

SIXTY YEARS OF PROFESSOR FRANTIŠEK NEUMAN

ONDŘEJ DOŠLÝ, Brno

An outstanding Czech mathematician, Prof. RNDr. František Neuman, DrSc., one of the leading personalities of Brno's mathematics and a prominent specialist in the theory of linear differential equations, celebrated his sixtieth birthday on May 28, 1997.



František Neuman was born in Brno, where he also attended primary and secondary school. Already as a secondary school student he showed his mathematical talent, being twice among the winners of the Czechoslovak Mathematical Olympiad. In 1960 he graduated at the Faculty of Science of J. E. Purkyně University (now Masaryk University) in Brno and started to work at the Department of Mathematics at this faculty. In 1965 he received his Candidate of Science degree (CSc.) and one year later he was appointed Associate Professor of Mathematical Analysis. In 1974 he left university for the Brno branch of Mathematical Institute of the Academy of Sciences just established, where since 1991 he has been working as its head. After the change F. Neuman has continued in his pedagogical activities at the Faculty of

Science by reading special lectures for undergraduate students and has devoted a lot of his time to postgraduate students. In 1980 he received the title Doctor of Science (DrSc.) and in 1991 was appointed Professor of Mathematical Analysis.

The scientific activities of F. Neuman are closely connected with the qualitative theory of differential equations. This orientation was strongly influenced by Professor Borůvka who was his supervisor during postgraduate studies. At the beginning of the scientific career F. Neuman concentrated his attention on the second order linear differential equations. He proved a series of new results concerning the distribution of zero points, periodicity, asymptotic behaviour and extremal properties of solutions of these equations. Among these results let us mention the paper [26], where conditions are established which guarantee that all solutions of a given second order linear differential equation are periodic and an explicit description of periodic solutions is offered. These papers attracted considerable attention of mathematical community since they reveal a close relationship between qualitative theory of linear differential equations, affine geometry and theory of functional equations.

In late sixties F. Neuman turned his attention to the third and higher order differential equations. Using an ingenious combination of algebraical and geometrical methods with methods typical for investigation of differential and functional equations he created in next 20 years a unified theory of global properties of linear differential equations. This global theory of linear differential equations made it possible to resolve several until that time open problems and its results find applications in many related mathematical disciplines, as theory of functional and functional-differential equations. It is rather difficult to describe the basic results of Neuman's theory in a few sentences. Let us mention here at least the paper [40] where, using a category theory approach, the algebraic structure of global transformations of linear differential equations is established, and papers [53, 59] where F. Neuman investigated the problem of global canonical forms of mutually transformable differential equations and offered (up to a certain particular exception) an effective criterion of global equivalence of two linear differential equations. These papers are valuable also from the historical point of view since they reveal the "localness" of investigation of linear differential equations in the 19-th century.

The main results of Neuman's global theory of linear differential equations are summarized in the monograph [M2]. This monograph has attracted a considerable international attention and became one of the basic references in the field—see the recent monograph [E].

Recently F. Neuman has been dealing with qualitative properties of functional-differential equations. From the transformation point of view he investigates algebraic and geometrical aspects of this problem. It is also worth mentioning that in addition to his fundamental work in the theory of differential equations, F. Neuman

has achieved remarkable results in other mathematical disciplines as well. For example, the paper [6] gives a complete characterization of the trees whose square is a Hamiltonian graph. A graph with this property is now usually called Neuman's tree. In the theory of functional equations his papers [49, 50, 51, 74] represent basic results for decomposition of functions of two variables into finite sums of products of functions of single variables.

František Neuman has been for many years one of the leading mathematical personalities in Brno. He organizes the seminar on differential equations at Masaryk University and acts as a member of Editorial Boards of several international journals. He also significantly contributes to the organization of Equadiff Conferences which are periodically held in Prague, Brno and Bratislava (he was the chairman of the Scientific and Organizing committee of Equadiff 9 held in Brno in August 97). F. Neuman was supervisor of several postgraduate students who now are well known mathematicians.

Based on his scientific and teaching activities, F. Neuman has obtained many invitations to lecture at universities abroad and to plenary lectures at international conferences. He has also obtained several scientific distinctions, among them let us mention the Bolzano medal awarded to distinguished scientists by the Presidium of the Czech Academy of Sciences.

On behalf of the whole Czech mathematical community we take the opportunity to wish Professor František Neuman good health and every success in his personal life and his scientific work.

References

- [E] *U. Elias*: Oscillation Theory of Two-Term Linear Differential Equations. Kluwer Academic Publishers, Dordrecht, 1996.

LIST OF SCIENTIFIC PUBLICATIONS OF FRANTIŠEK NEUMAN

Books:

- [M1] Functional Equations. SNTL, Praha, 1986, pp. 104. (In Czech.)
 [M2] Global Properties of Linear Ordinary Differential Equations. Mathematics and Its Applications, East European Series 52, Kluwer Academic Publishers (with Academia Praha), Dordrecht, 1991, pp. 334.

Scientific papers:

- [1] Sur les équations différentielles linéaires du second ordre dont les solutions ont des racines formant une suite convexe. Acta Math. Acad. Sci. Hungar. *13* (1962), 281–287.
 [2] On a certain ordering of the vertices of a tree. Časopis Pěst. Mat. *89* (1964), 323–339.

- [3] Sur les équations différentielles linéaires oscillatoires du deuxième ordre avec la dispersion fondamentale $\varphi(t) = t + \pi$. Bull. Polyt. Inst. Jassy 10 (14) (1964), 37–42.
- [4] (with M. Sekanina) Equivalent systems of sets and homeomorphic topologies. Czechoslovak Math. J. 15 (1965), 323–328.
- [5] Construction of second-order linear differential equations with solutions of prescribed properties. Arch. Math. (Brno) 1 (1965), 229–246.
- [6] On ordering vertices of infinite trees. Časopis Pěst. Mat. 91 (1966), 170–177.
- [7] Note on the second phase of the differential equation $y'' = q(t)y$. Arch. Math. (Brno) 2 (1966), 56–62.
- [8] Criterion of periodicity of solutions of a certain differential equation with a periodic coefficient. Ann. Mat. Pura Appl. 75 (1967), 385–396.
- [9] Extremal property of the equation $y'' = k^2y$. Arch. Math. (Brno) 3 (1967), 161–164.
- [10] Relation between the distribution of the zeros of the solutions of a 2nd order linear differential equation and the boundedness of these solutions. Acta Math. Acad. Sci. Hungar. 19 (1968), 1–6.
- [11] Centroaffine invariants of plane curves in connection with the theory of the second-order linear differential equations. Arch. Math. (Brno) 4 (1968), 201–216.
- [12] Note on a bounded non-periodic solutions of the second-order linear differential equations with periodic coefficients. Math. Nachrichten 39 (1969), 217–222.
- [13] On bounded solutions of a certain differential equation. Proceedings of the Conference on Differential Equations and Their Applications Equadiff 2 Bratislava 1966. Acta Fac. Rerum Natur. Univ. Comenian. Math. 17 (1969), 213–215.
- [14] On the coexistence of periodic solutions. J. Differential Equations 8 (1970), 277–282.
- [15] An explicit form of the differential equation $y'' = q(t)y$ with periodic solutions. Ann. Mat. Pura Appl. 85 (1970), 295–300.
- [16] On the Liouville transformation. Rend. Mat. 3 (1970), 133–140.
- [17] A note on differential equations with periodic solutions. Arch. Math. (Brno) 6 (1970), 189–192.
- [18] Construction of differential equations with coexisting periodic solutions. Bull. Polyt. Inst. Jassy 14 (20) (1970), 67–75.
- [19] Note on Kummer's transformation. Arch. Math. (Brno) 6 (1970), 185–188.
- [20] Closed plane curves and differential equations. Rend. Mat. 3 (1970), 423–433.
- [21] Periodic curvatures and closed curves. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) 48 (1970), 494–498.
- [22] Some results concerning Abel's equation in the theory of differential equations and consequences for the affine geometry of closed plane curves. Aequationes Math. 4 (1970), 282–284.
- [23] (with F. M. Arscott and M. Greguš) Three-point boundary value problems in differential equations. J. London Math. Soc. (2) 3 (1971), 429–436.
- [24] A role of Abel's equation in the stability theory of differential equations. Aequationes Math. 6 (1971), 66–70.
- [25] L^2 -solutions of $y'' = q(t)y$ and a functional equation. Aequationes Math. 6 (1971), 162–169.
- [26] Linear differential equations of the second order and their applications. Rend. Mat. 4 (1971), 559–617.
- [27] Some results on geometrical approach to linear differential equations of the n -th order. Comment. Math. Univ. Carolin. 12 (1971), 307–315.
- [28] A note on Santaló's isoperimetric theorem. Revista Mat. Fis. Teor. Tucuman 21 (1971), 203–206.

- [29] Geometrical approach to linear differential equations of the n -th order. *Rend. Mat.* 5 (1972), 579–602.
- [30] Oscillation in linear differential equations. Proceedings of the Conference Equadiff 3. J. E. Purkyně University, Brno, 1972, pp. 119–125.
- [31] Distribution of zeros of solutions of $y'' = q(t)y$ in relation to their behaviour in large. *Studia Sci. Math. Hungar.* 8 (1973), 177–185.
- [32] On n -dimensional closed curves and periodic solutions of linear differential equations of the n -th order. *Demonstratio Mat.* 6 (1973), 329–337.
- [33] On a problem of transformations between limit-circle and limit-point differential equations. *Proc. Roy. Soc. Edinburgh Sect. A* 72 (1973/74), 187–193.
- [34] On two problems about oscillation of linear differential equations of the third order. *J. Differential Equations* 15 (1974), 589–596.
- [35] Global transformation of linear differential equations of the n -th order. *Knížnice odb. a věd. spisů VUT Brno B-56* (1975), 165–171.
- [36] On solutions of the vector functional equation $y(\xi(x)) = f(x) \cdot A \cdot y(x)$. *Aequationes Math.* 16 (1977), 245–257.
- [37] (with J. Vosmanský) On functions (sequences) the derivatives (differences) of which are of constant sign. *Doklady Ak. Nauk Azer. SSR* 34 (1978), 8–12. (In Russian.)
- [38] (with S. Staněk) On the structure of second-order periodic differential equations with given characteristic multipliers. *Arch. Math. (Brno)* 13 (1977), 149–157.
- [39] Linear differential equations with periodic coefficients in the critical case. *An. Sti. Univ. Al. I. Cuza Jassy Sect. I a Mat.* 23 (1977), 325–328.
- [40] Categorical approach to global transformations of the n -th order linear differential equations. 1977 102, 350–355.
- [41] Limit circle classification and boundedness of solutions. *Proc. Roy. Soc. Edinburgh* 81 A (1978), 31–34.
- [42] Global properties of the n -th order linear differential equations. Proceedings of Equadiff 4 Praha 1977, Lecture Notes in Mathematics 703. Springer, Berlin, 1979, pp. 309–319.
- [43] Invariants of third order linear differential equations and Cartan's moving frame method. *Diferencial'nyje Uravnenija* 14 (1979), 398–404. (In Russian.)
- [44] A generalization of Floquet theory. *Acta Math. Univ. Comenian.* 39 (1980), 53–59.
- [45] Transformations of linear differential equations of the n -th order. Sborník 6. vědecké konference Vysoké školy dopravní v Žilině Sept. 1979. VŠD Žilina, 1979, pp. 11–19. (In Russian.)
- [46] On transformations of differential equations and systems with deviating argument. *Czechoslovak Math. J.* 31 (1981), 87–90.
- [47] Global theory of linear differential equations of the n -th order. Proceedings of the Colloquium on Qualitative Theory of Differential Equations Szeged-Hungary August 1979. Ser. Coll. Math. Soc. J. Bolyai, North-Holland Publ. Co., 1981, pp. 777–793.
- [48] Second order linear differential systems. *Ann. Sci. École Norm. Super. (Paris)* 13 (1980), 437–449.
- [49] Functions of two variables and matrices involving factorizations. *C. R. Math. Rep. Acad. Sci. Canada* 3 (1981), 7–11.
- [50] Factorizations of matrices and functions of two variables. *Czechoslovak Math. J.* 32 (1982), 582–588.
- [51] Functions of the form $\sum_{i=1}^N f_i(x)g_i(t)$ in L_2 . *Arch. Math. (Brno)* 18 (1982), 19–22.
- [52] Simultaneous solutions of a system of Abel equations and differential equations with several deviations. *Czechoslovak Math. J.* 32 (1982), 488–494.

- [53] Global canonical forms of linear differential equations. *Math. Slovaca* 33 (1983), 389–394.
- [54] Linear differential equations—global theory. *Proceedings of Equadiff 5 Bratislava 1981*. Teubner-Texte zur Mathematik, Leipzig, 1982, pp. 272–275.
- [55] Theory of global properties of ordinary differential equations of the n -th order. *Diferencial'nyje Uravnenija* 19 (1983), 799–808. (In Russian.)
- [56] A survey of global properties of linear differential equations of the n -th order. *Proceedings of the Conference on Ordinary and Partial Differential Equations, Dundee 1982*. Lecture Notes in Mathematics 964, Springer, Berlin, pp. 548–563.
- [57] (with W. N. Everitt) A concept of adjointness and symmetry of differential expressions based on the generalized Lagrange identity and Green's formula. *Proceedings: Ordinary Differential Equations and Operators, Dundee 1982*. Lecture Notes in Mathematics 1032, Springer, Berlin, pp. 161–169.
- [58] From local to global investigations of linear differential equations of the n -th order. *Jahrbuch Überblicke Mathematik* 1984, 55–80.
- [59] Criterion of global equivalence of linear differential equations. *Proc. Roy. Soc. Edinburgh* 97 A (1984), 217–221.
- [60] Stationary groups of linear differential equations. *Czechoslovak Math. J.* 34 (1984), 645–663; Abstract: *C. R. Acad. Sci. Paris Série I* 299 (1984), no. 8, 319–322.
- [61] A vector functional equation and linear differential equations. *Aequationes Math.* 29 (1985), 19–23.
- [62] A note on smoothness of the Stäckel transformation. *Prace Mat. WSP (Kraków)* 11 (1985), 147–151.
- [63] Covariant constructions in the theory of linear differential equations. *Časopis Pěst. Mat.* 111 (1986), 201–207.
- [64] Global theory of ordinary linear homogeneous differential equations in the real domain I, II. *Aequationes Math.* 33 (1987), 123–149; 34 (1987), 1–22.
- [65] Solution to the Problem No. 10 of N. Kamran. *Proceedings of the 23rd Intern. Symp. on Functional Equations Gargnano-Italy 1985*. Univ. of Waterloo, Ont. Canada, pp. 60–62.
- [66] Ordinary linear differential equations—a survey of the global theory. *Proceedings of Equadiff 6 Brno 1986*. Lecture Notes in Mathematics 1192, Springer, Berlin, pp. 59–70.
- [67] Oscillatory behavior of iterative linear ordinary differential equations depends on their order. *Arch. Math. (Brno)* 22 (1986), 187–192.
- [68] On iteration groups of certain functions. *Arch. Math. (Brno)* 25 (1989), 185–194.
- [69] Another proof of Borůvka's criterion on global equivalence of the second order ordinary linear differential equations. *Časopis Pěst. Mat.* 115 (1990), 73–80.
- [70] Smoothness as an invariant property of coefficients of linear differential equations. *Czechoslovak Math. J.* 39 (1989), 513–521.
- [71] On a canonical parametrization of continuous functions. *Opuscula Math. (Kraków)* 1335 (1990), 185–191.
- [72] On Halphen and Laguerre-Forsyth canonical forms of linear differential equations. *Arch. Math. (Brno)* 26 (1990), 147–154.
- [73] Transformations and canonical forms of functional-differential equations. *Proc. Roy. Soc. Edinburgh* 115 A (1990), 349–357.
- [74] Finite sums of products of functions in single variables. *Linear Algebra Appl.* 134 (1990), 153–164.
- [75] (with Th. M. Rassias) Functions decomposable into finite sums of products. *Constantin Carathéodory—An International Tribute, Vol. II*. World Scientific Publ. Co., Singapore, 1991, pp. 956–963.

- [76] Ordered groups commuting matrices and iterations of functions in transformations of differential equations. Constantin Carathéodory—An International Tribute, Vol. II. World Scientific Publ. Co., Singapore, 1991, pp. 942–955.
- [77] (with Á. Elbert and J. Vosmanský) Principal pairs of solutions of linear second order oscillatory differential equations. *Differential Integral Equations* 5 (1992), 945–960.
- [78] On transformation of quasilinear differential equations to canonical forms. *Recent Trends in Ordinary Differential Equations Vol. I*. World Scientific Series in Applicable Analysis World Sci., Singapore, 1992, pp. 457–461.
- [79] On the n -th order iterative linear ordinary differential equations. *Aequationes Math.* 46 (1993), 38–43.
- [80] (with Á. Elbert) Exceptional solutions of Hill equations. *J. Differential Equations* 116 (1995), 419–430.
- [81] Limit behavior of ordinary linear differential equations. *Proceedings of Georgian Academy of Sciences Mathematics* 1 (1993), 355–364.
- [82] (with M. Muldoon) Principal pairs for oscillatory second order linear differential equations. *Dynamical Systems and Applications*. World Scientific Series in Applicable Analysis World Sci. Vol. 4, Singapore, 1995, pp. 517–526.
- [83] On equivalence of linear functional-differential equations. *Results in Mathematics* 26 (1994), 354–359.
- [84] Nonextendable classes of linear differential equations. *J. Nonlinear Anal.* 25 (1995), 1045–1049.
- [85] Solutions of Abel’s equation in relation to asymptotic behaviour of linear differential equations. *Aequationes Math.* Accepted.
- [86] Transformation theory of linear ordinary differential equations—from local to global investigations. *Archivum Math. (Brno)* 33 (1997), 65–74.
- [87] Dispersions for linear differential equations of arbitrary order. *Archivum Math. (Brno)* 33 (1997), 147–155.
- [88] Asymptotic behaviour and zeros of solutions on n -th order linear differential equations. Submitted.