

## ON NEIGHBORHOODS OF A CERTAIN CLASS OF COMPLEX ORDER DEFINED BY RUSCHEWEYH DERIVATIVE OPERATOR

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Abstract

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## Abstract

In this paper, we introduce the subclass  $R_b^\lambda(A, B, \alpha, \mu)$  which is defined by concept of subordination. According to this, we obtain a necessary and sufficient condition which is equivalent to this class. Further, we apply to the  $\delta$ - neighborhoods for belonging to  $R_b^\lambda(A, B, \alpha, \mu)$  to this condition.

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*Key words:* Analytic function, Hadamard product,  $\delta$ - neighborhood, Subordination, Close-to-convex function.

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# 1. Introduction and Definitions

Let  $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$  and  $\mathcal{H}(\mathbb{U})$  be the set of all functions analytic in  $\mathbb{U}$ , and let

$$\mathcal{A} := \{f \in \mathcal{H}(\mathbb{U}) : f(0) = f'(0) - 1 = 0\}.$$

Given two functions  $f$  and  $g$ , which are analytic in  $\mathbb{U}$ . The function  $f$  is said to be *subordinate* to  $g$ , written

$$f \prec g \quad \text{and} \quad f(z) \prec g(z) \quad (z \in \mathbb{U}),$$

if there exists a Schwarz function  $\omega$  analytic in  $\mathbb{U}$ , with

$$\omega(0) = 0 \quad \text{and} \quad |\omega(z)| < 1 \quad (z \in \mathbb{U}),$$

and such that

$$f(z) = g(\omega(z)) \quad (z \in \mathbb{U}).$$

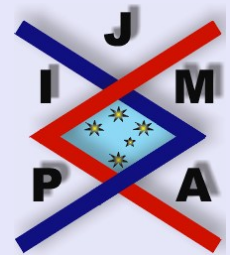
In particular, if  $g$  is univalent in  $\mathbb{U}$ , then  $f \prec g$  if and only if  $f(0) = g(0)$  and  $f(\mathbb{U}) \subset g(\mathbb{U})$  in [7].

Next, for the functions  $f_j$  ( $j = 1, 2$ ) given by

$$f_j(z) = z + \sum_{k=2}^{\infty} a_{k,j} z^k \quad (j = 1, 2).$$

Let  $f_1 * f_2$  denote the *Hadamard product (or convolution)* of  $f_1$  and  $f_2$ , defined by

$$(1.1) \quad (f_1 * f_2)(z) := z + \sum_{k=2}^{\infty} a_{k,1} a_{k,2} z^k =: (f_2 * f_1)(z).$$



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$(a)_v$  denotes the *Pochhammer symbol (or the shifted factorial)*, since

$$(1)_n = n! \quad \text{for } n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\},$$

defined (for  $a, v \in \mathbb{C}$  and in terms of the Gamma function) by

$$(a)_v := \frac{\Gamma(a+v)}{\Gamma(a)} = \begin{cases} 1; & (v = 0, a \in \mathbb{C} \setminus \{0\}), \\ a(a+1) \dots (a+n-1); & (v = n \in \mathbb{N}; a \in \mathbb{C}). \end{cases}$$

The earlier investigations by Goodman [1] and Ruscheweyh [9], we define the  $\delta$ -neighborhood of a function  $f \in \mathcal{A}$  by

$$\mathcal{N}_\delta(f) := \left\{ g \in \mathcal{A} : f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \right. \\ \left. g(z) = z + \sum_{k=2}^{\infty} b_k z^k \quad \text{and} \quad \sum_{k=2}^{\infty} k |a_k - b_k| \leq \delta \right\}$$

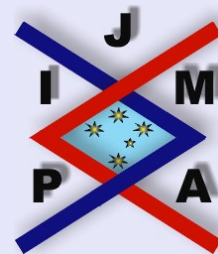
so that, obviously,

$$\mathcal{N}_\delta(e) := \left\{ g \in \mathcal{A} : g(z) = z + \sum_{k=2}^{\infty} b_k z^k \quad \text{and} \quad \sum_{k=2}^{\infty} k |b_k| \leq \delta \right\},$$

where  $e(z) := z$ .

Ruscheweyh [8] introduced an linear operator  $\mathcal{D}^\lambda : \mathcal{A} \rightarrow \mathcal{A}$ , defined by the Hadamard product as follows:

$$\mathcal{D}^\lambda f(z) := \frac{z}{(1-z)^{\lambda+1}} * f(z) \quad (\lambda > -1; z \in \mathbb{U}),$$

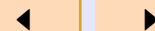


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which implies that

$$\mathcal{D}^n f(z) = \frac{z(z^{n-1}f(z))^{(n)}}{n!} \quad (n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}).$$

Clearly, we have

$$\mathcal{D}^0 f(z) = f(z), \quad \mathcal{D}^1 f(z) = zf'(z)$$

and

$$\mathcal{D}^n f(z) = \sum_{k=0}^{\infty} \frac{(\lambda+1)_k}{(1)_k} a_{k+1} z^{k+1} = \left( \sum_{k=0}^{\infty} \frac{(\lambda+1)_k}{(1)_k} z^{k+1} * f \right) (z),$$

where  $f \in \mathcal{A}$ .

Therefore, we can write the following equality, the easily verified result from the above definitions:

$$(1.2) \quad \left[ (1-\mu) \frac{\mathcal{D}^\lambda f(z)}{z} + \mu (\mathcal{D}^\lambda f(z))' \right] * \frac{1}{(1-z)^2} = (\mathcal{D}^\lambda f(z))' + \mu z (\mathcal{D}^\lambda f(z))'',$$

where  $f \in \mathcal{A}$ ,  $\lambda (\lambda > -1)$ ,  $\mu (\mu \geq 0)$  and for all  $z \in \mathbb{U}$ .

For each  $A$  and  $B$  such that  $-1 \leq B < A \leq 1$  and for all real numbers  $\alpha$  such that  $0 \leq \alpha < 1$ , we define the function

$$h(A, B, \alpha; z) := \frac{1 + \{(1-\alpha)A + \alpha B\}z}{1 + Bz} \quad (z \in \mathbb{U}).$$



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Also, let  $h(\alpha)$  denote the extremal function of functions with positive real part of order  $\alpha$  ( $0 \leq \alpha < 1$ ), defined by

$$h(\alpha) := h(1, -1, \alpha; z) = \frac{1 + (1 - 2\alpha)z}{1 - z} \quad (z \in \mathbb{U}).$$

The class  $\mathcal{R}^b(A, B)$  is studied by Premabai in [3]. According to this, we introduce the subclass  $\mathcal{R}_b^\lambda(A, B, \alpha, \mu)$  which is a generalization of this class, as follows:

$$(1.3) \quad 1 + \frac{1}{b} \left[ (\mathcal{D}^\lambda f(z))' + \mu z (\mathcal{D}^\lambda f(z))'' - 1 \right] \prec h(A, B, \alpha; z),$$

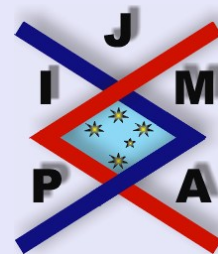
where  $f \in \mathcal{A}$ ,  $b \in \mathbb{C} \setminus \{0\}$ , for some real numbers  $A, B$  ( $-1 \leq B < A \leq 1$ ),  $\lambda$  ( $\lambda > -1$ ),  $\alpha$  ( $0 \leq \alpha < 1$ ),  $\mu$  ( $\mu \geq 0$ ) and for all  $z \in \mathbb{U}$  with  $\mathcal{R}_b(A, B, \alpha, \mu) := \mathcal{R}_b^0(A, B, \alpha, \mu)$  and  $\mathcal{R}^b(A, B) := \mathcal{R}_b(A, B, 0, 0)$ .

We note that the class  $\mathcal{R}^b(\mu) := \mathcal{R}_b(1, -1, 0, \mu)$  is studied by Altıntaş and Özkan in [4]. Therefore  $\mathcal{C}(b) := \mathcal{R}_b(1, -1, 0, 0)$  is the class of close-to-convex functions of complex order  $b$ .  $\mathcal{C}(\alpha) := \mathcal{R}_{1-\alpha}(1, -1, 0, 0)$  is the class of close-to-convex functions of order  $\alpha$  ( $0 \leq \alpha < 1$ ).

Also, let  $\mathcal{T}_b(A, B, \alpha)$  denote the class of functions  $\phi$  normalized by

$$(1.4) \quad \phi(z) := \frac{1 + \frac{1}{b} \left\{ \frac{1}{(1-z)^2} - 1 \right\} - \frac{1 + \{(1-\alpha)A + \alpha B\}e^{it}}{1 + Be^{it}}}{1 - \frac{1 + \{(1-\alpha)A + \alpha B\}e^{it}}{1 + Be^{it}}} \quad (t \in (0, 2\pi)),$$

where  $b \in \mathbb{C} \setminus \{0\}$ , for some real numbers  $A, B$  ( $-1 \leq B < A \leq 1$ ), for all  $\alpha$  ( $0 \leq \alpha < 1$ ) and for all  $z \in \mathbb{U}$  with  $\mathcal{T}_b(A, B) := \mathcal{T}_b(A, B, 0)$  and  $\mathcal{T}(b) := \mathcal{T}_b(1, -1, 0)$ .



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## 2. The Main Results

A theorem that contains the relationship between the above classes is given as follows:

**Theorem 2.1.**  $f \in \mathcal{R}_b^\lambda(A, B, \alpha, \mu)$  if and only if

$$\left[ (1 - \mu) \frac{\mathcal{D}^\lambda f(z)}{z} + \mu (\mathcal{D}^\lambda f(z))' \right] * \phi(z) \neq 0$$

for all  $\phi \in \mathcal{T}_b(A, B, \alpha)$  and for all  $f \in \mathcal{A}$ .

*Proof.* Firstly, let

$$F^\lambda(f, \mu; z) := (1 - \mu) \frac{\mathcal{D}^\lambda f(z)}{z} + \mu (\mathcal{D}^\lambda f(z))'$$

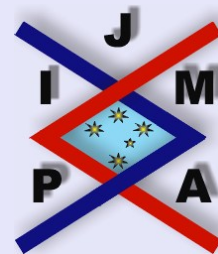
and we suppose that

$$(2.1) \quad F^\lambda(f, \mu; z) * \phi(z) \neq 0$$

for all  $f \in \mathcal{A}$  and for all  $\phi \in \mathcal{T}_b(A, B, \alpha)$ . In view of (1.4), we have

$$\begin{aligned} & F^\lambda(f, \mu; z) * \phi(z) \\ &= \frac{1 + \frac{1}{b} \left\{ F^\lambda(f, \mu; z) * \frac{1}{(1-z)^2} - 1 \right\} - \frac{1 + \{(1-\alpha)A + \alpha B\}e^{it}}{1 + Be^{it}}}{1 - \frac{1 + \{(1-\alpha)A + \alpha B\}e^{it}}{1 + Be^{it}}} \\ &= \frac{1 + \frac{1}{b} \left\{ (\mathcal{D}^\lambda f(z))' + \mu z (\mathcal{D}^\lambda f(z))'' - 1 \right\} - \frac{1 + \{(1-\alpha)A + \alpha B\}e^{it}}{1 + Be^{it}}}{1 - \frac{1 + \{(1-\alpha)A + \alpha B\}e^{it}}{1 + Be^{it}}} \end{aligned}$$

$\neq 0$ .



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From this inequality we find that

$$1 + \frac{1}{b} \left\{ (\mathcal{D}^\lambda f(z))' + \mu z (\mathcal{D}^\lambda f(z))'' - 1 \right\} \neq h(A, B, \alpha; e^{it}),$$

where  $t \in (0, 2\pi)$ .

This means that  $1 + \frac{1}{b} \left\{ (\mathcal{D}^\lambda f(z))' + \mu z (\mathcal{D}^\lambda f(z))'' - 1 \right\}$  does not take any value on the image of under  $h(A, B, \alpha; z)$  function of the boundary of  $\mathbb{U}$ . Therefore we note that  $1 + \frac{1}{b} \left\{ (\mathcal{D}^\lambda f(z))' + \mu z (\mathcal{D}^\lambda f(z))'' - 1 \right\}$  takes the value 1 for  $z = 0$ . Since  $0 \leq \alpha < 1$  and  $B < A$ , 1 is contained by the image under  $h(A, B, \alpha; z)$  function of  $\mathbb{U}$ . Thus, we can write

$$1 + \frac{1}{b} \left\{ (\mathcal{D}^\lambda f(z))' + \mu z (\mathcal{D}^\lambda f(z))'' - 1 \right\} \prec h(A, B, \alpha; z).$$

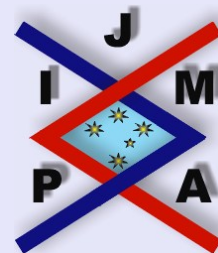
Hence  $f \in \mathcal{R}_b^\lambda(A, B, \alpha, \mu)$ .

Conversely, assume the function  $f$  is in the class  $\mathcal{R}_b^\lambda(A, B, \alpha, \mu)$ . From the definition of subordination, we can write the following inequality:

$$1 + \frac{1}{b} \left\{ (\mathcal{D}^\lambda f(z))' + \mu z (\mathcal{D}^\lambda f(z))'' - 1 \right\} \neq h(A, B, \alpha; e^{it}),$$

where  $t \in (0, 2\pi)$ . From (1.2) we can write

$$1 + \frac{1}{b} \left\{ F^\lambda(f, \mu; z) * \frac{1}{(1-z)^2} - 1 \right\} \neq h(A, B, \alpha; e^{it})$$



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or equivalently,

$$F^\lambda(f, \mu; z) * \left\{ \frac{1 + \frac{1}{b} \left( \frac{1}{(1-z)^2} - 1 \right) - h(A, B, \alpha; e^{it})}{1 - h(A, B, \alpha; e^{it})} \right\}.$$

Thus, from the definition of the function  $\phi$ , we can write

$$\left[ (1 - \mu) \frac{\mathcal{D}^\lambda f(z)}{z} + \mu (\mathcal{D}^\lambda f(z))' \right] * \phi(z) \neq 0$$

for all  $\phi \in \mathcal{T}_b(A, B, \alpha)$  and for all  $f \in \mathcal{A}$ . □

**Corollary 2.2.**  $f \in \mathcal{R}_b(A, B, \alpha, \mu)$  if and only if  $\left[ (1 - \mu) \frac{f(z)}{z} + \mu f'(z) \right] * \phi(z) \neq 0$  for all  $\phi \in \mathcal{T}_b(A, B, \alpha)$  and for all  $f \in \mathcal{A}$ .

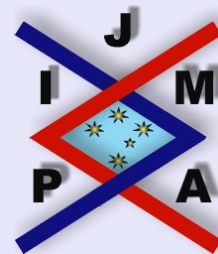
*Proof.* By putting  $\lambda = 0$  in Theorem 2.1. □

**Corollary 2.3.**  $f \in \mathcal{R}^b(A, B)$  if and only if  $\frac{f(z)}{z} * \phi(z) \neq 0$  for all  $\phi \in \mathcal{T}_b(A, B)$  and for all  $f \in \mathcal{A}$ .

*Proof.* By putting  $\alpha = 0, \mu = 0$  in Corollary 2.2. And, we obtain the result of Theorem 1 in [3]. □

**Corollary 2.4.**  $f \in \mathcal{C}(b)$  if and only if  $\frac{f(z)}{z} * \phi(z) \neq 0$  for all  $\phi \in \mathcal{T}(b)$  and for all  $f \in \mathcal{A}$ .

*Proof.* By putting  $A = 1, B = -1$  in Corollary 2.3. □




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**Theorem 2.5.** Let  $F_\epsilon(z) = \frac{f(z)+\epsilon z}{1+\epsilon}$  for  $\epsilon \in \mathbb{C}$  and  $f \in \mathcal{A}$ . If  $F_\epsilon \in \mathcal{R}_b^\lambda(A, B, \alpha, \mu)$  for  $|\epsilon| < \delta^*$ , then

$$(2.2) \quad \left| \left[ (1-\mu) \frac{\mathcal{D}^\lambda f(z)}{z} + \mu (\mathcal{D}^\lambda f(z))' \right] * \phi(z) \right| \geq \delta^* \quad (z \in \mathbb{U}),$$

where  $\phi \in \mathcal{T}_b(A, B, \alpha)$  and for all  $f \in \mathcal{A}$ .

*Proof.* Let  $\phi \in \mathcal{T}_b(A, B, \alpha)$  and  $F_\epsilon \in \mathcal{R}_b^\lambda(A, B, \alpha, \mu)$ . From Theorem 2.1, we can write

$$\left[ (1-\mu) \frac{\mathcal{D}^\lambda F_\epsilon(z)}{z} + \mu (\mathcal{D}^\lambda F_\epsilon(z))' \right] * \phi(z) \neq 0.$$

Using  $\mathcal{D}^\lambda(\epsilon z) = \epsilon z$ , we find that the following inequality

$$\frac{1}{1+\epsilon} \left\{ \left[ (1-\mu) \frac{\mathcal{D}^\lambda f(z)}{z} + \mu (\mathcal{D}^\lambda f(z))' \right] * \phi(z) + \epsilon \right\} \neq 0$$

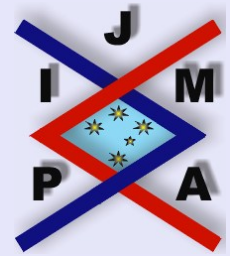
that is,

$$\left[ (1-\mu) \frac{\mathcal{D}^\lambda f(z)}{z} + \mu (\mathcal{D}^\lambda f(z))' \right] * \phi(z) \neq -\epsilon$$

or equivalently (2.2). □

**Lemma 2.6.** If  $\phi(z) = 1 + \sum_{k=1}^{\infty} c_k z^k \in \mathcal{T}_b(A, B, \alpha)$ , then we have

$$(2.3) \quad |c_k| \leq \frac{(k+1)(1+|B|)}{(1-\alpha)|b||B-A|} \quad (k = 1, 2, 3, \dots).$$



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*Proof.* We suppose that  $\phi(z) = 1 + \sum_{k=1}^{\infty} c_k z^k \in \mathcal{T}_b(A, B, \alpha)$ . From (1.4), we have

$$1 + \sum_{k=1}^{\infty} c_k z^k = \frac{1 + \frac{1}{b} \left\{ 1 + \sum_{k=2}^{\infty} k z^{k-1} - 1 \right\} - \frac{1 + \{(1-\alpha)A + \alpha B\} e^{it}}{1 + B e^{it}}}{1 - \frac{1 + \{(1-\alpha)A + \alpha B\} e^{it}}{1 + B e^{it}}} \quad (t \in (0, 2\pi)).$$

We write the following equality result which is easily verified result from the above equality:

$$c_k = \frac{(k+1)}{b(1-\alpha)} \cdot \frac{(1 + B e^{it})}{(B - A) e^{it}}.$$

Taking the modulus of both sides, we obtain inequality (2.3). □

**Theorem 2.7.** If  $F_\epsilon \in \mathcal{R}_b^\lambda(A, B, \alpha, \mu)$  for  $|\epsilon| < \delta^*$ , then

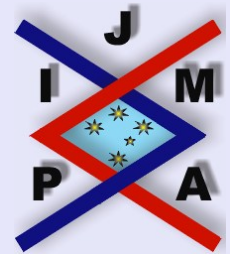
$$\mathcal{N}_\delta(f) \subset \mathcal{R}_b^\lambda(A, B, \alpha, \mu),$$

where  $\delta := \frac{(1-\alpha)|b||B-A|}{(1+\lambda)(1+\mu)(1+|B|)} \delta^*$ .

*Proof.* Let  $g \in \mathcal{N}_\delta(f)$  for  $\delta = \frac{(1-\alpha)|b||B-A|}{(1+\lambda)(1+\mu)(1+|B|)} \delta^*$ .

If we take

$$F^\lambda(g, \mu; z) := (1 - \mu) \frac{\mathcal{D}^\lambda g(z)}{z} + \mu (\mathcal{D}^\lambda g(z))',$$



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then

$$\begin{aligned} & |F^\lambda(g, \mu; z) * \phi(z)| \\ &= \left| \left[ (1 - \mu) \frac{\mathcal{D}^\lambda(f + g - f)(z)}{z} + \mu (\mathcal{D}^\lambda(f + g - f)(z))' \right] * \phi(z) \right| \\ &\geq |F^\lambda(f, \mu; z) * \phi(z)| - |F^\lambda(g - f, \mu; z) * \phi(z)| \end{aligned}$$

and using Theorem 2.5 we can write

$$(2.4) \quad \geq \delta^* - \left| \sum_{k=2}^{\infty} \Psi(k) (b_k - a_k) c_{k-1} z^{k-1} \right|,$$

where

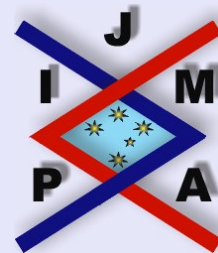
$$\Psi(k) = \frac{(\lambda + 1)_{(k-1)}}{(1)_k} (\mu k - \mu + 1).$$

We know that  $\Psi(k)$  is an increasing function of  $k$  and

$$0 < \Psi(2) = (1 + \lambda)(1 + \mu) \leq \Psi(k) \quad \left( \mu \geq 0; k \in \mathbb{N}; \lambda \geq \frac{-\mu k}{(1 + \mu k)} \right).$$

Since  $F_\epsilon \in \mathcal{R}_b^\lambda(A, B, \alpha, \mu)$  for  $|\epsilon| < \delta^*$  and using Lemma 2.6 in (2.4) we have

$$\begin{aligned} |F^\lambda(g, \mu; z) * \phi(z)| &> \delta^* - \Psi(2) |z| \sum_{k=2}^{\infty} |a_k - b_k| \frac{k(1 + |B|)}{(1 - \alpha) |b| |B - A|} \\ &> \delta^* - \frac{(1 + \lambda)(1 + \mu)(1 + |B|)}{(1 - \alpha) |b| |B - A|} \sum_{k=2}^{\infty} k |a_k - b_k| \end{aligned}$$



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$$> \delta^* - \delta \frac{(1 + \lambda)(1 + \mu)(1 + |B|)}{(1 - \alpha)|b||B - A|} > 0.$$

That is, we can write

$$\left[ (1 - \mu) \frac{\mathcal{D}^\lambda g(z)}{z} + \mu (\mathcal{D}^\lambda g(z))' \right] * \phi(z) \neq 0 \quad (z \in \mathbb{U}).$$

Thus, from Theorem 2.1 we can find that  $g \in \mathcal{R}_b^\lambda(A, B, \alpha, \mu)$ . □

**Corollary 2.8.** *If  $F_\epsilon \in \mathcal{R}_b(A, B, \alpha, \mu)$  for  $|\epsilon| < \delta^*$ , then*

$$\mathcal{N}_\delta(f) \subset \mathcal{R}_b(A, B, \alpha, \mu),$$

where  $\delta := \frac{(1-\alpha)|b||B-A|}{(1+\mu)(1+|B|)} \delta^*$ .

*Proof.* By putting  $\lambda = 0$  in Theorem 2.7. □

**Corollary 2.9.** *If  $F_\epsilon \in \mathcal{R}_b(A, B)$  for  $|\epsilon| < \delta^*$ , then*

$$\mathcal{N}_\delta(f) \subset \mathcal{R}_b(A, B)$$

where  $\delta := \frac{|b||B-A|}{(1+|B|)} \delta^*$ .

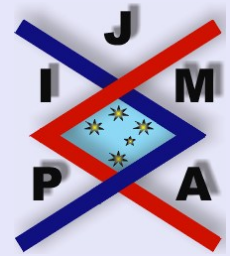
*Proof.* By putting  $\alpha = 0, \mu = 0$  in Corollary 2.8. Thus, we obtain the result of Theorem 2.7 in [3]. □

**Corollary 2.10.** *If  $F_\epsilon \in \mathcal{C}(b)$  for  $|\epsilon| < \delta^*$ , then*

$$\mathcal{N}_\delta(f) \subset \mathcal{C}(b),$$

where  $\delta := |b| \delta^*$ .

*Proof.* By putting  $A = 1, B = -1$  in Corollary 2.9. □



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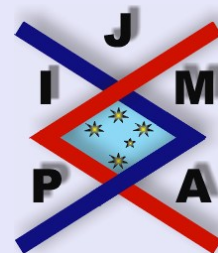
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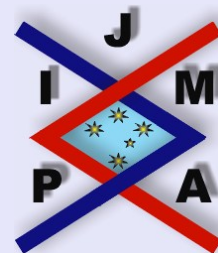
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