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CORRIGENDUM ON THE PAPER: 'LOWER BOUNDS FOR THE INFIMUM OF THE SPECTRUM OF THE SCHRŐDINGER OPERATOR IN $\mathbb{R}^N$ AND THE SOBOLEV INEQUALITIES' PUBLISHED IN JIPAM, VOL. 3, NO. 4. (2002), ARTICLE 63

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Abstract

This paper is a corrigendum on a paper published in an earlier volume of JIPAM, 'Lower Bounds for the Infimum of the Spectrum of the Schrődinger Operator in $\mathbb{R}^N$ and the Sobolev Inequalities' published in JIPAM, vol. 3, no. 4. (2002), Article 63. It concerns of a number of misprints.

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Key words: Optimal lower bound, infimum spectrum Schrődinger operator, Sobolev inequality.

Contents

1. Results

The following list of misprints have been brought to the attention of the author by the review in Mathematical Reviews #1923362 by Jan Kříž. It appeared that these misprints had crept in during the process of text-editing of an earlier concept.

1. Page 2, formula (1.4) (definition of the form domain $Q(h)$):

   $$Q(h) = H^1(\mathbb{R}^N) \cap \{u \mid u \in L^2(\mathbb{R}^N), \quad q_+^{1/2} \in L^2(\mathbb{R}^N)\}.$$

   to be replaced by [see second condition]

   $$Q(h) = H^1(\mathbb{R}^N) \cap \{u \mid u \in L^2(\mathbb{R}^N), \quad q_+^{1/2} u \in L^2(\mathbb{R}^N)\}.$$

2. Page 3, brackets between formulas (1.12) and (1.13): the line with

   $$(P = 1/\theta, Q = 1/(1-\theta), a = \eta \|\nabla w\|_2^{2\theta}, b = \|w\|_2^{2\theta}/\eta).$$

   to be replaced by [see exponent in expression for b]

   $$(P = 1/\theta, Q = 1/(1-\theta), a = \eta \|\nabla w\|_2^{2\theta}, b = \|w\|_2^{2(1-\theta)}/\eta).$$

3. Page 4, formula (1.22), integral in the numerator:

   $$l(N, \theta) = \inf_{q_- \in L^p(\mathbb{R}^N)} \inf_{u \in H^1(\mathbb{R}^N)} \frac{\|\nabla u\|_2^2 + \int_{\mathbb{R}^N} q \|u\|_2^2 dx}{\|u\|_2^2} \|q_-\|_p^{-1/(1-\theta)}.$$

   to be replaced by [see $|u|^2$ in integrand in integral in numerator]

   $$l(N, \theta) = \inf_{q_- \in L^p(\mathbb{R}^N)} \inf_{u \in H^1(\mathbb{R}^N)} \frac{\|\nabla u\|_2^2 + \int_{\mathbb{R}^N} q |u|^2 dx}{\|u\|_2^2} \|q_-\|_p^{-1/(1-\theta)}.$$

4. Page 5, formulas (1.28) and (1.29), integrals (0,infinity) in the numerators:

$$\inf_{q_-\in L^p(\mathbb{R}^+)}\ \inf_{u\in\mathcal{D}(T_0)} \frac{\|u'\|_2^2+\int_0^\infty q|u|_2^2dx}{\|u\|_2^2}\|q_-\|_p^{-2p/(2p-1)} = l(1,1/(2p)),$$

$$\inf_{q_-\in L^p(\mathbb{R}^+)}\ \inf_{u\in\mathcal{D}(T_{\pi/2})} \frac{\|u'\|_2^2+\int_0^\infty q|u|_2^2dx}{\|u\|_2^2}\|q_-\|_p^{-2p/(2p-1)} = 2^{2/(2p-1)}l(1,1/(2p)).$$

to be replaced by [see $|u|^2$ in integrand integral numerator], respectively

$$\inf_{q_-\in L^p(\mathbb{R}^+)}\ \inf_{u\in\mathcal{D}(T_0)} \frac{\|u'\|_2^2+\int_0^\infty q|u|^2dx}{\|u\|_2^2}\|q_-\|_p^{-2p/(2p-1)} = l(1,1/(2p)),$$

$$\inf_{q_-\in L^p(\mathbb{R}^+)}\ \inf_{u\in\mathcal{D}(T_{\pi/2})} \frac{\|u'\|_2^2+\int_0^\infty q|u|^2dx}{\|u\|_2^2}\|q_-\|_p^{-2p/(2p-1)} = 2^{2/(2p-1)}l(1,1/(2p)).$$

5. Page 7, Lemma 2.1: “defined in (6)” should be replaced by “defined in (1.6)”.

6. Page 9, formula (2.15):

$$h(u,u) = -b^Q/Q = -(1-\theta)\theta^{\theta/(1-\theta)}\lambda_{N,\theta}^{-2/(1-\theta)}\|q_-\|_p^{1/(1-\theta)}\|u\|_2^2,$$

to be replaced by [first equality sign to be replaced by inequality sign]

$$h(u,u) \geq -b^Q/Q = -(1-\theta)\theta^{\theta/(1-\theta)}\lambda_{N,\theta}^{-2/(1-\theta)}\|q_-\|_p^{1/(1-\theta)}\|u\|_2^2,$$

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7. Page 10, formula (2.25), integral in the numerator:

$$\frac{\|u_j'\|_2^2 + \int_{-\infty}^{\infty} q|u_j|_2^2 dx}{\|u_j\|_2^2}\|q_j\|_1^{-2} = -(1+1/j)^2/4 > -1/4 = l(1,1/2).$$

to be replaced by [see q_j and $|u_j|^2$ in integrand integral numerator]

$$\frac{\|u_j'\|_2^2 + \int_{-\infty}^{\infty} q_j|u_j|^2 dx}{\|u_j\|_2^2}\|q_j\|_1^{-2} = -(1+1/j)^2/4 > -1/4 = l(1,1/2).$$

Moreover, corrections have to be made in the following lines.

1. Page 1, Abstract, line 4: change "$\Lambda_{N,\theta}(\nu) = \|\nabla v\|_2^\theta \|v\|_2^{1-\theta}\|v\|_r^{-1}$, with ν element of the Sobolev space $H^1(\mathbb{R}^N)$", into "$\Lambda_{N,\theta}(v) = \|\nabla v\|_2^\theta \|v\|_2^{1-\theta}\|v\|_r^{-1}$, with v element of the Sobolev space $H^1(\mathbb{R}^N)$".
2. Page 1, line -2 : change "$q = q_+ + q_-$" into "$q = q_+ - q_-$" .
3. Page 6, line 12: change "$l(3,3/4) = -1.750180_{10-4}$" into "$l(3,3/4) \simeq -1.750180_{10-4}$".
4. Page 8: label (2.4) refers the expression, one line higher; label (2.5) refers to the expression two lines higher.
5. Page 9: change formula (2.16) "$q = q_-$" into " $q = -q_-$".
6. Page 11, line 9: change "side of (31)" into "side of (1.31)".