბნევის ამოცანებისათვის, რომლებიც აგრეთვე ცნობილია, როგორც სითხე-მყარი სხეულის (სითხე-სტრუქტურის) ურთიერთქმედების ამოცანები. იგეგმისხმება, რომ წინაღობას აქვს ლიფშიცური საზღვარი. საძიებელი ველის ფუნქციები ეკუთვნის სივრცეებს, რომლებსაც აქვთ L_2 -ინტეგრებადი არამხები მაქსიმალური ფუნქციები ურთიერთქმედების ზედაპირზე, ხოლო გადაცემის პირობები გვესმის თითქმის ყველგან არამხები კრებადობის აზრით. შესწავლილია ერთადერთობის და არსებობის საკითხები. ამონახსნები წარმოდგენილია პიტენციალის ტიპის ინტეგრალების სახით. პირდაპირი ამოცანის ამოხსნადობა ნაჩვენებია დაცემული ტალღების ნებისმიერი რაოდენობისა და ნებისმიერი ტალღური ფუნქციისათვის. დადგენილია, რომ სკალარული აკუსტიკური (წნევის) ველი გარე არეში ცალსახად განისაზღვრება, ხოლო დრეკადი (დეფორმაციის) ვექტორული ველი შიდა არეში საზოგადოდ განისაზღვრება ჯონსის მოდების სიზუსტით. მიღებული შედეგების საფუძველზე დამტკიცებულია, რომ სითხე-სტრუქტურის ურთიერთქმედების შებრუნებულ ამოცანას აქვს არაუმეტეს ერთი ამონახსნისა.

1. INTRODUCTION

Direct and inverse problems related to the interaction between vector fields of different dimension have received much attention in the mathematical and engineering scientific literature and have been intensively investigated for the past years. They arise in many physical and mechanical models describing the interaction of two different media where the whole process is characterized by a vector-function of dimension k in one medium and by a vector-function of dimension n in another one (for example, fluid-structure interaction where a streamlined body is an elastic obstacle, scattering of acoustic and electromagnetic waves by an elastic obstacle, interaction between an elastic body and seismic waves, etc.).

Quite many authors have considered and studied in detail the direct problems of the interaction between an elastic *isotropic* body which occupies a bounded region $\overline{\Omega^+}$ (where a three-dimensional elastic vector field is to be defined) and some *isotropic* medium (fluid say) which occupies the unbounded exterior region, the complement of Ω^+ (where a scalar field is to be defined). The time-harmonic dependent unknown vector and scalar fields are coupled by some kinematic and dynamic conditions on the boundary $\partial\Omega^+$. Main attention has been given to the problems determining the manner in which an incoming acoustic wave is scattered by an elastic body immersed in a *compressible inviscid fluid*. An exhaustive information in this direction concerning theoretical and numerical results can be found in [3], [4], [5], [6], [7], [21], [12], [13], [14], [17], [19], [20], [29], [40].

The case of *anisotropic* obstacle have been treated in [37], [22], [23], [36]. In [36] the corresponding inverse problem is also considered. This kind of problems arise in detecting and identifying submerged objects.

In all the above papers the boundary of the region occupied by an elastic obstacle is assumed to be *sufficiently smooth* and the transmission conditions are considered either in the classical, or in the usual Sobolev or generalized functional trace sense (in the case of weak setting).

In the present paper our main goal is to generalize the results of the above cited works to *Lipschitz domains* when the transmission conditions are understood in the sense of nontangential convergence almost everywhere.

Following the approach for the case of smooth interface, we propose as solution an ansatz of combinations of single and double layer potentials. By the special representation formulas of the sought for acoustic and elastic fields we reduce *equivalently* the original transmission problem to the system of integral equations. In the case of Lipschitz interface, however, the lack of smoothness introduce essential difficulties in the analysis of the integral equations obtained. These are overcome through the use of harmonic analysis technique together with a careful study of the properties of the boundary integral operators generated by the single and double layer acoustic and elastic potentials. We essentially employ the results obtained in papers [43], [11], [41], [31], [38], [32], [33], [1], [2].

In particular, by the potential method we have derived the necessary and sufficient conditions of solvability of the original transmission problem and shown that the direct scattering problems are solvable for arbitrary values of the frequency parameter and for arbitrary incident wave functions. It is established that the scalar radiating acoustic (pressure) field in the exterior domain is defined uniquely, while the elastic (displacement) vector field in the interior domain is defined modulo Jones modes, in general.

On the basis of the results obtained and applying the approach developed in [9], [27], and [36] we have proved the uniqueness of solution to the inverse fluid-structure interaction (scattering) problem.

2. MATHEMATICAL FORMULATION OF THE INTERFACE PROBLEM. PROPERTIES OF POTENTIALS

2.1. Elastic field. Let $\Omega^+ \subset \mathbb{R}^3$ be a bounded domain ($\text{diam } \Omega^+ < +\infty$) with a connected boundary $S = \partial\Omega^+$ and $\Omega^- = \mathbb{R}^3 \setminus \overline{\Omega^+}$, $\overline{\Omega^+} = \Omega^+ \cup S$.

Throughout the paper we assume that the boundary S is a *Lipschitz surface* (if not otherwise stated).

The region Ω^+ is supposed to be filled up by a homogeneous *isotropic medium* with the elastic coefficients (Lamé constants) λ and μ , and the density $\varrho_1 = \text{const} > 0$.

The homogeneous system of steady state oscillation equations of the linear elasticity reads as follows (see, e.g., [28])

$$\begin{aligned} A(\partial, \omega) u(x) &:= A(\partial) u(x) + \varrho_1 \omega^2 u(x) = \\ &= \mu \Delta u + (\lambda + \mu) \text{grad div } u + \varrho_1 \omega^2 u(x) = 0, \end{aligned} \quad (2.1)$$

where $u = (u_1, u_2, u_3)^\top$ is the complex-valued displacement vector (amplitude), $\omega > 0$ is the oscillation (frequency) parameter,

$$\begin{aligned} A(\partial, \omega) &:= A(\partial) + \varrho_1 \omega^2 I_3, \quad A(\partial) := [A_{kj}(\partial)]_{3 \times 3}, \\ A_{kj}(\partial) &= \mu \delta_{kj} \Delta + (\lambda + \mu) \partial_k \partial_j, \quad \partial = (\partial_1, \partial_2, \partial_3), \quad \partial_j = \frac{\partial}{\partial x_j}; \end{aligned}$$

here and in what follows I_3 stands for the unit 3×3 matrix, δ_{kp} is the Kronecker delta, $\Delta = \partial_1^2 + \partial_2^2 + \partial_3^2$ is the Laplace operator, the superscript \top denotes transposition.

The stress tensor $\{\sigma_{kj}\}$ and the strain tensor $\{\varepsilon_{kj}\}$ are related by Hook's law

$$\sigma_{kj} = \delta_{kj} \lambda (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) + 2\mu \varepsilon_{kj}, \quad \varepsilon_{kj} = 2^{-1} (\partial_k u_j + \partial_j u_k).$$

As usual, the quadratic form corresponding to the density of potential energy is assumed to be positive definite in the symmetric real variables $\varepsilon_{kj} = \varepsilon_{jk}$ (see, e.g., [28], [15])

$$E(u, u) = \sigma_{kj} \varepsilon_{kj} \geq \delta_1 \varepsilon_{kj} \varepsilon_{kj}, \quad \delta_1 = \text{const} > 0, \quad (2.2)$$

implying the inequalities $\mu > 0$, $3\lambda + 2\mu > 0$. Clearly, we also have

$$E(u, \bar{u}) = \sigma_{kj} \overline{\varepsilon_{kj}} \geq \delta_1 [\varepsilon'_{kj} \varepsilon'_{kj} + \varepsilon''_{kj} \varepsilon''_{kj}], \quad (2.3)$$

where an over-bar denotes complex conjugation, and where $\varepsilon_{kj} = \varepsilon'_{kj} + i \varepsilon''_{kj}$ are the entries of the complex strain tensor corresponding to the vector $u = u' + i u''$, $i = \sqrt{-1}$. Here and in what follows we employ the summation over repeated indices from 1 to 3, unless otherwise stated.

The inequality (2.2) implies the positive definiteness of the matrix $A(\xi)$ for $\xi \in \mathbb{R}^3 \setminus \{0\}$

$$A(\xi) \zeta \cdot \zeta = A_{kj}(\xi) \zeta_j \bar{\zeta}_k \geq \delta_2 |\xi|^2 |\zeta|^2, \quad \delta_2 = \text{const} > 0,$$

where ζ is an arbitrary three-dimensional complex vector, $\zeta \in \mathbb{C}^3$. Throughout the paper $a \cdot b = \sum_{k=1}^m a_k \bar{b}_k$ denotes the scalar product of two vectors in \mathbb{C}^m .

Further we introduce the stress operator

$$T(\partial, n) = [T_{kj}(\partial, n)]_{3 \times 3}, \quad T_{kj}(\partial, n) = \lambda n_k \partial_j + \mu n_j \partial_k + \mu \delta_{kj} \partial_n,$$

where $n = (n_1, n_2, n_3)$ is a unit vector and ∂_n denotes the directional derivative along the vector n .

The k -th component of the stress vector acting on a surface element with the unit normal vector n is calculated by the formula

$$[T(\partial, n) u]_k = \sigma_{kj} n_j = \left[2\mu \partial_n u + \lambda n \operatorname{div} u + \mu [n \times \operatorname{curl} u] \right]_k, \quad (2.4)$$

where $[\cdot \times \cdot]$ denotes the cross product of two vectors.

Note that throughout the paper we will employ the notation L_2 , W_2^s ($s \geq 0$), and H_2^r ($r \in \mathbb{R}$) for the usual Lebesgue, Sobolev-Slobodetski, and Bessel potential function spaces respectively. Recall that $L_2 = W_2^0 = H_2^0$ and $W_2^s = H_2^s$ for $s \geq 0$, and for a Lipschitz surface S the space $H_2^r(S)$ is defined correctly only for $-1 \leq r \leq 1$. By $\|u\|_X$ we denote the norm of the element u in the space X .

2.2. Scalar field. We assume that the exterior, simply connected domain Ω^- is filled up by a homogeneous anisotropic medium (compressible viscid fluid say) with the constant density ϱ_2 . Further, let some physical process (the propagation of acoustic waves say) in Ω^- be described by a complex-valued scalar function (scalar pressure field) $w(x)$ being a solution of the homogeneous "wave equation" (generalized Helmholtz equation)

$$a(\partial, \omega) w := a(\partial) w + \varrho_2 \omega^2 w = 0, \quad (2.5)$$

where $a(\partial) = a_{kj} \partial_k \partial_j$, the real constants $a_{kj} = a_{jk}$ define a positive definite matrix $\tilde{a} = [a_{kj}]_{3 \times 3}$, i.e.,

$$\tilde{a} \zeta \cdot \zeta = a_{kj} \zeta_j \bar{\zeta}_k \geq \delta_3 |\zeta|^2, \quad \delta_3 = \text{const} > 0, \quad (2.6)$$

for arbitrary $\zeta \in \mathbb{C}^3$.

Denote by S_ω the characteristic surface (ellipsoid) given by the equation

$$\tilde{a} \xi \cdot \xi - \varrho_2 \omega^2 = 0, \quad \xi \in \mathbb{R}^3.$$

For an arbitrary vector $\eta \in \mathbb{R}^3$ with $|\eta| = 1$ there exists only one point $\xi(\eta) \in S_\omega$ such that the outward unit normal vector $n(\xi(\eta))$ to S_ω at the point $\xi(\eta)$ has the same direction as η , i.e., $n(\xi(\eta)) = \eta$. Note that $\xi(-\eta) = -\xi(\eta) \in S_\omega$ and $n(-\xi(\eta)) = -\eta$.

It can be easily verified that

$$\xi(\eta) = \omega \sqrt{\varrho_2} (\tilde{a}^{-1} \eta \cdot \eta)^{-1/2} \tilde{a}^{-1} \eta, \quad (2.7)$$

where \tilde{a}^{-1} is the matrix inverse to \tilde{a} .

Now we are in the position to define the class $\text{Som}(\Omega^-)$ of complex-valued functions satisfying the generalized Sommerfeld type radiation conditions (see, e.g., [42]).

A function w belongs to $\text{Som}(\Omega^-)$ if $w \in C^1(\Omega^-)$ and for sufficiently large $|x|$

$$w(x) = \mathcal{O}(|x|^{-1}), \quad \partial_k w(x) - i \xi_k(\eta) w(x) = \mathcal{O}(|x|^{-2}), \quad k = 1, 2, 3, \quad (2.8)$$

where $\xi(\eta) \in S_\omega$ corresponds to the vector $\eta = x/|x|$ (i.e., $\xi(\eta)$ is given by (2.7) with $\eta = \hat{x} := x/|x|$).

The conditions (2.8) are equivalent to the classical Sommerfeld radiation conditions for the Helmholtz equation if the $a(\partial)$ is the Laplace operator (see, e.g., [42], [8]). In the sequel, elements of the class $\text{Som}(\Omega^-)$ will also be referred to as *radiating functions*.

We have the following analogue of the classical Rellich-Vekua lemma (for details see [22]).

Lemma 2.1. *Let $w \in \text{Som}(\Omega^-)$ be a solution of (2.5) in Ω^- and let*

$$\lim_{R \rightarrow +\infty} \Im \left\{ \int_{\Sigma_R} [\Lambda(\partial_x, n(x)) w(x)] [\overline{w(x)}] d\Sigma_R \right\} = 0,$$

where Σ_R is the sphere centered at the origin and radius R , and $\Lambda(\partial, n)$ denotes the co-normal differentiation

$$\Lambda(\partial_x, n(x)) := a_{kj} n_k(x) \partial_j.$$

Then $w = 0$ in Ω^- .

Note that, if w is a solution of the homogeneous equation (2.5), then w is an analytic function of the real variable x in the domain Ω^- . Moreover, if, in addition, $w \in C^1(\overline{\Omega^-}) \cap \text{Som}(\Omega^-)$ and the boundary surface $S = \partial\Omega^\pm$ is sufficiently smooth ($C^{1,\alpha}$ smooth say), then the following integral representation formula holds (cf. [42], [23])

$$\begin{aligned} \int_S \gamma(x-y, \omega) [\Lambda(\partial, n) w(y)]^- dS_y - \int_S [\Lambda(\partial_y, n(y)) \gamma(y-x, \omega)] [w(y)]^- dS_y = \\ = \begin{cases} w(x) & \text{for } x \in \Omega^-, \\ 0 & \text{for } x \in \Omega^+, \end{cases} \quad (2.9) \end{aligned}$$

where

$$\gamma(x, \omega) = -\frac{\exp\{i\omega\sqrt{\varrho_2}(\tilde{a}^{-1}x \cdot x)^{1/2}\}}{4\pi|\tilde{a}|^{1/2}(\tilde{a}^{-1}x \cdot x)^{1/2}}, \quad |\tilde{a}| = \det \tilde{a}, \quad (2.10)$$

is a radiating fundamental solution to the equation (2.5) (see, e.g., Lemma 1.1 in [23]), the symbols $[\cdot]^\pm$ denote limits on S from Ω^\pm .

Here and throughout the paper $n(y)$ stands for the *outward unit normal vector* to S at the point $y \in S$.

For sufficiently large $|x|$ we have the following asymptotic representation

$$\begin{aligned} \gamma(x-y, \omega) &= c(\xi) \frac{\exp\{i\xi \cdot (x-y)\}}{|x|} + \mathcal{O}(|x|^{-2}), \\ c(\xi) &= -\frac{|\tilde{a}\xi|}{4\pi\omega(\varrho_2|\tilde{a}|)^{1/2}}, \end{aligned} \quad (2.11)$$

where y varies in a bounded subset of \mathbb{R}^3 and $\xi = \xi(\eta) \in S_\omega$ corresponds to the direction $\eta = x/|x|$; the asymptotic formula (2.11) can be differentiated any times with respect to x and y (see [23]).

From (2.9) with the help of (2.11) we get the asymptotic representation (for sufficiently large $|x|$) of a radiating solution to the equation (2.5)

$$w(x) = w_\infty(\xi) \frac{\exp\{i\xi \cdot x\}}{|x|} + \mathcal{O}(|x|^{-2}), \quad (2.12)$$

where

$$w_\infty(\xi) = c(\xi) \int_S e^{-i\xi \cdot y} ([\Lambda(\partial, n)w(y)]^- + i(\tilde{a}\xi \cdot n(y))[w(y)]^-) dS_y$$

with ξ and $c(\xi)$ as in (2.11); $w_\infty(\xi)$ is the so-called *far-field pattern* of the radiating solution w (cf. [9]).

2.3. Formulation of direct and inverse interaction problems. We recall that any Lipschitz surface S satisfies the uniform cone condition and vice versa [18], i.e., each point $x \in S$ is the vertex of two truncated cones $\gamma^{(\pm)}(x)$ with common axis that are congruent to a fixed cone

$$\{x = (x_1, x_2, x_3) \in \mathbb{R}^3 : 0 \leq x_3 \leq h, \sqrt{x_1^2 + x_2^2} \leq c^*(h - x_3)\}, \quad c^* > 0, \quad h > 0,$$

and such that all points of these cones except x lie in the respective domains $\gamma^{(\pm)}(x) \subset \Omega^\pm$. Usually, these cones $\gamma^{(\pm)}(x)$ are called *nontangential approach regions* and are subjected to some regularity conditions described, e.g., in [43]. Note that the exterior normal vector $n(x)$ exists almost everywhere on S and belongs to the space $L_\infty(S)$.

In what follows the boundary values $[\cdot]^\pm$ on the surface S are taken in the sense of *point-wise nontangential convergence* at *almost every point* with

respect to the surface measure (if not otherwise stated). In particular,

$$\begin{aligned} [u(x)]^\pm &= \lim_{\gamma^{(\pm)}(x) \ni y \rightarrow x \in S} u(y), \\ [w(x)]^\pm &= \lim_{\gamma^{(\pm)}(x) \ni y \rightarrow x \in S} w(y), \\ [T(\partial_x, n(x)) u(x)]^\pm &= \lim_{\gamma^{(\pm)}(x) \ni y \rightarrow x \in S} T(\partial_y, n(x)) u(y), \\ [\Lambda(\partial_x, n(x)) w(x)]^\pm &= \lim_{\gamma^{(\pm)}(x) \ni y \rightarrow x \in S} \Lambda(\partial_y, n(x)) w(y), \end{aligned}$$

for almost all $x \in S$.

Further, we denote by $\mathcal{M}^\pm(v)$ the *nontangential maximal functions* on S corresponding to a function v

$$\mathcal{M}^\pm(v)(x) = \sup_{y \in \gamma^{(\pm)}(x)} |v(y)| \quad \text{for almost all } x \in S$$

(for details see [43], [11]).

Remark 2.2. Denote by $\mathbb{H}_{a,\omega}(\Omega^\pm)$ the subspace of $C^2(\Omega^\pm)$ consisting of functions w that satisfy the homogeneous equation (2.5) in Ω^\pm and such that the nontangential boundary values $[w]^\pm$ and $[\Lambda(\partial, n)w]^\pm$ exist almost everywhere on S , and the maximal nontangential functions $\mathcal{M}^\pm(w)$ and $\mathcal{M}^\pm(\partial_j w)$ ($j = 1, 2, 3$) are in $L_2(S)$. Then automatically, $[w]^\pm \in H_2^1(S)$, $[\Lambda(\partial, n)w]^\pm \in L_2(S)$, and $w \in H_2^{\frac{3}{2}}(\Omega^+)$ and $w \in H_{2,loc}^{\frac{3}{2}}(\Omega^-)$.

Analogously, let $\mathbb{H}_{A,\omega}(\Omega^\pm)$ be the subspace of $[C^2(\Omega^\pm)]^3$ consisting of vectors u that satisfy the homogeneous equation (2) in Ω^\pm and such that the nontangential boundary values $[u]^\pm$ and $[T(\partial, n)u]^\pm$ exist almost everywhere on S , and the maximal nontangential functions $\mathcal{M}^\pm(u_k)$ and $\mathcal{M}^\pm(\partial_j u_k)$ ($k, j = 1, 2, 3$) are in $L_2(S)$. Then automatically, $[u]^\pm \in [H_2^1(S)]^3$, $[T(\partial, n)u]^\pm \in [L_2(S)]^3$, and $u \in [H_2^{\frac{3}{2}}(\Omega^+)]^3$ and $u \in [H_{2,loc}^{\frac{3}{2}}(\Omega^-)]^3$.

Note that for solutions w and u of the homogeneous equations (2.5) and (2), respectively, the following equivalences hold

$$\begin{aligned} \mathcal{M}^+(w) \in L_2(S) &\Leftrightarrow w \in H_2^{\frac{1}{2}}(\Omega^+), \\ \mathcal{M}^-(w) \in L_2(S) &\Leftrightarrow w \in H_{2,loc}^{\frac{1}{2}}(\Omega^-), \\ \mathcal{M}^+(u) \in [L_2(S)]^3 &\Leftrightarrow u \in [H_2^{\frac{1}{2}}(\Omega^+)]^3, \\ \mathcal{M}^-(u) \in [L_2(S)]^3 &\Leftrightarrow u \in [H_{2,loc}^{\frac{1}{2}}(\Omega^-)]^3, \\ \mathcal{M}^+(\partial_j w) \in L_2(S) &\Leftrightarrow w \in H_2^{\frac{3}{2}}(\Omega^+), \\ \mathcal{M}^-(\partial_j w) \in L_2(S) &\Leftrightarrow w \in H_{2,loc}^{\frac{3}{2}}(\Omega^-), \\ \mathcal{M}^+(\partial_j u) \in [L_2(S)]^3 &\Leftrightarrow u \in [H_2^{\frac{3}{2}}(\Omega^+)]^3, \\ \mathcal{M}^-(\partial_j u) \in [L_2(S)]^3 &\Leftrightarrow u \in [H_{2,loc}^{\frac{3}{2}}(\Omega^-)]^3, \end{aligned}$$

where $j = 1, 2, 3$ (for details see [11], [32], [33], [1], [2]).

Note that for functions of the class $\mathbb{H}_{a,\omega}(\Omega^+)$ (respect. $\mathbb{H}_{A,\omega}(\Omega^\pm)$) there hold standard Green's formulas where the boundary limiting values on the boundary S are understood in the above described point-wise nontangential convergence sense.

First we set the direct fluid-structure interaction problem.

Let a total wave field in Ω^- is represented as a sum of incident and scattered fields

$$w^{tot}(x) = w^{inc}(x) + w^{sc}(x),$$

where the incident field w^{inc} is taken in the form of a plane wave

$$w^{inc}(x) = w^{inc}(x; d) = \exp\{i x \cdot d\}, \quad x \in \mathbb{R}^3, \quad d \in S_\omega, \quad (2.13)$$

while the scattered field (scattered acoustic pressure) $w^{sc}(x) = w^{sc}(x; d)$ is a radiating solution of equation (2.5); here $d = (d_1, d_2, d_3)$ denotes the direction of propagation of the plane wave.

Problem $P^{(dir)}$. Find a vector $u = (u_1, u_2, u_3)^\top \in \mathbb{H}_{A,\omega}(\Omega^+)$ and a radiating function $w^{sc} \in \mathbb{H}_{a,\omega}(\Omega^-) \cap \text{Som}(\Omega^-)$, satisfying the following (kinematic and dynamic) coupling conditions in the sense of point-wise nontangential convergence at almost every point on S :

$$[u(x) \cdot n(x)]^+ = b_1 [\Lambda(\partial, n) w^{tot}(x)]^- = b_1 [\Lambda(\partial, n) w^{sc}(x)]^- + f_0(x), \quad (2.14)$$

$$[T(\partial, n) u(x)]^+ = b_2 [w^{tot}(x)]^- n(x) = b_2 [w^{sc}(x)]^- n(x) + f(x), \quad (2.15)$$

where $T(\partial, n)u$ is the stress vector given by formula (2.4), $\Lambda(\partial, n)w = a_{pq} n_p \partial_q w$ is the co-normal derivative, $n(x)$ denotes the unit outward normal vector to S at the point $x \in S$,

$$b_1 = [\varrho_2 \omega^2]^{-1}, \quad b_2 = -1. \quad (2.16)$$

Remark that all the arguments below are valid if b_1 and b_2 are given complex constants satisfying the conditions $b_1 b_2 \neq 0$ and $\Im[\overline{b_1} b_2] = 0$.

Here the boundary scalar function f_0 and the vector-valued function f are defined as follows:

$$f_0(x) = f_0(x; d) = b_1 \Lambda(\partial, n) w^{inc}(x; d), \quad (2.17)$$

$$f(x) = (f_1(x), f_2(x), f_3(x))^\top = f(x; d) = b_2 w^{inc}(x; d) n(x). \quad (2.18)$$

As it follows from the above statement, in the direct problem the domains Ω^+ and Ω^- are fixed and we look for the displacement vector u and the radiating scalar function w^{sc} (scattered field). The inverse fluid-structure acoustic interaction problem consists in finding the surface S (i.e., the scatterer Ω^+) if the corresponding far-field pattern $w_\infty^{sc}(\cdot; d)$ is known for several or all direction vectors $d \in S_\omega$. More rigorous mathematical formulation of the inverse problem considered in this paper reads as follows.

Problem $P^{(inv)}$. Find an elastic scatterer Ω^+ with a compact, connected, Lipschitz boundary surface S provided that the conditions of Problem $P^{(dir)}$ are satisfied on S and the far-field pattern $w_\infty^{sc}(\xi; d)$ is a known

function of ξ on S_ω

$$w_\infty^{sc}(\xi; d) = G(\xi; d)$$

for several (or all) direction vectors $d \in S_\omega$; here $G(\cdot; d)$ is a given function of ξ on S_ω and ξ corresponds to the vector $\eta = x/|x|$ (see (2.7)).

In the both problems the oscillation parameter ω is an arbitrarily fixed *positive number*. The investigation of the inverse problem becomes complicated due to the fact that, in general, the direct interaction problem for arbitrary scatterer Ω^+ is not unconditionally solvable for all ω . For exceptional values of the parameter ω , i.e., for those values of ω for which the corresponding homogeneous direct problem possesses nontrivial solutions, the boundary data f_0 and f , involved in the equations (2.14) and (2.15), have to satisfy special compatibility (necessary) conditions. However, as we shall show below for functions given by (2.17) and (2.18) these necessary conditions are fulfilled automatically and Problem $P^{(dir)}$ is always solvable. Moreover, the scalar field w^{sc} is defined uniquely in Ω^- for all ω , while the elastic field u is defined modulo Jones modes, in general (see Section 3). This makes meaningful and justifies the above setting of the inverse problem with arbitrary ω .

We shall study the above problems by the layer potentials (boundary integral equations) method. The properties of the corresponding potential type operators partly can be found in [43], [10], [11], [41], [31], [32], [33], but for the readers convenient we bring together needed material in the forthcoming subsection.

2.4. Scalar potentials. Steklov-Poincaré type relations. Let us introduce the single and double layer scalar potentials related to the operator $a(\partial, \omega)$:

$$V_{a, \omega}(g)(x) = \int_S \gamma(x-y, \omega) g(y) dS_y, \quad x \in \mathbb{R}^3 \setminus S,$$

$$W_{a, \omega}(g)(x) = \int_S [\Lambda(\partial_y, n(y)) \gamma(y-x, \omega)] g(y) dS_y, \quad x \in \mathbb{R}^3 \setminus S,$$

where g is a scalar density function.

For a solution $w \in \mathbb{H}_{a, \omega}(\Omega^+)$ of equation (2.5) in Ω^+ we have the following integral representation

$$W_{a, \omega}([w]^+)(x) - V_{a, \omega}([\Lambda w]^+)(x) = \begin{cases} w(x) & \text{for } x \in \Omega^+, \\ 0 & \text{for } x \in \Omega^-. \end{cases} \quad (2.19)$$

The similar representation holds also for a radiating solution $w \in \mathbb{H}_{a, \omega}(\Omega^-) \cap \text{SK}(\Omega^-)$ to the equation (2.5) in Ω^- ,

$$V_{a, \omega}([\Lambda w]^-)(x) - W_{a, \omega}([w]^-)(x) = \begin{cases} w(x) & \text{for } x \in \Omega^-, \\ 0 & \text{for } x \in \Omega^+. \end{cases} \quad (2.20)$$

These representations can be derived by standard arguments from the corresponding Green's formulas which hold for functions of the classes $\mathbb{H}_{a,\omega}(\Omega^+)$ and $\mathbb{H}_{a,\omega}(\Omega^-) \cap \text{Som}(\Omega^-)$ (cf. Lemma 5.1 in [41], Proposition 6.6 in [1], and Proposition 5.5 in [2]).

In what follows we will essentially use the following properties of the layer potentials.

Lemma 2.3. *Let $g \in L_2(S)$, $h \in H_2^1(S)$, and $-\frac{1}{2} \leq r \leq \frac{1}{2}$. Then*

(i) *the potentials $V_{a,\omega}(g)$, $W_{a,\omega}(g)$, and $W_{a,\omega}(h)$ are radiating solutions of equation (2.5) in $\mathbb{R}^3 \setminus S$ and*

$$\begin{aligned} \mathcal{M}^\pm(W_{a,\omega}(g)) &\in L_2(S), & \mathcal{M}^\pm(\partial_j W_{a,\omega}(h)) &\in L_2(S), \quad j=1,2,3, \\ \mathcal{M}^\pm(V_{a,\omega}(g)) &\in L_2(S), & \mathcal{M}^\pm(\partial_j V_{a,\omega}(g)) &\in L_2(S), \quad j=1,2,3, \\ W_{a,\omega}(g) &\in H_2^{\frac{1}{2}}(\Omega^+), & W_{a,\omega}(g) &\in H_{2,\text{loc}}^{\frac{1}{2}}(\Omega^-) \cap \text{Som}(\Omega^-), \\ W_{a,\omega}(h) &\in \mathbb{H}_{a,\omega}(\Omega^\pm) \cap \text{Som}(\Omega^-), & V_{a,\omega}(g) &\in \mathbb{H}_{a,\omega}(\Omega^\pm) \cap \text{Som}(\Omega^-); \end{aligned}$$

(ii) *the following jump relations hold on S for almost all $z \in S$*

$$[V_{a,\omega}(g)(z)]^\pm = \int_S \gamma(z-y, \omega) g(y) dS_y =: \mathcal{H}_{a,\omega} g(z),$$

$$\begin{aligned} [\Lambda(\partial, n) V_{a,\omega}(g)(z)]^\pm &= \mp 2^{-1} g(z) + \int_S [\Lambda(\partial_z, n(z)) \gamma(z-y, \omega)] g(y) dS_y =: \\ &=: [\mp 2^{-1} \mathcal{I} + \mathcal{K}_{a,\omega}^{(1)}] g(z), \end{aligned} \quad (2.21)$$

$$\begin{aligned} [W_{a,\omega}(g)(z)]^\pm &= \pm 2^{-1} g(z) + \int_S [\Lambda(\partial_y, n(y)) \gamma(y-z, \omega)] g(y) dS_y =: \\ &=: [\pm 2^{-1} \mathcal{I} + \mathcal{K}_{a,\omega}^{(2)}] g(z), \end{aligned} \quad (2.22)$$

$$[\Lambda(\partial, n) W_{a,\omega}(h)(z)]^+ = [\Lambda(\partial, n) W_{a,\omega}(h)(z)]^- =: \mathcal{L}_{a,\omega} h(z), \quad (2.23)$$

where \mathcal{I} stands for the identical operator;

(iii) *the operators*

$$\begin{aligned} \mathcal{H}_{a,\omega} &: H_2^{-\frac{1}{2}+r}(S) \rightarrow H_2^{\frac{1}{2}+r}(S), \\ \mathcal{K}_{a,\omega}^{(2)} &: H_2^{\frac{1}{2}+r}(S) \rightarrow H_2^{\frac{1}{2}+r}(S), \\ \mathcal{K}_{a,\omega}^{(1)} &: H_2^{-\frac{1}{2}+r}(S) \rightarrow H_2^{-\frac{1}{2}+r}(S), \\ \mathcal{L}_{a,\omega} &: H_2^{\frac{1}{2}+r}(S) \rightarrow H_2^{-\frac{1}{2}+r}(S), \end{aligned}$$

are continuous;

(iv) the operators

$$\begin{aligned}\mathcal{H}_{a,\omega} &: H_2^{-\frac{1}{2}+r}(S) \rightarrow H_2^{\frac{1}{2}+r}(S), \\ \pm 2^{-1}\mathcal{I} + \mathcal{K}_{a,\omega}^{(1)}, \pm 2^{-1}\mathcal{I} + \mathcal{K}_{a,\omega}^{(2)} &: L_2(S) \rightarrow L_2(S), \\ \mathcal{L}_{a,\omega} &: H_2^1(S) \rightarrow L_2(S),\end{aligned}$$

are bounded Fredholm operators with zero index;

(v) the following operator equations

$$\begin{aligned}\mathcal{H}_{a,\omega} \mathcal{K}_{a,\omega}^{(1)} = \mathcal{K}_{a,\omega}^{(2)} \mathcal{H}_{a,\omega}, \quad \mathcal{L}_{a,\omega} \mathcal{H}_{a,\omega} = -4^{-1}\mathcal{I} + [\mathcal{K}_{a,\omega}^{(1)}]^2, \\ \mathcal{L}_{a,\omega} \mathcal{K}_{a,\omega}^{(2)} = \mathcal{K}_{a,\omega}^{(1)} \mathcal{L}_{a,\omega}, \quad \mathcal{H}_{a,\omega} \mathcal{L}_{a,\omega} = -4^{-1}\mathcal{I} + [\mathcal{K}_{a,\omega}^{(2)}]^2,\end{aligned}\tag{2.24}$$

hold in appropriate function spaces.

Proof. The proof of items (i)-(iv) (except the relations involving the operator $\mathcal{L}_{a,\omega}$) based on the harmonic analysis technique can be found in the reference [43] for $\omega = 0$ (see also [10] where a variational approach is used and the analogous results are obtained for $-\frac{1}{2} < r < \frac{1}{2}$ with the help of duality and interpolation arguments based on the Sobolev trace theorem). The estimates

$$\begin{aligned}|\gamma(x,\omega) - \gamma(x,0)| &< \omega C_0(\lambda,\mu), \\ |\partial_l [\gamma(x,\omega) - \gamma(x,0)]| &< \omega^2 C_1(\lambda,\mu), \\ \partial_l \partial_m [\gamma(x,\omega) - \gamma(x,0)] &= \mathcal{O}(|x|^{-1}),\end{aligned}\tag{2.25}$$

show that the potential and boundary operators corresponding to $\omega \neq 0$ and $\omega = 0$ differ by smoothing (compact) operators. Therefore, the results obtained in [43] can be extended to the operators corresponding to arbitrary ω (for details see, e.g., [41]).

The properties of the normal derivative of the double layer potential $W_{a,\omega}$ and the operator $\mathcal{L}_{a,\omega}$ are studied in detail in [41]. However, we give here a simpler proof of (2.23) which does not require invertibility of any boundary operator and can be extended to more general cases (e.g., to the case of elastic double layer vector potential). Since the double layer potential $W_{a,\omega}(h)$ with $h \in H_2^1(S)$ belongs to the class $\mathbb{H}_{a,\omega}(\Omega^\pm) \cap \text{Som}(\Omega^-)$, we can write the integral representation formulas (2.19) and (2.20) with $W_{a,\omega}(h)$ for w . Add termwise these formulas and apply the jump relations (2.22) to obtain

$$V_{a,\omega} ([\Lambda W_{a,\omega}(h)]^+ - [\Lambda W_{a,\omega}(h)]^-) = 0 \quad \text{in } \mathbb{R}^3 \setminus S.$$

Whence we arrive at (2.23) by jump relations (2.21).

To prove the item (v) let us remark that the representation formula (2.19) implies

$$\begin{aligned}[-2^{-1}\mathcal{I} + \mathcal{K}_{a,\omega}^{(2)}][w]^+ = \mathcal{H}_{a,\omega}[\Lambda w]^+, \\ \mathcal{L}_{a,\omega}[w]^+ = [2^{-1}\mathcal{I} + \mathcal{K}_{a,\omega}^{(1)}][\Lambda w]^+.\end{aligned}\tag{2.26}$$

The operator equalities (2.24) can be then obtained by substitution into (2.26) single and double layer potentials with densities from the spaces $L_2(S)$ and $H_2^1(S)$, respectively. It is evident that the first and second equations in (2.24) originally hold in $L_2(S)$ but they can be continuously extended to the space $H_2^{-1}(S)$ due to the mapping properties of the operators involved; analogously, the third and fourth equations which originally hold in $H_2^1(S)$, can be continuously extended to the space $L_2(S)$. \square

To obtain a boundary integral formulation equivalent to the basic oscillation problems in exterior domains we need the following

Lemma 2.4. *Let $g \in H_2^1(S)$ and*

$$w(x) = W_{a,\omega}(g)(x) - i V_{a,\omega}(g)(x), \quad x \in \Omega^-. \quad (2.27)$$

If w vanishes in Ω^- , then $g = 0$ on S .

Proof. If the function (2.27) vanishes in Ω^- due to the jump properties of the layer potentials we conclude

$$[\Lambda w]^+ - i [w]^+ = 0 \quad \text{on } S. \quad (2.28)$$

Since $g \in H_2^1(S)$ we have $w \in \mathbb{H}_{a,\omega}(\Omega^+)$ and there holds Green's formula (cf. [41])

$$\int_{\Omega^+} [a_{kj} \partial_j w \overline{\partial_k w} - \varrho_2 \omega^2 |w|^2] dx = \int_S [\Lambda w]^+ [\overline{w}]^+ dS. \quad (2.29)$$

With the help of (2.6) and (2.28) we conclude from (2.29) that $[w]^+ = 0$ which yields $[w]^+ - [w]^- = g = 0$ on S . \square

Further, let us introduce the boundary operators

$$\mathcal{D}_{a,\omega} g := [-2^{-1} \mathcal{I} + \mathcal{K}_{a,\omega}^{(2)}] - i \mathcal{H}_{a,\omega}, \quad (2.30)$$

$$\mathcal{N}_{a,\omega} g := \mathcal{L}_{a,\omega} - i [2^{-1} \mathcal{I} + \mathcal{K}_{a,\omega}^{(1)}]. \quad (2.31)$$

These operators are generated by the limiting values on S (from Ω^-) of the superposition of potentials (2.27) and its co-normal derivative.

Lemma 2.5. (i) *The operators*

$$\mathcal{D}_{a,\omega} : L_2(S) \rightarrow L_2(S), \quad (2.32)$$

$$: H_2^1(S) \rightarrow H_2^1(S), \quad (2.33)$$

$$\mathcal{N}_{a,\omega} : H_2^1(S) \rightarrow L_2(S) \quad (2.34)$$

are isomorphisms.

(ii) *The exterior Dirichlet BVP*

$$a(\partial, \omega) w(x) = 0 \quad \text{in } \Omega^-, \quad w \in \text{Som}(\Omega^-),$$

$$[w(z)]^- = \varphi(z) \quad \text{on } S, \quad \varphi \in L_2(S),$$

$$\mathcal{M}^-(w) \in L_2(S),$$

is uniquely solvable and the solution is representable in the form

$$w(x) = (W_{a,\omega} - iV_{a,\omega}) (\mathcal{D}_{a,\omega}^{-1} \varphi)(x), \quad x \in \Omega^-.$$

Moreover, $w \in H_{2,loc}^{\frac{1}{2}}(\Omega^-)$ and for arbitrary R there is a positive constant $C_0(R)$ independent of w and φ such that

$$\|w\|_{H_2^{\frac{1}{2}}(\Omega_R^-)} \leq C_0(R) \|\varphi\|_{L_2(S)}$$

with $\Omega_R^- := \Omega^- \cap B_R$, where B_R is the ball centered at the origin and radius R .

If $\varphi \in H_2^1(S)$ then $\mathcal{M}^-(\partial_j w) \in L_2(S)$ ($j = 1, 2, 3$), $w \in H_{2,loc}^{\frac{3}{2}}(\Omega^-)$, and

$$\|w\|_{H_2^{\frac{3}{2}}(\Omega_R^-)} \leq C_1(R) \|\varphi\|_{H_2^1(S)}.$$

(iii) The exterior Neumann BVP

$$a(\partial, \omega) w(x) = 0 \quad \text{in } \Omega^-, \quad w \in \text{Som}(\Omega^-),$$

$$[\Lambda(\partial, n) w(z)]^- = \psi(z) \quad \text{on } S, \quad \psi \in L_2(S),$$

$$\mathcal{M}^-(\nabla w) \in L_2(S),$$

is uniquely solvable and the solution is representable in the form

$$w(x) = (W_{a,\omega} - iV_{a,\omega}) (\mathcal{N}_{a,\omega}^{-1} \psi)(x), \quad x \in \Omega^-.$$

Moreover, $w \in H_{2,loc}^{\frac{3}{2}}(\Omega^-)$ and for arbitrary R there is a positive constant $C_2(R)$ independent of w and ψ such that

$$\|w\|_{H_2^{\frac{3}{2}}(\Omega_R^-)} \leq C_2(R) \|\psi\|_{L_2(S)}.$$

(iv) If two functions $g \in H_2^1(S)$ and $h \in L_2(S)$ are related by the equation $\mathcal{N}_{a,\omega}^{-1} h = \mathcal{D}_{a,\omega}^{-1} g$ on S , then g and h are Cauchy data on S of some radiating solution w of the homogeneous equation (2.5) in Ω^- , namely, $g = [w]^-$ and $h = [\Lambda(\partial, n) w]^-$ on S . Consequently, the Dirichlet and Neumann data for an arbitrary radiating solution w of the equation (2.5) are related on S by the following generalized Steklov-Poincaré type relation

$$\mathcal{N}_{a,\omega}^{-1} [\Lambda(\partial, n) w]^- = \mathcal{D}_{a,\omega}^{-1} [w]^-.$$

Proof. First we show the invertibility of the operator (2.32). Due to the results in [43] the operator $\mathcal{D}_a^{(0)} := -2^{-1} \mathcal{I} + \mathcal{K}_{a,0}^{(2)} : L_2(S) \rightarrow L_2(S)$ is Fredholm with index zero. In accordance with (2.10) and (2.25) the operator $\mathcal{D}_{a,\omega} - \mathcal{D}_a^{(0)} : L_2(S) \rightarrow L_2(S)$ is compact. Therefore it remains only to prove the injectivity of (2.32). To this end we show that the null space of the corresponding adjoint operator (without complex conjugation) is trivial.

Let $\psi \in L_2(S)$ be a solution to the homogeneous adjoint equation

$$[-2^{-1} \mathcal{I} + \mathcal{K}_{a,\omega}^{(1)}] \psi - i \mathcal{H}_{a,\omega} \psi = 0 \quad \text{on } S.$$

It then follows that the single layer potential $V_{a,\omega}(\psi) \in \mathbb{H}_{a,\omega}(\Omega^+)$ solves the homogeneous Robin type problem with boundary condition (2.28). Therefore, $V_{a,\omega}(\psi) = 0$ in Ω^+ . This implies $[V_{a,\omega}(\psi)]^- = 0$ on S . Thus $V_{a,\omega}(\psi)$ solves the homogeneous exterior Dirichlet BVP. Write Green's formula (2.29) for the function $w = V_{a,\omega}(\psi)$ and for the region $\Omega_R^- = \Omega^- \cap B_R$ where B_R is the ball centered at the origin and radius R , and $\partial B_R = \Sigma_R$,

$$\begin{aligned} \int_{\Omega_R^-} [a_{kj} \partial_j V_{a,\omega}(\psi) \overline{\partial_k V_{a,\omega}(\psi)} - \rho_2 \omega^2 |V_{a,\omega}(\psi)|^2] dx = \\ = \int_{\Sigma_R} [\Lambda V_{a,\omega}(\psi)] [\overline{V_{a,\omega}(\psi)}] dS. \end{aligned}$$

Evidently

$$\Im \left\{ \int_{\Sigma_R} [\Lambda(\partial, n) V_{a,\omega}(\psi)] [\overline{V_{a,\omega}(\psi)}] d\Sigma_R \right\} = 0,$$

and in accordance with Lemma 2.1 we get $V_{a,\omega}(\psi) = 0$ in Ω^- . Therefore $[\Lambda(\partial, n) V_{a,\omega}(\psi)]^- - [\Lambda(\partial, n) V_{a,\omega}(\psi)]^+ = \psi = 0$ from which the injectivity and, consequently, the invertibility of the operator (2.32) follows.

As it is shown in the references [43] and [41] the operator $\mathcal{D}_a^{(0)} =: H_2^1(S) \rightarrow H_2^1(S)$ is Fredholm with index zero as well. Therefore the invertibility of the operator (2.33) follows from its injectivity.

Analogously it can be shown that the operator (2.34), as a compact perturbation of an invertible operator, is Fredholm with zero index (cf. [41]). On the other hand with the help of the same arguments as above we easily derive that the null space of the operator $\mathcal{N}_{a,\omega}$ is trivial. Therefore (2.34) is an isomorphism.

Proof of the items (ii) and (iii) are quite similar to the proofs of Theorem 6.2 and Proposition 6.8 in [1] (see also the proof of Lemma 4.1 in [16] and Theorem 5.6 in [41]).

The item (iv) immediately follows from the item (i) and the uniqueness theorems for the exterior Dirichlet and Neumann boundary value problems. \square

2.5. Special Robin type problem. Properties of plane waves.

Let us consider the interior Robin type BVP

$$a(\partial, \omega) w(x) = 0 \quad \text{in } \Omega^+, \quad w \in \mathbb{H}_{a,\omega}(\Omega^+), \quad (2.35)$$

$$[\Lambda(\partial, n) w(z) - i w(z)]^- = \psi \quad \text{on } S, \quad \psi \in L_2(S). \quad (2.36)$$

If we look for a solution in the form of a single layer potential $w(x) = V_{a,\omega}(g)(x)$, we arrive at the integral equation on S

$$\mathcal{P}_{a,\omega} g := \left[-2^{-1} \mathcal{I} + \mathcal{K}_{a,\omega}^{(1)} - i \mathcal{H}_{a,\omega} \right] g = \psi,$$

where $\mathcal{P}_{a,\omega} : L_2(S) \rightarrow L_2(S)$ is a Fredholm operator with zero index due to Lemma 2.2.(iv).

Lemma 2.6. (i) *The BVP (2.35)-(2.36) is uniquely solvable.*

(ii) *The operator $\mathcal{P}_{a,\omega} : L_2(S) \rightarrow L_2(S)$ is invertible.*

(iii) *An arbitrary solution $w \in \mathbb{H}_{a,\omega}(\overline{\Omega^+})$ of the equation (2.35) is uniquely representable in the form*

$$w(x) = V_{a,\omega} \left(\mathcal{P}_{a,\omega}^{-1} [\Lambda(\partial, n) w - i w]^+ \right) (x), \quad x \in \overline{\Omega^+}.$$

Moreover, for $\overline{\Omega_0^+} \in \Omega^+$ there holds the uniform estimate

$$|w(x)| \leq C \delta^{-1} \| [\Lambda(\partial, n) w - i w]^+ \|_{L_2(S)} \quad \text{for all } x \in \overline{\Omega_0^+},$$

where C is a positive constant independent of w and δ . Here δ is the distance between Ω_0^+ and S .

Proof. The items (i) and (ii) have been shown as intermediate steps in the proof of Lemma 2.5. The item (iii) is then a direct consequence of the invertibility of the operator $\mathcal{P}_{a,\omega}$. \square

From Lemma 2.6 it follows that the plane wave $\exp\{i d \cdot x\}$, where $d \in S_\omega$, can be uniquely represented in the form

$$e^{i d \cdot x} = V_{a,\omega} \left(\mathcal{P}_{a,\omega}^{-1} [(\Lambda(\partial, n) - i) e^{i d \cdot \zeta}]_S^+ \right) (x), \quad x \in \Omega^+.$$

Note, that $\exp\{i d \cdot x\}$ with $d \in S_\omega$ is a non-radiating solution to the homogeneous equation (2.5) in \mathbb{R}^3 . Let

$$P(S) := \{p(x; d) \equiv (\Lambda(\partial_x, n(x)) - i) e^{i d \cdot x}, \quad x \in S : d \in S_\omega\},$$

$$P_{sp}(S) := \left\{ \sum_{q=1}^m c_q p(x; d^{(q)}), \quad x \in S : p(x; d^{(q)}) \in P(S), \right. \\ \left. c_q \in \mathbb{C}, \quad d^{(q)} \in S_\omega, \quad m \in \mathbb{N} \right\},$$

$$P_{sp}(\mathbb{R}^3) := \left\{ \sum_{q=1}^m c_q e^{i d^{(q)} \cdot x}, \quad x \in \mathbb{R}^3 : c_q \in \mathbb{C}, \quad d^{(q)} \in S_\omega, \quad m \in \mathbb{N} \right\};$$

here \mathbb{N} and \mathbb{C} are the sets of all natural and complex numbers, respectively.

Lemma 2.7. *The set $P(S)$ is complete in $L_2(S)$.*

Proof. Let $f \in L_2(S)$ and

$$\int_S [(\Lambda(\partial_y, n(y)) - i) e^{i d \cdot y}] f(y) dS_y = 0 \quad (2.37)$$

for all $d \in S_\omega$.

Let us consider the function

$$w(x) = (W_{a,\omega} - i V_{a,\omega})(f)(x), \quad x \in \mathbb{R}^3 \setminus S.$$

Clearly, in view of (2.11) we have

$$w(x) = c(\xi) \frac{\exp\{i \xi \cdot x\}}{|x|} \int_S [(\Lambda(\partial_y, n(y)) - i) e^{-i \xi \cdot y}] f(y) dS_y + \mathcal{O}(|x|^{-2})$$

as $|x| \rightarrow +\infty$, where $\xi \in S_\omega$ corresponds to \hat{x} and $c(\xi)$ is defined by (2.11).

By (2.37) we conclude $w(x) = \mathcal{O}(|x|^{-2})$, which implies $w(x) = 0$ in Ω^- due to Lemma 2.1. Therefore, we obtain $[w(x)]^- = \mathcal{D}_{a,\omega} f = 0$ on S . By Lemma 2.5.(i) then we have $f = 0$ on S . This completes the proof. \square

Lemma 2.8. *Let Ω^+ be a bounded Lipschitz domain such that $\Omega^- = \mathbb{R}^3 \setminus \overline{\Omega^+}$ be connected and let $w \in \mathbb{H}_{a,\omega}(\Omega^+)$ be a solution to the homogeneous equation (2.35) in Ω^+ .*

Then there exists a sequence $v_m \in P_{sp}(\mathbb{R}^3)$ such that $v_m \rightarrow w$ and $\partial^\beta v_m \rightarrow \partial^\beta w$ as $m \rightarrow \infty$ uniformly on compact subsets of Ω^+ ($\beta = (\beta_1, \beta_2, \beta_3)$ is an arbitrary multi-index and $\partial^\beta = \partial_1^{\beta_1} \partial_2^{\beta_2} \partial_3^{\beta_3}$).

Proof. From Lemma 2.7 it follows that there exists in $P_{sp}(S)$ a sequence of type

$$\sum_{q=1}^m c_q (\Lambda(\partial_x, n(x)) - i) e^{i d^{(q)} \cdot x}, \quad x \in S,$$

which converges (in the L_2 -sense) to the function $[(\Lambda(\partial, n) - i)w]^+ \in L_2(S)$.

We set

$$v_m(x) = \sum_{q=1}^m c_q e^{i d^{(q)} \cdot x}, \quad x \in \overline{\Omega^+}.$$

Hence, $(\Lambda(\partial, n) - i)v_m(x) \rightarrow [(\Lambda(\partial, n) - i)w(x)]^+$ in $L_2(S)$. By Lemma 2.6 the functions v_m and w can be represented in the form

$$v_m(x) = V_{a,\omega} (\mathcal{P}_{a,\omega}^{-1} [(\Lambda(\partial, n) - i)v_m]^+) (x), \quad x \in \Omega^+,$$

$$w(x) = V_{a,\omega} (\mathcal{P}_{a,\omega}^{-1} [(\Lambda(\partial, n) - i)w]^+) (x), \quad x \in \Omega^+.$$

Now, let $\overline{\Omega_0^+} \subset \Omega^+$ and $x \in \overline{\Omega_0^+}$. Denote by δ the distance between $\overline{\Omega_0^+}$ and $S = \partial\Omega^+$. The above representations of v_m and w together with Lemma 2.6 then imply

$$\begin{aligned} |\partial^\beta w(x) - \partial^\beta v_m(x)| &\leq \\ &\leq C \delta^{-1} \|\mathcal{P}_{a,\omega}^{-1} [(\Lambda(\partial, n) - i)v_m]^+ - \mathcal{P}_{a,\omega}^{-1} [(\Lambda(\partial, n) - i)w]^+\|_{L_2(S)} \leq \\ &\leq C_1 \delta^{-1} \|[(\Lambda(\partial, n) - i)v_m]^+ - [(\Lambda(\partial, n) - i)w]^+\|_{L_2(S)} \rightarrow 0 \end{aligned}$$

as $m \rightarrow +\infty$ (uniformly in $\overline{\Omega_0^+}$) for arbitrary multi-index β . \square

Corollary 2.9. *Let $x_0 \notin \overline{\Omega^+}$. Then there exists a sequence $v_m \in P_{sp}(\mathbb{R}^3)$ such that (for arbitrary multi-index β)*

$$\partial^\beta v_m(x) \rightarrow \partial^\beta \gamma(x - x_0, \omega)$$

uniformly in $\overline{\Omega^+}$, i.e.,

$$\|v_m(x) - \gamma(x - x_0, \omega)\|_{C^k(\overline{\Omega^+})} \rightarrow 0 \quad \text{as } m \rightarrow \infty$$

for arbitrary integer $k \geq 0$.

2.6. Vector-valued potential operators of the theory of steady state elastic oscillations. Denote by $\Gamma(x, \omega)$ the fundamental matrix (Kupradze matrix) of the steady state elastic oscillation operator $A(\partial, \omega)$, i.e., $A(\partial, \omega) \Gamma(x, \omega) = I_3 \delta(x)$ (for details see [28], Ch. II),

$$\Gamma(x, \omega) = [\Gamma_{kj}(x, \omega)]_{3 \times 3},$$

$$\Gamma_{kj}(x, \omega) = \sum_{l=1}^2 (\delta_{kj} \alpha_l + \beta_l \partial_k \partial_j) \frac{\exp\{i k_l |x|\}}{|x|}, \quad (2.38)$$

where

$$k_1^2 = \varrho_1 \omega^2 (\lambda + 2\mu)^{-1}, \quad k_2^2 = \varrho_1 \omega^2 \mu^{-1},$$

$$\alpha_l = -\delta_{2l} (4\pi\mu)^{-1}, \quad \beta_l = (-1)^{l+1} (4\pi\varrho_1 \omega^2)^{-1}.$$

Note that the principal singular part of $\Gamma(x, \omega)$ in a vicinity of the origin is the fundamental matrix $\Gamma(x)$ (Kelvin's matrix) of the operator $A(\partial)$ of elastostatics,

$$\Gamma(x) = [\Gamma_{kj}(x)]_{3 \times 3}, \quad \Gamma_{kj}(x) = \lambda' \delta_{kj} |x|^{-1} + \mu' x_k x_j |x|^{-3}, \quad (2.39)$$

where

$$\lambda' = -(\lambda + 3\mu) [8\pi\mu(\lambda + 2\mu)]^{-1}, \quad \mu' = -(\lambda + \mu) [8\pi\mu(\lambda + 2\mu)]^{-1}.$$

It is easy to show that in a vicinity of the origin ($|x| < 1$ say)

$$|\Gamma_{kj}(x, \omega) - \Gamma_{kj}(x)| < \omega C_0(\lambda, \mu),$$

$$|\partial_l [\Gamma_{kj}(x, \omega) - \Gamma_{kj}(x)]| < \omega^2 C_1(\lambda, \mu), \quad (2.40)$$

$$\partial_l \partial_m [\Gamma_{kj}(x, \omega) - \Gamma_{kj}(x)] = \mathcal{O}(|x|^{-1}),$$

where $k, j, l, m = 1, 2, 3$, and $C_0(\lambda, \mu)$ and $C_1(\lambda, \mu)$ are positive constants depending only on the Lamé parameters.

A vector u belongs to the class $\text{SK}(\Omega^-)$ if $u \in [C^1(\Omega^-)]^3$ and for sufficiently large $|x|$ the following relations hold:

$$u(x) = u^{(p)}(x) + u^{(s)}(x),$$

$$\Delta u^{(p)}(x) + k_1^2 u^{(p)}(x) = 0, \quad \partial_j u^{(p)}(x) - i \hat{x}_j k_1 u^{(p)}(x) = \mathcal{O}(|x|^{-2}),$$

$$\Delta u^{(s)}(x) + k_2^2 u^{(s)}(x) = 0, \quad \partial_j u^{(s)}(x) - i \hat{x}_j k_2 u^{(s)}(x) = \mathcal{O}(|x|^{-2}),$$

where $\hat{x} = x/|x|$ and $j = 1, 2, 3$. These conditions are *the Sommerfeld-Kupradze type radiation conditions* in the elasticity theory (for details see [28]).

From (2.38) it follows that each column of the matrix $\Gamma(\cdot, \omega)$ belongs to $\text{SK}(\mathbb{R}^3 \setminus \{0\})$.

The analogue of Rellich's lemma in the elasticity theory reads as follows (for details see [28], [34]).

Lemma 2.10. *Let $u \in \text{SK}(\Omega^-)$ be a solution of (2) in Ω^- and let*

$$\lim_{R \rightarrow +\infty} \Im \left\{ \int_{\Sigma_R} T(\partial, n) u \cdot u \, d\Sigma_R \right\} = 0,$$

where Σ_R is the same as in Lemma 2.1. Then $u = 0$ in Ω^- .

This lemma implies that the basic exterior homogeneous BVPs of steady state elastic oscillations (with given zero displacements or stresses on the boundary) have only the trivial solution (see [28]).

Further, we construct the single and double layer vector potentials,

$$\begin{aligned} V_{A,\omega}(g)(x) &= \int_S \Gamma(x-y, \omega) g(y) dS_y, \\ W_{A,\omega}(g)(x) &= \int_S [T(\partial_y, n(y)) \Gamma(y-x, \omega)]^\top g(y) dS_y. \end{aligned}$$

For a solution $u \in \mathbb{H}_{A,\omega}(\Omega^+)$ of equation (2) in Ω^+ we have the following integral representation

$$W_{A,\omega}([u]^+)(x) - V_{A,\omega}([Tu]^+)(x) = \begin{cases} u(x) & \text{for } x \in \Omega^+, \\ 0 & \text{for } x \in \Omega^-. \end{cases} \quad (2.41)$$

The similar representation holds also for a radiating solution $u \in \mathbb{H}_{A,\omega}(\Omega^-) \cap \text{SK}(\Omega^-)$ to the equation (2) in Ω^- ,

$$V_{A,\omega}([Tu]^-)(x) - W_{A,\omega}([u]^-)(x) = \begin{cases} u(x) & \text{for } x \in \Omega^-, \\ 0 & \text{for } x \in \Omega^+. \end{cases}$$

These representations follow from the corresponding Green's formulas which hold for functions of the classes $\mathbb{H}_{A,\omega}(\Omega^+)$ and $\mathbb{H}_{A,\omega}(\Omega^-) \cap \text{SK}(\Omega^-)$ (cf. Ch. VII in [28], Proposition 6.6 in [1], and Proposition 5.5 in [2]).

Further, we introduce the boundary operators on S generated by the above vector potentials

$$\mathcal{H}_{A,\omega} g(z) := \int_S \Gamma(z-y, \omega) g(y) dS_y, \quad (2.42)$$

$$\mathcal{K}_{A,\omega}^{(1)} g(z) := \int_S [T(\partial_z, n(z)) \Gamma(z-y, \omega)] g(y) dS_y, \quad (2.43)$$

$$\mathcal{K}_{A,\omega}^{(2)} g(z) := \int_S [T(\partial_y, n(y)) \Gamma(y-z, \omega)]^\top g(y) dS_y,$$

$$\mathcal{L}_{A,\omega} g(z) := [T(\partial_z, n(z)) W_{A,\omega}(g)(z)]^\pm.$$

Note that the potential and boundary operators $V_{A,\omega}$, $W_{A,\omega}$, $\mathcal{H}_{A,\omega}$, $\mathcal{K}_{A,\omega}^{(1)}$, $\mathcal{K}_{A,\omega}^{(2)}$, and $\mathcal{L}_{A,\omega}$ have quite the same jump and mapping properties as the corresponding scalar operators considered in Subsection 2.4 (see Lemma 2.3).

Lemma 2.11. *Let $g \in [L_2(S)]^3$, $h \in [H_2^1(S)]^3$, and $-\frac{1}{2} \leq r \leq \frac{1}{2}$. Then*

(i) *the potentials $V_{A,\omega}(g)$, $W_{A,\omega}(g)$, and $W_{A,\omega}(h)$ are radiating solutions of equation (2) in $\mathbb{R}^3 \setminus S$ and*

$$\begin{aligned} \mathcal{M}^\pm(W_{A,\omega}(g)) &\in [L_2(S)]^3, & \mathcal{M}^\pm(\partial_j W_{A,\omega}(h)) &\in [L_2(S)]^3, \\ \mathcal{M}^\pm(V_{A,\omega}(g)) &\in [L_2(S)]^3, & \mathcal{M}^\pm(\partial_j V_{A,\omega}(g)) &\in [L_2(S)]^3, \\ W_{A,\omega}(g) &\in [H_2^{\frac{1}{2}}(\Omega^+)]^3, & W_{A,\omega}(g) &\in [H_{2,\text{loc}}^{\frac{1}{2}}(\Omega^-)]^3 \cap \text{SK}(\Omega^-), \\ W_{A,\omega}(h) &\in \mathbb{H}_{A,\omega}(\Omega^\pm) \cap \text{SK}(\Omega^-), & V_{A,\omega}(g) &\in \mathbb{H}_{A,\omega}(\Omega^\pm) \cap \text{SK}(\Omega^-); \end{aligned}$$

(ii) *the following jump relations hold on S for almost all $z \in S$*

$$[V_{A,\omega}(g)(z)]^\pm = \mathcal{H}_{A,\omega} g(z),$$

$$[T(\partial, n) V_{A,\omega}(g)(z)]^\pm = \left[\mp 2^{-1} I_3 + \mathcal{K}_{A,\omega}^{(1)} \right] g(z),$$

$$[W_{A,\omega}(g)(z)]^\pm = \left[\pm 2^{-1} I_3 + \mathcal{K}_{A,\omega}^{(2)} \right] g(z),$$

$$[T(\partial, n) W_{A,\omega}(h)(z)]^+ = [T(\partial, n) W_{a,\omega}(h)(z)]^- =: \mathcal{L}_{A,\omega} h(z); \quad (2.44)$$

(iii) *the operators*

$$\mathcal{H}_{A,\omega} : [H_2^{-\frac{1}{2}+r}(S)]^3 \rightarrow [H_2^{\frac{1}{2}+r}(S)]^3,$$

$$\mathcal{K}_{A,\omega}^{(2)} : [H_2^{\frac{1}{2}+r}(S)]^3 \rightarrow [H_2^{\frac{1}{2}+r}(S)]^3,$$

$$\mathcal{K}_{A,\omega}^{(1)} : [H_2^{-\frac{1}{2}+r}(S)]^3 \rightarrow [H_2^{-\frac{1}{2}+r}(S)]^3,$$

$$\mathcal{L}_{A,\omega} : [H_2^{\frac{1}{2}+r}(S)]^3 \rightarrow [H_2^{-\frac{1}{2}+r}(S)]^3,$$

are continuous;

(iv) *the operators*

$$\mathcal{H}_{A,\omega} : [H_2^{-\frac{1}{2}+r}(S)]^3 \rightarrow [H_2^{\frac{1}{2}+r}(S)]^3,$$

$$\pm 2^{-1} I_3 + \mathcal{K}_{A,\omega}^{(1)}, \pm 2^{-1} I_3 + \mathcal{K}_{A,\omega}^{(2)} : [L_2(S)]^3 \rightarrow [L_2(S)]^3,$$

$$\mathcal{L}_{A,\omega} : [H_2^1(S)]^3 \rightarrow [L_2(S)]^3,$$

are bounded Fredholm operators with zero index;

(v) *the following operator equations*

$$\mathcal{H}_{A,\omega} \mathcal{K}_{A,\omega}^{(1)} = \mathcal{K}_{A,\omega}^{(2)} \mathcal{H}_{A,\omega}, \quad \mathcal{L}_{A,\omega} \mathcal{H}_{A,\omega} = -4^{-1} I_3 + [\mathcal{K}_{A,\omega}^{(1)}]^2, \quad (2.45)$$

$$\mathcal{L}_{A,\omega} \mathcal{K}_{A,\omega}^{(2)} = \mathcal{K}_{A,\omega}^{(1)} \mathcal{L}_{A,\omega}, \quad \mathcal{H}_{A,\omega} \mathcal{L}_{A,\omega} = -4^{-1} I_3 + [\mathcal{K}_{A,\omega}^{(2)}]^2,$$

hold in appropriate function spaces.

Proof. The proof of the item (i) can be found in [2], Subsection 5.2 (see also [26]). The relations (2.40) show that the corresponding potential and boundary operators with subscripts $\omega \neq 0$ and $\omega = 0$ differ by smoothing (compact) operators. Therefore, the items (i)-(iv) immediately follow from the results obtained in [11] and [16] for the operators with $\omega = 0$ (see also [32]). Note that the relation (2.44) can be shown by the same arguments as (2.23) in the proof of Lemma 2.3.

The item (v) follows from the representation formula (2.41) which implies

$$\begin{aligned} [-2^{-1} I_3 + \mathcal{K}_{A,\omega}^{(2)}] [u]^+ &= \mathcal{H}_{A,\omega} [Tu]^+, \\ \mathcal{L}_{A,\omega} [u]^+ &= [2^{-1} I_3 + \mathcal{K}_{A,\omega}^{(1)}] [Tu]^+. \end{aligned} \quad (2.46)$$

The operator equalities (2.45) can be then obtained by substitution into (2.46) single and double layer potentials with densities from the spaces $[L_2(S)]^3$ and $[H_2^1(S)]^3$, respectively. It is evident that the first and second equations in (2.45) originally hold in $[L_2(S)]^3$ but they can be continuously extended to the space $[H_2^{-1}(S)]^3$ due to the mapping properties of the operators involved; analogously, the third and fourth equations which originally hold in $[H_2^1(S)]^3$, can be continuously extended to the space $[L_2(S)]^3$. \square

As in the scalar case we have the following

Lemma 2.12. *Let $g \in [H_2^1(S)]^3$ and*

$$u(x) = W_{A,\omega}(g)(x) - i V_{A,\omega}(g)(x), \quad x \in \Omega^-. \quad (2.47)$$

If u vanishes in Ω^- , then $g = 0$ on S .

Proof. If the function (2.47) vanishes in Ω^- due to the jump properties of the elastic layer potentials we conclude

$$[Tu]^+ - i [u]^+ = 0 \quad \text{on } S. \quad (2.48)$$

Since $g \in [H_2^1(S)]^3$ we have $u \in \mathbb{H}_{A,\omega}(\Omega^+)$ and there holds Green's formula (cf. [28], [1])

$$\int_{\Omega^+} [E(u, \bar{u}) - \varrho_1 \omega^2 |u|^2] dx = \int_S [Tu]^+ \cdot [u]^+ dS. \quad (2.49)$$

With the help of (2.3) and (2.48) we conclude from (2.49) that $[u]^+ = 0$ which yields $[u]^+ - [u]^- = g = 0$ on S . \square

Further, let

$$\begin{aligned} \mathcal{D}_{A,\omega} g &:= [-2^{-1} I_3 + \mathcal{K}_{A,\omega}^{(2)}] - i \mathcal{H}_{A,\omega}, \\ \mathcal{N}_{A,\omega} g &:= \mathcal{L}_{A,\omega} - i [2^{-1} I_3 + \mathcal{K}_{A,\omega}^{(1)}]. \end{aligned} \quad (2.50)$$

These operators are generated by the limiting values on S (from Ω^-) of the displacement vector (2.47) and the corresponding stress vector, i.e., $[u]^- = \mathcal{D}_{A,\omega} g$ and $[Tu]^- = \mathcal{N}_{A,\omega} g$.

Lemma 2.13. (i) *The operators*

$$\mathcal{D}_{A,\omega} : [L_2(S)]^3 \rightarrow [L_2(S)]^3, \quad (2.51)$$

$$: [H_2^1(S)]^3 \rightarrow [H_2^1(S)]^3, \quad (2.52)$$

$$\mathcal{N}_{A,\omega} : [H_2^1(S)]^3 \rightarrow [L_2(S)]^3, \quad (2.53)$$

are isomorphisms.

(ii) *The exterior Dirichlet BVP (with prescribed displacement vector)*

$$A(\partial, \omega) u(x) = 0 \quad \text{in } \Omega^-, \quad u \in \text{SK}(\Omega^-),$$

$$[u(z)]^- = \varphi(z) \quad \text{on } S, \quad \varphi \in [L_2(S)]^3,$$

$$\mathcal{M}^-(u) \in L_2(S),$$

is uniquely solvable and the solution is representable in the form

$$u(x) = (W_{A,\omega} - iV_{A,\omega}) (\mathcal{D}_{A,\omega}^{-1} \varphi)(x), \quad x \in \Omega^-.$$

Moreover, $u \in [H_{2,loc}^{\frac{1}{2}}(\Omega^-)]^3$ and for arbitrary R there is a positive constant $C_0(R)$ independent of u and φ such that

$$\|u\|_{[H_2^{\frac{1}{2}}(\Omega_R^-)]^3} \leq C_0(R) \|\varphi\|_{[L_2(S)]^3}$$

with $\Omega_R^- := \Omega^- \cap B_R$, where B_R is the ball centered at the origin and radius R .

If $\varphi \in [H_2^1(S)]^3$ then $\mathcal{M}^-(\partial_j u) \in [L_2(S)]^3$ ($j=1, 2, 3$), $u \in [H_{2,loc}^{\frac{3}{2}}(\Omega^-)]^3$, i.e., $u \in \mathbb{H}_{A,\omega}(\Omega^-)$ and

$$\|u\|_{[H_2^{\frac{3}{2}}(\Omega_R^-)]^3} \leq C_1(R) \|\varphi\|_{[H_2^1(S)]^3}.$$

(iii) *The exterior Neumann BVP (with prescribed stress vector)*

$$A(\partial, \omega) u(x) = 0 \quad \text{in } \Omega^-, \quad u \in \text{SK}(\Omega^-),$$

$$[T(\partial, n)u(z)]^- = \psi(z) \quad \text{on } S, \quad \psi \in [L_2(S)]^3,$$

$$\mathcal{M}^-(\partial_j u) \in [L_2(S)]^3, \quad j = 1, 2, 3,$$

is uniquely solvable and the solution is representable in the form

$$u(x) = (W_{A,\omega} - iV_{A,\omega}) (\mathcal{N}_{A,\omega}^{-1} \psi)(x), \quad x \in \Omega^-.$$

Moreover, $u \in [H_{2,loc}^{\frac{3}{2}}(\Omega^-)]^3$ and for arbitrary R there is a positive constant $C_2(R)$ independent of u and ψ such that

$$\|u\|_{[H_2^{\frac{3}{2}}(\Omega_R^-)]^3} \leq C_2(R) \|\psi\|_{[L_2(S)]^3}.$$

(iv) *If two vector functions $g \in [H_2^1(S)]^3$ and $h \in [L_2(S)]^3$ are related by the equation $\mathcal{N}_{A,\omega}^{-1} h = \mathcal{D}_{A,\omega}^{-1} g$ on S , then g and h are Cauchy data on S of some radiating solution u of the homogeneous equation (2) in Ω^- , namely, $g = [u]^-$ and $h = [T(\partial, n)u]^-$ on S . Consequently, the Dirichlet*

and Neumann data for an arbitrary radiating solution u of the equation (2) are related on S by the following Steklov-Poincaré type relation

$$\mathcal{N}_{A,\omega}^{-1} [T(\partial, n) u]^- = \mathcal{D}_{A,\omega}^{-1} [u]^-.$$

Proof. First we show the invertibility of the operator (2.51). Due to the results in [11] the operator $\mathcal{D}_A^{(0)} := -2^{-1} I_3 + \mathcal{K}_{A,0}^{(2)} : [L_2(S)]^3 \rightarrow [L_2(S)]^3$ is Fredholm with index zero. In accordance with (2.38), (2.39), and (2.40) the operator $\mathcal{D}_{A,\omega} - \mathcal{D}_A^{(0)} : [L_2(S)]^3 \rightarrow [L_2(S)]^3$ is compact. Therefore it remains only to prove that (2.51) is one to one. To this end we show that the null space of the corresponding adjoint operator (without complex conjugation) is trivial.

Let $\psi \in [L_2(S)]^3$ be a solution to the homogeneous adjoint equation

$$[-2^{-1} I_3 + \mathcal{K}_{A,\omega}^{(1)}] \psi - i \mathcal{H}_{A,\omega} \psi = 0 \quad \text{on } S.$$

It then follows that the single layer potential $V_{A,\omega}(\psi) \in \mathbb{H}_{A,\omega}(\Omega^+)$ solves the homogeneous Robin type problem with boundary condition (2.48). Therefore, $V_{A,\omega}(\psi) = 0$ in Ω^+ . This implies $[V_{A,\omega}(\psi)]^- = 0$ on S . Thus $V_{A,\omega}(\psi)$ solves the homogeneous exterior Dirichlet BVP. Write Green's formula (2.49) for the function $u = V_{A,\omega}(\psi)$ and for the region $\Omega_R^- = \Omega^- \cap B_R$ where as above B_R is the ball centered at the origin and radius R , and $\partial B_R = \Sigma_R$,

$$\int_{\Omega_R^-} [E(V_{A,\omega}(\psi), \overline{V_{A,\omega}(\psi)}) - \varrho_1 \omega^2 |V_{A,\omega}(\psi)|^2] dx = \int_{\Sigma_R} T V_{A,\omega}(\psi) \cdot V_{A,\omega}(\psi) dS.$$

Evidently, by (2.3)

$$\Im \left\{ \int_{\Sigma_R} T V_{A,\omega}(\psi) \cdot V_{A,\omega}(\psi) d\Sigma_R \right\} = 0,$$

and in accordance with Lemma 2.10 we get $V_{A,\omega}(\psi) = 0$ in Ω^- . Therefore by jump formulas $[T(\partial, n) V_{A,\omega}(\psi)]^- - [T(\partial, n) V_{A,\omega}(\psi)]^+ = \psi = 0$, from which the injectivity and, consequently, invertibility of the operator (2.32) follows.

As it is shown in [11] the operator $\mathcal{D}_A^{(0)} = : [H_2^1(S)]^3 \rightarrow [H_2^1(S)]^3$ is Fredholm with index zero as well. Therefore the invertibility of the operator (2.52) follows from its injectivity.

It can be proved that the operator (2.53) is a compact perturbation of the invertible operator $\mathcal{N}_{A,\tau}$ with $\Im \tau > 0$. Therefore (2.53) is Fredholm with zero index. On the other hand with the help of the same arguments as above we easily derive that the null space of the operator (2.53) is trivial and, consequently, it is an isomorphism.

Proof of the items (ii) and (iii) are quite similar to the proofs of Theorem 6.2 and Proposition 6.8 in [1] (see also the proof of Lemma 4.1 in [16]).

The item (iv) immediately follows from the item (i) and the uniqueness theorems for the exterior Dirichlet and Neumann boundary value problems. \square

Remark 2.14. Note that for $-\frac{1}{2} \leq r \leq \frac{1}{2}$ the operators

$$\mathcal{D}_{a,\omega} : H_2^{\frac{1}{2}+r}(S) \rightarrow H_2^{\frac{1}{2}+r}(S), \quad (2.54)$$

$$\mathcal{N}_{a,\omega} : H_2^{\frac{1}{2}+r}(S) \rightarrow H_2^{-\frac{1}{2}+r}(S), \quad (2.55)$$

$$\mathcal{D}_{A,\omega} : [H_2^{\frac{1}{2}+r}(S)]^3 \rightarrow [H_2^{\frac{1}{2}+r}(S)]^3, \quad (2.56)$$

$$\mathcal{N}_{A,\omega} : [H_2^{\frac{1}{2}+r}(S)]^3 \rightarrow [H_2^{-\frac{1}{2}+r}(S)]^3 \quad (2.57)$$

are continuous due to Lemmas 2.3.(iii) and 2.11.(iii).

Moreover, these operators are invertible. In fact, invertibility of the operators (2.54) and (2.56) follows from Lemmas 2.5.(i) and 2.13.(i) by duality and interpolation arguments (cf. [26], Section 2).

To show that (2.55) is an isomorphism, we proceed as follows. It can easily be shown that $\mathcal{L}_{a,\omega} : H_2^{\frac{1}{2}+r}(S) \rightarrow H_2^{-\frac{1}{2}+r}(S)$ and its adjoint $\mathcal{L}_{a,\omega}^* : H_2^{\frac{1}{2}-r}(S) \rightarrow H_2^{-\frac{1}{2}-r}(S)$ coincide on the space $H_2^1(S)$. Therefore, it is reasonable to use the same symbol $\mathcal{L}_{a,\omega}$ for the operator $\mathcal{L}_{a,\omega}^*$. In particular, for $r = 0$ the operator $\mathcal{L}_{a,\omega}$ is self-adjoint.

Further, the invertibility of the operator (2.34) implies that $\mathcal{L}_{a,\omega} : H_2^1(S) \rightarrow H_2^0(S)$ and $\mathcal{L}_{a,\omega} : H_2^0(S) \rightarrow H_2^{-1}(S)$ are Fredholm operators with zero index. Consequently, by interpolation $\mathcal{L}_{a,\omega} : H_2^{\frac{1}{2}+r}(S) \rightarrow H_2^{-\frac{1}{2}+r}(S)$ is Fredholm with zero index.

Some further analysis show that the null spaces of the operator (2.55) is the same for all $-\frac{1}{2} \leq r \leq \frac{1}{2}$. Therefore (2.55) is injective, since its kernel is trivial for $r = \frac{1}{2}$. Whence, we conclude that (2.55) is an isomorphism. The proof for (2.57) is word for word.

It should be mentioned that for $g \in H_2^{\frac{1}{2}+r}(S)$ with $-\frac{1}{2} \leq r < \frac{1}{2}$ the non-tangential limits on S of $\partial_j W_{a,\omega}(g)$ do not exist, in general, and $[\Lambda(\partial, n)w]_S^- = \mathcal{L}_{a,\omega}(g)$ is to be understood as a generalized (functional) trace on S of the co-normal derivative of the double layer potential (cf. [39] for smooth domains).

3. THE DIRECT FLUID-STRUCTURE INTERACTION PROBLEM

3.1. Uniqueness theorem. Jones modes and Jones eigenfrequencies. We denote by $J(\Omega^+)$ the set of values of the frequency parameter $\omega > 0$ for which the following boundary value problem

$$\begin{aligned} A(\partial, \omega) u(x) &= 0, \quad x \in \Omega^+, \\ [T(\partial, n)u(x)]^+ &= 0, \quad [u(x) \cdot n(x)]^+ = 0, \quad x \in S, \end{aligned}$$

admits a nontrivial solution $u \in \mathbb{H}_{A,\omega}(\Omega^+)$. Such solutions (vectors) are called *Jones modes*, while the corresponding values of ω are called *Jones eigenfrequencies* (cf., e.g., [29], [22]). The space of Jones modes corresponding to ω we denote by $X_\omega(\Omega^+)$. Note that $J(\Omega^+)$ is at most enumerable, and for each $\omega \in J(\Omega^+)$ the space of associated Jones modes is of finite dimension (see [34], [1]). Clearly, if $u \in X_\omega(\Omega^+)$, then $\bar{u} \in X_\omega(\Omega^+)$.

The uniqueness result for the homogeneous direct problem ($f_0 = 0$ and $f = 0$ in (2.14) and (2.15)) is given by the following assertion.

Theorem 3.1. *Let a pair $(u, w^{sc}) \in \mathbb{H}_{A,\omega}(\Omega^+) \times [\mathbb{H}_{a,\omega}(\Omega^-) \cap \text{Som}(\Omega^-)]$ be a solution of the homogeneous direct problem (2), (2.5), (2.14), and (2.15), where $b_1 b_2 \neq 0$ and $\Im [\bar{b}_1 b_2] = 0$. Then $w^{sc} = 0$ in Ω^- and $u \in X_\omega(\Omega^+)$.*

Proof. Let $(u, w^{sc}) \in \mathbb{H}_{A,\omega}(\Omega^+) \times [\mathbb{H}_{a,\omega}(\Omega^-) \cap \text{Som}(\Omega^-)]$ be a solution pair to the homogeneous problem $(P)^{dir}$. By Green's formulas (2.29) and (2.49) we then have

$$\begin{aligned} \int_{\Omega_R^-} [a_{kj} \partial_j w \overline{\partial_k w} - \varrho_2 \omega^2 |w|^2] dx &= \\ &= - \int_S [\bar{w}]^- [\Lambda w]^- dS + \int_{\Sigma_R} \bar{w} \Lambda w d\Sigma_R, \end{aligned} \quad (3.1)$$

$$\int_{\Omega^+} [E(u, \bar{u}) - \varrho_1 \omega^2 |u|^2] dx = \int_S [Tu]^+ \cdot [u]^+ dS, \quad (3.2)$$

where $R > 0$ is a sufficiently large number.

Since $[Tu]^+ \cdot [u]^+ = \bar{b}_1 b_2 [w]^- [\overline{\Lambda(\partial, n)w}]^-$ and $\bar{b}_1 b_2$ is a real number different of zero, from (3.1) and (3.2) we get

$$\Im \int_{\Sigma_R} w \overline{\Lambda w} d\Sigma_R = 0.$$

Whence $w = 0$ in Ω^- by Lemma 2.1. From the homogeneous conditions (2.14) and (2.15) it follows that $u \in X_\omega(\Omega^+)$, which completes the proof. \square

Corollary 3.2. *Let $\omega \notin J(\Omega^+)$. Then the homogeneous direct problem possesses only the trivial solution.*

3.2. Existence results. First we prove the following

Lemma 3.3. *An arbitrary solution $u \in \mathbb{H}_{A,\omega}(\Omega^+)$ of equation (2) is representable in the form of a single layer potential.*

Proof. Let us consider the following boundary value problem

$$A(\partial, \omega) u(x) = 0 \quad \text{in } \Omega^+, \quad u \in \mathbb{H}_{A,\omega}(\Omega^+), \quad (3.3)$$

$$[T(\partial, n)u(x)]^+ - i[u(x)]^+ = \Phi(x), \quad x \in S, \quad (3.4)$$

where $\Phi = (\Phi_1, \Phi_2, \Phi_3)^\top \in [L_2(S)]^3$ is an arbitrary vector-function.

We look for a solution to the BVP (3.3)-(3.4) in the form of a single layer potential

$$u(x) = V_{A,\omega}(g)(x) \quad x \in \Omega^+,$$

where $g = (g_1, g_2, g_3)^\top \in [L_2(S)]^3$ is a sought for density.

The boundary condition (3.4) leads then to the Fredholm system of integral equations with index equal to zero

$$[-2^{-1}I_3 + \mathcal{K}_{A,\omega}^{(1)} - i\mathcal{H}_{A,\omega}]g = \Phi, \quad (3.5)$$

where $\mathcal{K}_{A,\omega}^{(1)}$ and $\mathcal{H}_{A,\omega}$ are given by (2.43) and (2.42), respectively.

Further we show that the operator

$$\mathcal{P}_{A,\omega} := -2^{-1}I_3 + \mathcal{K}_{A,\omega}^{(1)} - i\mathcal{H}_{A,\omega} : [L_2(S)]^3 \rightarrow [L_2(S)]^3 \quad (3.6)$$

is invertible. To this end we first prove that the homogeneous BVP (3.3)-(3.4) has only the trivial solution. With the help of Green's identity (3.2) and the condition (3.4) with $\Phi = 0$ we arrive at the equation

$$\int_{\Omega^+} \{E(u, \bar{u}) - \omega^2 |u|^2\} dx = -i \int_S |[u]^+|^2 dS.$$

Whence it follows that $[u]^+ = 0$ on S . Therefore, $[Tu]^+ = 0$ on S in view of (3.4). With the help of the general integral representation (2.41) we conclude that $u = 0$ in Ω^+ , which shows that the homogeneous BVP in question has only the trivial solution.

Let $g_0 \in [L_2(S)]^3$ be an arbitrary solution to the homogeneous system (3.5) ($\Phi = 0$). The potential $V_{A,\omega}(g_0) \in \mathbb{H}_{A,\omega}(\Omega^+)$ solves then the homogeneous BVP (3.3)-(3.4) and therefore

$$V_{A,\omega}(g_0)(x) = 0, \quad x \in \Omega^+. \quad (3.7)$$

Using the continuity property of the single layer potential, from (3.7) we have

$$[V_{A,\omega}(g_0)(x)]^+ = [V_{A,\omega}(g_0)(x)]^- = 0.$$

Evidently, $V_{A,\omega}(g_0)(x) \in \mathbb{H}_{A,\omega}(\Omega^-) \cap \text{SK}(\Omega^-)$ solves the homogeneous Dirichlet type exterior BVP (with zero displacements on S). By Lemma 2.13.(ii) we get $V_{A,\omega}(g_0)(x) = 0$ in Ω^- and taking into consideration the jump relation $[TV_{A,\omega}(g_0)]^- - [TV_{A,\omega}(g_0)]^+ = g_0$, we conclude that $g_0 = 0$ on S , i.e., $\ker \mathcal{P}_{A,\omega}$ is trivial.

Since $\mathcal{P}_{A,\omega}$ is a Fredholm operator of zero index in accordance with Lemma 2.11.(iv), it follows that the operator (3.6) is invertible. Therefore, from (3.5) we have

$$g = \mathcal{P}_{A,\omega}^{-1} \Phi = \mathcal{P}_{A,\omega}^{-1} \{[T(\partial, n)u]^+ - i[u]^+\}.$$

In turn, this proves that an arbitrary solution $u \in \mathbb{H}_{A,\omega}(\Omega^+)$ to equation (2) in Ω^+ can be represented as a single layer potential

$$u(x) = V_{A,\omega}(\mathcal{P}_{A,\omega}^{-1} F)(x) \quad \text{with} \quad F(x) := [T(\partial, n)u(x)]^+ - i[u(x)]^+.$$

This completes the proof. \square

Further, we show that the nonhomogeneous problem $P^{(dir)}$ is solvable for arbitrary incident wave and for arbitrary value of the oscillation parameter ω .

Let us look for a solution to the problem $(P)^{dir}$ in the following form:

$$u(x) = V_{A,\omega}(g)(x), \quad x \in \Omega^+, \quad (3.8)$$

$$w^{sc}(x) = W_{a,\omega}(g_4)(x) - i V_{a,\omega}(g_4)(x), \quad x \in \Omega^-, \quad (3.9)$$

where $g = (g_1, g_2, g_3)^\top \in [L_2(S)]^3$ and $g_4 \in H_2^1(S)$ are sought for densities.

The boundary conditions (2.14) and (2.15) lead to the system of integral equations on S ,

$$\left[-2^{-1}I_3 + \mathcal{K}_{A,\omega}^{(1)} \right] g(x) - b_2 n(x) \mathcal{D}_{a,\omega} g_4(x) = f(x), \quad (3.10)$$

$$[\mathcal{H}_{A,\omega} g(x)] \cdot n(x) - b_1 \mathcal{N}_{a,\omega} g_4(x) = f_0(x), \quad (3.11)$$

where $\mathcal{K}_{A,\omega}^{(1)}$, $\mathcal{H}_{A,\omega}$, $\mathcal{D}_{a,\omega}$, $\mathcal{N}_{a,\omega}$, f , and f_0 are given by (2.43), (2.42), (2.30), (2.31), (2.18), and (2.17), respectively. The constants b_1 and b_2 are defined by (2.16).

Let us remark that by the above approach Problem $P^{(dir)}$ is *equivalently* reduced to the system of integral equations (3.10)-(3.11).

The matrix operator generated by the left-hand side expressions in (3.10) and (3.11) reads as

$$\mathcal{K} := \begin{bmatrix} \left[-2^{-1}I_3 + \mathcal{K}_{A,\omega}^{(1)} \right]_{3 \times 3} & \left[-b_2 n_k(x) \mathcal{D}_{a,\omega} \right]_{3 \times 1} \\ \left[n_j(x) (\mathcal{H}_{A,\omega})_{jk} \right]_{1 \times 3} & -b_1 \mathcal{N}_{a,\omega} \end{bmatrix}_{4 \times 4}. \quad (3.12)$$

Therefore, the system (3.10) and (3.11) can be rewritten in the matrix form

$$\mathcal{K} G = F,$$

where $G = (g_1, g_2, g_3, g_4)^\top \in [L_2(S)]^3 \times H_2^1(S)$ and $F = (f_1, f_2, f_3, f_0)^\top \in [L_2(S)]^4$.

From the mapping properties of boundary integral operators described in Subsections 2.4 and 2.6 it follows that

$$\mathcal{K} : [L_2(S)]^3 \times H_2^1(S) \rightarrow [L_2(S)]^4. \quad (3.13)$$

Further, we show that the operator (3.13) is Fredholm with zero index and establish the necessary and sufficient conditions for solvability of the system (3.10)-(3.11).

To this end, we represent the operator \mathcal{K} as

$$\mathcal{K} = \mathcal{T}_1 + \mathcal{T}_2,$$

where

$$\mathcal{T}_1 := \begin{bmatrix} [\mathcal{P}_{A,\omega}]_{3 \times 3} & [0]_{3 \times 1} \\ [0]_{1 \times 3} & -b_1 \mathcal{N}_{a,\omega} \end{bmatrix}_{4 \times 4}, \quad (3.14)$$

$$\begin{aligned} \mathcal{T}_2 &:= \begin{bmatrix} [i\mathcal{H}_{A,\omega}]_{3 \times 3} & [-b_2 n(x) \mathcal{D}_{a,\omega}]_{3 \times 1} \\ [n_j(x) (\mathcal{H}_{A,\omega})_{jk}]_{1 \times 3} & 0 \end{bmatrix}_{4 \times 4} = \\ &= \begin{bmatrix} [I_3]_{3 \times 3} & [n(x)]_{3 \times 1} \\ [n(x)]_{1 \times 3} & 0 \end{bmatrix}_{4 \times 4} \begin{bmatrix} [i\mathcal{H}_{A,\omega}]_{3 \times 3} & [0]_{3 \times 1} \\ [0]_{1 \times 3} & -b_2 \mathcal{D}_{a,\omega} \end{bmatrix}_{4 \times 4}, \end{aligned} \quad (3.15)$$

the operator $\mathcal{P}_{A,\omega}$ is given by (3.6).

It is evident that the operators \mathcal{T}_1 and \mathcal{T}_2 have the same mapping properties as \mathcal{K} (see (3.13)).

Note that the first (matrix) multiplier in (3.15) as operator from $[L_2(S)]^4$ into $[L_2(S)]^4$ is continuous, while the second one maps $[L_2(S)]^3 \times H_2^1(S)$ into $[H_2^1(S)]^4$ due to Lemmas 2.3.(iv) and 2.13.(i). Therefore, \mathcal{T}_2 as operator from $[L_2(S)]^3 \times [H_2^1(S)]$ into $[L_2(S)]^4$ is compact (as a composition of bounded and compact operators). Further, from invertibility of the operators (2.34) and (3.6) the invertibility of the operator $\mathcal{T}_1 : [L_2(S)]^3 \times H_2^1(S) \rightarrow [L_2(S)]^4$ follows. Consequently, the operator (3.13) is Fredholm with zero index as a compact perturbation of the invertible operator (3.14).

Let us analyze the null spaces of the operator (3.13) and its adjoint one. Applying the uniqueness Theorem 3.1 and Lemma 2.4 we can prove that the homogeneous system (3.10)-(3.11) (with $f = 0$ and $f_0 = 0$) has only the trivial solution ($g = 0, g_4 = 0$) if $\omega \notin J(\Omega^+)$, and the operator (3.13) is then invertible. If $\omega \in J(\Omega^+)$, then $g_4 = 0$ and $g = (g_1, g_2, g_3)^\top$ is a nontrivial vector such that $V_{A,\omega}(g) \in X_\omega(\Omega^+)$. Below we show that $\dim \ker \mathcal{K} = \dim X_\omega(\Omega^+)$. Thus, if ω is a Jones eigenfrequency then $\ker \mathcal{K}$ is not trivial and the operator (3.13) is not invertible.

The operator formally adjoint to \mathcal{K} with respect to the usual L_2 -duality (without complex conjugation) reads as

$$\mathcal{K}^* := \begin{bmatrix} [-2^{-1}I_3 + \mathcal{K}_{A,\omega}^{(2)}]_{3 \times 3} & [(\mathcal{H}_{A,\omega})_{jk} n_j]_{3 \times 1} \\ [-b_2 \mathcal{P}_{a,\omega} n_k]_{1 \times 3} & -b_1 \{ \mathcal{L}_{a,\omega} - i [2^{-1} \mathcal{I} + \mathcal{K}_{a,\omega}^{(2)}] \} \end{bmatrix}_{4 \times 4}.$$

This means that

$$\langle \mathcal{K}\Phi, \Psi \rangle = \langle \Phi, \mathcal{K}^*\Psi \rangle \quad (3.16)$$

for all $\Phi, \Psi \in [L_2(S)]^3 \times H_2^1(S)$, where

$$\langle \Phi, \Psi \rangle = \int_S \sum_{j=1}^4 \Phi_j \Psi_j dS.$$

The relation (3.16) can be extended by continuity to the case $\Phi \in [L_2(S)]^3 \times H_2^1(S)$ and $\Psi \in [L_2(S)]^4$, defining the operator adjoint to (3.12) with mapping property

$$\mathcal{K}^* : [L_2(S)]^4 \rightarrow [L_2(S)]^3 \times H_2^{-1}(S).$$

Consider the homogeneous adjoint equation

$$\mathcal{K}^* \Psi^* = 0 \quad \text{with } \Psi^* \in [L_2(S)]^4. \quad (3.17)$$

Put

$$\Psi^* = (\psi^*, \psi_4^*)^\top, \quad \psi^* = (\psi_1^*, \psi_2^*, \psi_3^*)^\top.$$

Let us first show that an arbitrary solution $\Psi^* \in [L_2(S)]^4$ of (3.17) belongs actually to the space $[H_2^1(S)]^4$. In fact, (3.17) implies:

$$\begin{aligned} [-2^{-1}I_3 + \mathcal{K}_{A,\omega}^{(2)}] \psi^* + \mathcal{H}_{A,\omega}(\psi_4^* n) &= 0, \\ b_2 \{ -2^{-1}\mathcal{I} + \mathcal{K}_{a,\omega}^{(1)} - i\mathcal{H}_{a,\omega} \} (\psi^* \cdot n) + \\ &+ b_1 \{ \mathcal{L}_{a,\omega} - i[2^{-1}\mathcal{I} + \mathcal{K}_{a,\omega}^{(2)}] \} \psi_4^* = 0 \end{aligned}$$

which with the help of (2.50) and (2.31) can be rewritten as

$$\mathcal{D}_{A,\omega} \psi^* = -i\mathcal{H}_{A,\omega} \psi^* - \mathcal{H}_{A,\omega} \psi_4^*, \quad (3.18)$$

$$\begin{aligned} b_1 \mathcal{N}_{a,\omega} \psi_4^* &= -b_2 \{ -2^{-1}\mathcal{I} + \mathcal{K}_{a,\omega}^{(1)} - i\mathcal{H}_{a,\omega} \} (\psi^* \cdot n) + \\ &+ i b_1 \{ \mathcal{K}_{a,\omega}^{(2)} - \mathcal{K}_{a,\omega}^{(1)} \} \psi_4^*. \end{aligned} \quad (3.19)$$

By Lemmas 2.5.(i) and 2.13.(i) we conclude that $\psi^* \in [H_2^1(S)]^3$ and $\psi_4^* \in H_2^1(S)$ since the right hand side expressions in equations (3.18) and (3.19) are in $[H_2^1(S)]^3$ and $L_2(S)$, respectively.

Next, we show that if $\Psi^* = (\psi^*, \psi_4^*)^\top$ is an arbitrary solution of (3.17) then

$$\psi_4^*(x) = 0 \quad \text{and} \quad (\psi^*(x) \cdot n(x)) = 0 \quad \text{on } S. \quad (3.20)$$

Let $\Psi^* = (\psi^*, \psi_4^*)^\top \in \ker \mathcal{K}^*$ and let

$$u^*(x) = W_{A,\omega}(\psi^*)(x) + V_{A,\omega}(\psi_4^* n)(x),$$

$$w^*(x) = b_2 V_{a,\omega}(\psi^* \cdot n)(x) + b_1 W_{a,\omega}(\psi_4^*)(x).$$

It is evident that $u^* \in \mathbb{H}_{A,\omega}(\Omega^\pm) \cap \text{SK}(\Omega^-)$ and $w^* \in \mathbb{H}_{a,\omega}(\Omega^\pm) \cap \text{Som}(\Omega^-)$ due to the above mentioned regularity property of Ψ^* . Simple calculations show that

$$\begin{aligned} [u^*]_k^- &= [\mathcal{K}^* \Psi^*]_k = 0, \quad k = 1, 2, 3, \\ [\Lambda(\partial, n) w^* - i w^*]^+ &= [\mathcal{K}^* \Psi^*]_4 = 0 \quad \text{on } S. \end{aligned}$$

Due to Lemmas 2.13.(ii) and 2.6.(i) we derive that $u^* = 0$ in Ω^- and $w^* = 0$ in Ω^+ . In accordance with the jump relations of the scalar and vector layer potentials we get

$$\begin{aligned} [u^*]^+ - [u^*]^- &= \psi^*, \quad [T(\partial, n) u^*]^+ - [T(\partial, n) u^*]^- = -\psi_4^* n, \\ [w^*]^+ - [w^*]^- &= b_1 \psi_4^*, \quad [\Lambda(\partial, n) w^*]^+ - [\Lambda(\partial, n) w^*]^- = -b_2 \psi^* \cdot n, \end{aligned}$$

i.e.,

$$\begin{aligned} [u^*]^+ &= \psi^*, \quad [T(\partial, n) u^*]^+ = -\psi_4^* n, \\ [w^*]^- &= -b_1 \psi_4^*, \quad [\Lambda(\partial, n) w^*]^- = b_2 \psi^* \cdot n. \end{aligned} \quad (3.21)$$

Therefore

$$[u^* \cdot n]^+ = b_2^{-1} [\Lambda(\partial, n) w^*]^- , \quad [T(\partial, n) u^*]^+ = b_1^{-1} [w^*]^- n.$$

By Theorem 3.1 we then have $w^* = 0$ in Ω^- and $u^* \in X_\omega(\Omega^+)$. Whence in view of equations (3.21) the relations (3.20) follow. Moreover, the first equality in (3.21) yields that ψ^* belongs to the space of traces (boundary values) on S of Jones modes. We denote this space by

$$[X_\omega(\Omega^+)]_S := \{ [u]_S^+ : u \in X_\omega(\Omega^+) \}.$$

With the help of the integral representation formula (2.41) and definition of Jones modes we can show that the reverse assertion is also valid, i.e., if $\psi^* \in [X_\omega(\Omega^+)]_S$, then $(\psi^*, 0)^\top \in \ker \mathcal{K}^*$. Therefore we have the evident equalities

$$\dim \ker \mathcal{K} = \dim \ker \mathcal{K}^* = \dim [X_\omega(\Omega^+)]_S = \dim X_\omega(\Omega^+).$$

These results lead to the following

Lemma 3.4. *The operator (3.13) is Fredholm with zero index.*

If $\omega \notin J(\Omega^+)$ then (3.13) is invertible and the system (3.10)-(3.11) is solvable for arbitrary right hand side functions $f_k \in L_2(S)$, $k = 0, 1, 2, 3$.

If $\omega \in J(\Omega^+)$ then the condition

$$\begin{aligned} \int_S f(y) \cdot h(y) dS &= \int_S \sum_{j=1}^3 f_j(y) \overline{h_j(y)} dS = 0 \quad \text{for all} \quad (3.22) \\ h &= (h_1, h_2, h_3)^\top \in [X_\omega(\Omega^+)]_S \end{aligned}$$

is necessary and sufficient for system (3.10)-(3.11) to be solvable (recall that $h \in [X_\omega(\Omega^+)]_S$ yields $\bar{h} \in [X_\omega(\Omega^+)]_S$).

Note that, if $f(x) = n(x) \varphi(x)$, where $\varphi \in L_2(S)$ is some scalar function and, as above, n is the unit normal vector to S , then the condition (3.22) is automatically satisfied. Therefore, finally we have the following main existence result.

Theorem 3.5. *The direct scattering problem $P^{(dir)}$ is solvable for arbitrary incident wave w^{inc} and for arbitrary value of the oscillation parameter ω .*

Moreover, a solution is representable in the form of (3.8) and (3.9), where g_4 and w^{sc} are defined uniquely, while g and u are defined uniquely if $\omega \notin J(\Omega^+)$ and, if ω is exceptional ($\omega \in J(\Omega^+)$), then g is defined modulo vector-functions of $\ker \mathcal{K}$ and u is defined modulo Jones modes ($X_\omega(\Omega^+)$).

When ω is exceptional then the boundary values of the stress vector $[T(\partial, n) u]^+ = (-\frac{1}{2}I_3 + \mathcal{K}_{A, \omega}^{(1)}) g$ and the normal component of the displacement vector $[u \cdot n]^+ = (\mathcal{H}_{A, \omega} g) \cdot n$ are determined uniquely.

Remark 3.6. It is evident that the far field pattern $w_\infty^{sc}(\xi)$ is defined uniquely as well and due to the representation (3.9) we have

$$w_\infty^{sc}(\xi) = c(\xi) \int_S [(\Lambda(\partial_y, n(y)) - i) e^{-i\xi \cdot y}] g_4(y) dS_y, \quad (3.23)$$

where $\xi \in S_\omega$ corresponds to the vector $\hat{x} = x/|x|$ and $c(\xi)$ is given by (2.11).

Corollary 3.7. *Let $G = (g_1, g_2, g_3, g_4)^\top$ be a solution to the system (3.10)-(3.11). Then there are constants $C_1 > 0$ and $C_2 > 0$ independent of g_4, f , and f_4 , such that*

$$\begin{aligned} \|g_4\|_{H_2^1(S)} &\leq C_1 \{ \|f\|_{[L_2(S)]^3} + \|f_0\|_{L_2(S)} \}, \\ \|w_\infty^{sc}(\xi)\|_{L_2(S_\omega)} &\leq C_1 \{ \|f\|_{[L_2(S)]^3} + \|f_0\|_{L_2(S)} \}, \end{aligned}$$

and

$$|\partial^\beta w^{sc}(x)| \leq \delta^{-1} C_2 \{ \|f\|_{[L_2(S)]^3} + \|f_0\|_{L_2(S)} \}, \quad x \in \overline{\Omega_0^-},$$

uniformly for any subset $\overline{\Omega_0^-} \subset \Omega^-$ and arbitrary multi-index β , where $\delta := \text{dist} \{ \overline{\Omega_0^-}, S \}$.

Moreover, let $\{(g^{(q)}, 0)^\top\}_{q=1}^N$ with $g^{(q)} = (g_1^{(q)}, g_2^{(q)}, g_3^{(q)})^\top$ be a complete system of linearly independent solutions to the homogeneous version of the simultaneous equations (3.10)-(3.11) and $(\tilde{g}, 0)^\top = (\tilde{g}_1, \tilde{g}_2, \tilde{g}_3, 0)^\top$ be its particular solution orthogonal to all $(g^{(q)}, 0)^\top$ ($q = \overline{1, n}$). Then there is a constant $C_3 > 0$ independent of \tilde{g}, f , and f_4 , such that

$$\|\tilde{g}\|_{L_2(S)} \leq C_3 \{ \|f\|_{[L_2(S)]^3} + \|f_0\|_{L_2(S)} \}.$$

The proof is a consequence of the following - more general assertions (see [36], Lemmas 2.8 and 2.11).

Lemma 3.8. *Let a Banach space X be the direct product of two Banach spaces X_1 and X_2 , i.e., $X = X_1 \times X_2$ with the norm $\|x\|_X = \|x_1\|_{X_1} + \|x_2\|_{X_2}$, where $x = (x_1, x_2) \in X$, $x_k \in X_k$, $k = 1, 2$.*

Let $\mathcal{T} : X \rightarrow Y$ be a linear continuous operator from X into Banach space Y and assume that the linear equation

$$\mathcal{T}x = y, \quad (3.24)$$

where $y \in Y$ is a given element and $x \in X$ is an unknown, is normally solvable, i.e., the range $R(\mathcal{T})$ is closed in Y .

Moreover, let $\ker \mathcal{T} \subset X_1 \times \{\theta_2\}$ where θ_k , $k = 1, 2$, are zero elements of X_k .

If $x = (x_1, x_2) \in X$ is a solution of equation (3.24) then there exists a constant $C > 0$, independent of y , such that

$$\|x_2\|_{X_2} \leq C \|y\|_Y = C \|\mathcal{T}x\|_Y.$$

Lemma 3.9. *Let X and Y be Banach spaces, $\mathcal{T} : X \rightarrow Y$ be a linear continuous operator, and the equation*

$$\mathcal{T} x = y \quad (3.25)$$

be normally solvable.

Moreover, let $\dim \ker \mathcal{T} = N < \infty$, $\{e_j\}_{j=1}^N$ be a basis in $\ker \mathcal{T}$, and $\{f_j\}_{j=1}^N$ be a corresponding bi-orthogonal system in the adjoint space X^* , i.e., $f_j \in X^*$ and $f_i(e_j) = \delta_{ij}$, where δ_{ij} is the Kronecker's delta.

If x is an arbitrary solution to (3.25), then $\tilde{x} = x - \sum_{i=1}^N f_i(x)e_i$ is a particular solution of the same equation satisfying the inequality

$$\|\tilde{x}\|_X \leq C \|y\|_Y,$$

where C does not depend on x and y .

4. INVERSE PROBLEM. UNIQUENESS THEOREM

This section deals with the uniqueness of solution to the inverse fluid-structure interaction problem (see Subsection 1.3).

Theorem 4.1. *Let $\overline{\Omega_j^+}$, $j = 1, 2$, be two bounded elastic scatterers with Lipschitz boundaries $\partial\Omega_j^+ = S_j$ and with simply connected complements $\Omega_j^- = \mathbb{R}^3 \setminus \overline{\Omega_j^+}$, and let for a fixed wave number ω the far-field patterns $w_\infty^{(j)sc}(\cdot; d)$ for the both scatterers coincide for all incident directions $d \in S_\omega$. Then $\Omega_1^+ = \Omega_2^+$.*

Proof. Step 1. We denote the elastic vector field in the domain Ω_j^+ by $u^{(j)}(x; d)$ and the scattered scalar field in the domain Ω_j^- by $w^{(j)sc}(x; d) =: w^{(j)}(x; d)$ $j = 1, 2$.

In the both cases the incident field is represented in the form of a plane wave (see (2.13)). Thus, the pair $(u^{(j)}, w^{(j)})$ is a solution to Problem $P^{(dir)}$ for the scatterer Ω_j^+ ($j = 1, 2$) with a fixed oscillation parameter ω (see (2.14)-(2.18)).

Let $\Omega_1^+ \neq \Omega_2^+$ and $\Omega_{12}^- := \mathbb{R}^3 \setminus \{\overline{\Omega_1^+ \cup \Omega_2^+}\}$.

Since $w^{(1)}(x; d)$ and $w^{(2)}(x; d)$ are radiating solutions of the equation (2.5) in Ω_{12}^- and have the same far field patterns $w_\infty^{(1)}(\xi; d) = w_\infty^{(2)}(\xi; d)$ for all $d \in S_\omega$, we conclude that

$$w^{(1)}(x; d) = w^{(2)}(x; d) \quad \text{in } \Omega_{12}^-, \quad (4.1)$$

due to the asymptotic relation (2.12) and Lemma 2.1.

Step 2. Let us consider Problem $P^{(dir)}$ with the domains Ω_j^+ , Ω_j^- ($j = 1, 2$), where the incident field is taken in the form $w^{inc}(x) = v_m(x) \in P_{sp}(\mathbb{R}^3)$, i.e., $f_0(x) = b_1 \Lambda(\partial, n) v_m(x)$, $f(x) = b_2 v_m(x) n(x)$. The corresponding elastic field in Ω_j^+ and the scattered field in Ω_j^- we denote by

$u^{(j,m)}(x)$ and $w^{(j,m)}(x)$, respectively. From the conditions of Theorem 4.1 and the equality (4.1) it follows that

$$w_{\infty}^{(1,m)}(\xi) = w_{\infty}^{(2,m)}(\xi), \quad \xi \in S_{\omega}, \quad \text{and} \quad w^{(1,m)}(x) = w^{(2,m)}(x), \quad x \in \Omega_{12}^{-},$$

since the direct problem is linear.

Step 3. Let x_0 be an arbitrary point in Ω_{12}^{-} and let us consider Problem $P^{(dir)}$ with the same domains Ω_j^+ , Ω_j^- ($j = 1, 2$), where the interface data are given as follows

$$f_0(x) = f_0(x; x_0) := b_1 \Lambda(\partial, n) \gamma(x - x_0, \omega), \quad (4.2)$$

$$f(x) = f(x; x_0) := b_2 \gamma(x - x_0, \omega) n(x); \quad (4.3)$$

here $\gamma(\cdot, \omega)$ is the fundamental function defined by (2.10). The corresponding elastic field (in Ω_j^+) and scalar scattered field (in Ω_j^-) we denote by $u^{(j)}(x; x_0)$ and $w^{(j)}(x; x_0)$.

Due to Corollary 2.9 there exists a sequence $v_m \in P_{sp}(\mathbb{R}^3)$ such that (for arbitrary multi-index β)

$$\partial^{\beta} v_m(x) \rightarrow \partial^{\beta} \gamma(x - x_0, \omega) \quad (4.4)$$

uniformly in $\overline{\Omega_1^+ \cup \Omega_2^+}$.

Applying the linearity of the direct problem, equation (3.23), Corollaries 2.9 and 3.7, and the results obtained in Step 2, we get

$$\begin{aligned} & \|w_{\infty}^{(1)}(\xi; x_0) - w_{\infty}^{(2)}(\xi; x_0)\|_{L_2(S_{\omega})} = \\ & = \|w_{\infty}^{(1)}(\xi; x_0) - w_{\infty}^{(1,m)}(\xi) + w_{\infty}^{(2,m)}(\xi) - w_{\infty}^{(2)}(\xi; x_0)\|_{L_2(S_{\omega})} \leq \\ & \leq \|w_{\infty}^{(1)}(\xi; x_0) - w_{\infty}^{(1,m)}(\xi)\|_{L_2(S_{\omega})} + \|w_{\infty}^{(2)}(\xi; x_0) - w_{\infty}^{(2,m)}(\xi)\|_{L_2(S_{\omega})} \leq \\ & \leq C \{ \|\gamma(x - x_0, \omega) - v_m(x)\|_{L_2(S_1)} + \\ & \quad + \|\Lambda(\partial, n) [\gamma(x - x_0, \omega) - v_m(x)]\|_{L_2(S_1)} + \\ & \quad + \|\gamma(x - x_0, \omega) - v_m(x)\|_{L_2(S_2)} + \\ & \quad + \|\Lambda(\partial, n) [\gamma(x - x_0, \omega) - v_m(x)]\|_{L_2(S_2)} \} \rightarrow 0 \end{aligned}$$

as $m \rightarrow \infty$; here $w_{\infty}^{(j,m)}(\xi)$ denotes the far field pattern of the scattered field $w^{(j,m)}(x)$ corresponding to the incident wave function $v_m \in P_{sp}(\mathbb{R}^3)$ involved in (4.4).

This implies $w_{\infty}^{(1)}(\xi; x_0) = w_{\infty}^{(2)}(\xi; x_0)$ for $\xi \in S_{\omega}$ and, consequently, by Lemma 2.1

$$w^{(1)}(x; x_0) = w^{(2)}(x; x_0) \quad \text{in} \quad \Omega_{12}^{-}. \quad (4.5)$$

Step 4. Since $\Omega_1^+ \neq \Omega_2^+$, there exists a point $x^* \in \partial(\overline{\Omega_1^+ \cup \Omega_2^+})$ such that the closed ball $B(x^*, 2\delta)$ centered at x^* and radius $2\delta > 0$ does not intersect either $\overline{\Omega_1^+}$ or $\overline{\Omega_2^+}$. Without restriction of generality, we assume

that $\overline{B(x^*, 2\delta)} \cap \overline{\Omega_2^+} = \emptyset$ and that $n(x^*)$ exists. Evidently, $S_1^* := \partial\Omega_1^+ \cap \overline{B(x^*, 2\delta)} \subset S_1$ and $\text{dist}\{\overline{B(x^*, \delta)}, \overline{\Omega_2^+}\} \geq \delta$.

Further we choose a sequence $x^p \in B(x^*, \delta) \cap \gamma^-(x^*) \cap \Omega_{12}^-$ such that $|x^* - x^p| \rightarrow 0$ as $p \rightarrow \infty$. Here $\gamma^-(x^*)$ is a nontangential approach region (cone) at the point $x^* \in S_1$.

Now let us consider the problem described in Step 3 with the point x^p for x_0 .

For the domains Ω_1^+ and Ω_1^- , the interface conditions of type (2.14) on S_1 reads as follows:

$$\begin{aligned} [u^{(1)}(x; x^p) \cdot n(x)]_{S_1^+}^+ &= b_1 [\Lambda(\partial, n) w^{(1)}(x; x^p)]_{S_1^-}^- + \\ &+ b_1 \Lambda(\partial, n) \gamma(x - x^p; \omega), \quad x \in S_1. \end{aligned}$$

Taking into account the fact that $w^{(2)}(\cdot; x^p)$ is bounded in $\overline{B(x^*, \delta)} \subset \Omega_2^-$ together with its derivatives uniformly with respect to $x^p \in \overline{B(x^*, \delta)}$ (see Corollary 3.7) and applying the equation (4.5) with x^p for x_0 , we arrive at the relation

$$\begin{aligned} &| [u^{(1)}(x; x^p) \cdot n(x)]_{S_1^*}^+ - b_1 [\Lambda(\partial, n(x)) \gamma(x - x^p, \omega)]_{S_1^*} | = \\ &= | b_1 [\Lambda(\partial, n(x)) w^{(1)}(x; x^p)]_{S_1^*}^- | = | b_1 [\Lambda(\partial, n(x)) w^{(2)}(x; x^p)]_{S_1^*}^- | \leq C_1, \end{aligned}$$

where C_1 does not depend on $u^{(1)}$ and x^p .

In particular,

$$\begin{aligned} &\| [u^{(1)}(x; x^p) \cdot n(x)]^+ - b_1 [\Lambda(\partial, n(x)) \gamma(x - x^p, \omega)] \|_{L_2(S_1^*)} \leq \\ &\leq C_1 |S_1^*|^{1/2}, \end{aligned} \quad (4.6)$$

where $|S_1^*|$ is the area of the sub-manifold S_1^* and $p = 1, 2, 3, \dots$.

Step 5. Here we prove that

$$\| [u^{(1)}(x; x^p) \cdot n(x)]^+ \|_{L_2(S)} \leq C_2 \quad (4.7)$$

with a constant $C_2 > 0$ independent of x^p and $u^{(1)}$. Note that $u^{(1)}(x; x^p)$ and $w^{(1)}(x; x^p)$ can be represented in the form (3.8) and (3.9), where the densities g and g_4 are to be defined from the system (3.10)-(3.11) with f_0 and f given by (4.2) and (4.3), and with S_1 for S .

Moreover,

$$g(x; x^p) = \tilde{g}(x; x^p) - \sum_{q=1}^N c_q g^{(q)}(x),$$

where c_q ($q = \overline{1, N}$) are arbitrary constants, $\{(g^{(q)}, 0)^\top\}_{q=1}^N$ is a complete (orthonormal) system of linearly independent solutions of the corresponding homogeneous equations, and $(\tilde{g}, g_4)^\top$ is a fixed particular solution orthogonal to this system. Remark that $V_{A, \omega}(g^{(q)}) \in X_\omega(\Omega^+)$ if $\omega \in J(\Omega^+)$ and $[u^{(1)}(x; x^p) \cdot n(x)]_S^+ = [\mathcal{H}_{A, \omega} \tilde{g}(x, x^p)] \cdot n(x)$.

We proceed as follows. From the interface condition

$$[T(\partial, n) u^{(1)}(x; x^p)]^+ = b_2 n(x) [w^{(1)}(x; x^p)]^- + b_2 n(x) \gamma(x - x^p, \omega), \quad x \in S_1,$$

we conclude that $g := g(x, x^p)$ solves then the integral equation

$$[-2^{-1}g + \mathcal{K}_{A,\omega}^{(1)}g](x) = \psi(x, x^p) + b_2 n(x) \gamma(x - x^p, \omega) \quad \text{on } S_1, \quad (4.8)$$

where $\psi(x, x^p) := b_2 n(x) [w^{(1)}(x; x^p)]^- \in [L_2(S_1)]^3$. A simple analysis implies that the norm $\|[w^{(1)}(x; x^p)]^-\|_{L_2(S_1 \setminus S_1^*)}$ is uniformly bounded with respect to $x^p \in B(x^*, \delta)$. Due to the relation (4.5) it is evident that the norm $\|[w^{(1)}(x; x^p)]^-\|_{L_2(S_1^*)} = \|[w^{(2)}(x; x^p)]_{S_1^*}\|_{L_2(S_1^*)}$ is also uniformly bounded with respect to $x^p \in B(x^*, \delta)$. Therefore $\|\psi(\cdot, x^p)\|_{[L_2(S)]^3} \leq C_3$ with a positive constant C_3 independent of x^p . Apply to equation (4.8) the operator $\mathcal{H}_{A,\omega}$ and use the first equality in (2.45) to obtain

$$[-2^{-1}\mathcal{H}_{A,\omega}g + \mathcal{K}_{A,\omega}^{(2)}\mathcal{H}_{A,\omega}g](x) = \Psi(x, x^p) \quad \text{on } S_1,$$

where

$$\Psi(\cdot, x^p) = \mathcal{H}_{A,\omega} \{ \psi(x, x^p) + b_2 n(x) \gamma(x - x^p, \omega) \} \in [L_2(S)]^3$$

and $\|\Psi(\cdot, x^p)\|_{[L_2(S)]^3} \leq C_4$ with a positive constant C_4 independent of x^p . From these relations with the help of the invertibility of the operator (2.51) with $\mathcal{D}_{A,\omega}$ given by (2.50) we derive that $\mathcal{H}_{A,\omega}g(\cdot, x^p) \in [L_2(S)]^3$ and by Lemma 3.9 we finally get $\|\mathcal{H}_{A,\omega}g(\cdot, x^p)\|_{[L_2(S)]^3} \leq C_5$ with a positive constant C_5 independent of x^p . Whence (4.7) follows directly.

Step 6. The inequality (4.7) contradicts to (4). In fact, we easily derive that

$$\Lambda(\partial, n(x)) \gamma(x - x^p; \omega) = \frac{n(x) \cdot \zeta^p}{4\pi|\tilde{a}^{1/2}| [\tilde{a}^{-1} \zeta^p \cdot \zeta^p]^{3/2}} \frac{1}{|x - x^p|^2} + \mathcal{O}(1),$$

where $\zeta^p = \frac{x - x^p}{|x - x^p|}$, $x \in S_1^*$ and $x^p \in \gamma^-(x^*)$. Whence it follows that the left-hand side in (4) is not bounded as x^p approaches x^* . This completes the proof. \square

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