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ON THE DETERMINANTAL INEQUALITIES

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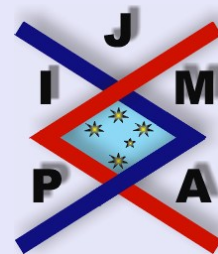


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Abstract

In this paper, we discuss the determinantal inequalities over arbitrary complex matrices, and give some sufficient conditions for

$$d[A + B]^t \geq d[A]^t + d[B]^t,$$

where $t \in \mathbb{R}$ and $t \geq \frac{2}{n}$. If B is nonsingular and $\operatorname{Re} \lambda(B^{-1}A) \geq 0$, the sufficient and necessary condition is given for the above equality at $t = \frac{2}{n}$. The famous Minkowski inequality and many recent results about determinantal inequalities are extended.

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1. Preliminaries

We use conventional notions and notations, as in [2]. Let $A \in M_n(C)$, $d[A]$ stands for the modulus of $\det(A)$ (or $|A|$), where $\det(A)$ is the determinant of A . $\sigma(A)$ is the spectrum of A , namely the set of eigenvalues of matrix A . A matrix $X \in M_n(C)$ is called complex (semi-) positive definite if $\operatorname{Re}(x^*Ax) > 0$ ($\operatorname{Re}(x^*Ax) \geq 0$) for all nonzero $x \in C^n$ or if $\frac{1}{2}(X + X^*)$ is a complex (semi-) positive definite matrix (see [4, 7, 8, 2]). Throughout this paper, we denote $C = B^{-1}A$ for $A, B \in M_n(C)$ and B is invertible.

The famous Minkowski inequality states:

If $A, B \in M_n(R)$ are real positive definite symmetric matrices, then

$$(1.1) \quad |A + B|^{\frac{1}{n}} \geq |A|^{\frac{1}{n}} + |B|^{\frac{1}{n}}.$$

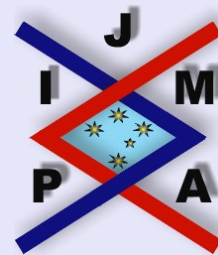
It is a very interesting work to generalize the Minkowski inequality. Obviously, (1.1) holds if $A, B \in M_n(C)$ are positive definite Hermitian matrices. Recently, (1.1) has been generalized for $A, B \in M_n(C)$ positive definite matrices (see [8], [9], [10], [3]).

In this paper, we discuss determinantal inequalities over arbitrary complex matrices, and give some sufficient conditions for

$$(1.2) \quad d[A + B]^t \geq d[A]^t + d[B]^t,$$

where $t \in \mathbb{R}$.

If B is nonsingular and $\operatorname{Re} \lambda(B^{-1}A) \geq 0$, a sufficient and necessary condition has been given for equality as $t = \frac{2}{n}$ in (1.2). The famous Minkowski inequality and many results about determinantal inequalities are extended.



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For $c \in C$, $\text{Re}(c)$ denotes the real part of c and $|c|$ denotes the modulus of c . Let $t > 0$ be fixed, we have

Lemma 1.1. *If $A, B \in M_n(C)$ and B is invertible, $\sigma(C) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$, then inequality (1.2) is true if and only if*

$$(1.3) \quad \prod_{i=1}^n |\lambda_i + 1|^t \geq \prod_{i=1}^n |\lambda_i|^t + 1,$$

with equality holding in (1.2) if and only if it holds in (1.3).

Proof. Since $d[A + B]^t = d[B]^t d[C + I]^t$ and $d[A]^t + d[B]^t = d[B]^t (1 + d[C]^t)$, formula (1.2) is equivalent to

$$(1.4) \quad d[C + I]^t \geq 1 + d[C]^t.$$

Notice $\sigma(C + I) = \{\lambda_k + 1 : k = 1, 2, \dots, n\}$,

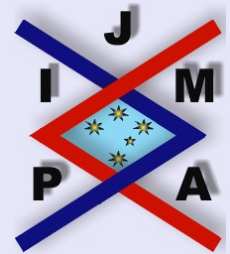
$$d[C + I]^t = \prod_{i=1}^n |\lambda_i + 1|^t \quad \text{and} \quad d[C]^t = \prod_{i=1}^n |\lambda_i|^t,$$

we obtain that formula (1.4) is equivalent to (1.3). Similarly, it is easy to see that the case of equality is true. Thus the lemma is proved. \square

Lemma 1.2 (see [6]). *If $x_t, y_t \geq 0$ ($t = 1, 2, \dots, n$), then*

$$\prod_{t=1}^n (x_t + y_t)^{\frac{1}{n}} \geq \prod_{t=1}^n x_t^{\frac{1}{n}} + \prod_{t=1}^n y_t^{\frac{1}{n}},$$

with equality if and only if there is linear dependence between (x_1, x_2, \dots, x_n) and (y_1, y_2, \dots, y_n) or $x_t + y_t = 0$ for a certain number t .



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Lemma 1.3 (Jensen's inequality). If a_1, a_2, \dots, a_m are positive numbers, then

$$\left(\sum_{i=1}^n a_i^s \right)^{\frac{1}{s}} \leq \left(\sum_{i=1}^n a_i^r \right)^{\frac{1}{r}} \quad \text{for } 0 < r \leq s, n \geq 2.$$

Lemma 1.4. If P_1, P_2, \dots, P_m are positive numbers and $T \geq \frac{1}{m}$, then

$$(1.5) \quad \prod_{k=1}^m (P_k + 1)^T \geq \prod_{k=1}^m P_k^T + 1,$$

with equality if and only if P_k ($k = 1, 2, \dots, m$) is constant as $T = \frac{1}{m}$.

Proof. By Lemma 1.2, we have

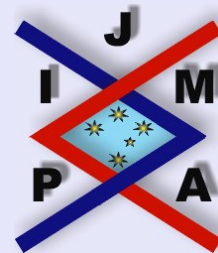
$$\prod_{k=1}^m (P_k + 1)^T = \left[\prod_{k=1}^m (P_k + 1)^{\frac{1}{m}} \right]^{mT} \geq \left[\prod_{k=1}^m (P_k^T)^{\frac{1}{mT}} + 1 \right]^{mT}.$$

On noting that $0 < \frac{1}{mT} \leq 1$, by Lemma 1.3, we obtain

$$\left[\prod_{k=1}^m (P_k^T)^{\frac{1}{mT}} + 1 \right]^{mT} \geq \prod_{k=1}^m P_k^T + 1,$$

and inequality (1.5) is demonstrated. By Lemma 1.2, it is easy to see that equality holds if and only if P_k ($k = 1, 2, \dots, m$) is constant as $T = \frac{1}{m}$. \square

Remark 1. Apparently, Lemma 1.3 is tenable for $a_i \geq 0$ ($i = 1, 2, \dots, n$), and Lemma 1.4 is tenable for $P_i \geq 0$ ($i = 1, 2, \dots, m$).



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2. Main Results

Theorem 2.1. Let $A, B \in M_n(C)$. If B is nonsingular and $\operatorname{Re} \lambda_k \geq 0$ ($k = 1, 2, \dots, n$), where $\sigma(C) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$, then for $t \geq \frac{2}{n}$

$$(2.1) \quad d[A + B]^t \geq d[A]^t + d[B]^t,$$

Proof. By Lemma 1.1, we need to prove inequality (1.3). Note that $\operatorname{Re} \lambda_k \geq 0$ ($k = 1, 2, \dots, n$) and $|\lambda_k + 1|^2 \geq 1 + |\lambda_k|^2$,

$$\prod_{k=1}^n |\lambda_k + 1|^t = \left(\prod_{k=1}^n |\lambda_k + 1|^2 \right)^{\frac{t}{2}} \geq \prod_{k=1}^n (|\lambda_k|^2 + 1)^{\frac{t}{2}}.$$

Applying Lemma 1.4, we can show that

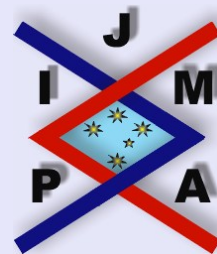
$$\prod_{k=1}^n (|\lambda_k|^2 + 1)^{\frac{t}{2}} \geq \prod_{k=1}^n |\lambda_k|^t + 1 \quad \text{for } t \geq \frac{2}{n},$$

with equality if and only if $|\lambda_k|^2$ ($k = 1, 2, \dots, n$) is constant as $t = \frac{2}{n}$. The above two inequalities imply formula (1.3). \square

When $t = 1$, we have

Corollary 2.2. Let $A, B \in M_n(C)$ ($n \geq 2$). If B is invertible and $\operatorname{Re} \lambda_k \geq 0$ ($k = 1, 2, \dots, n$), where $\sigma(C) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$, then

$$(2.2) \quad d[A + B] \geq d[A] + d[B].$$



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Corollary 2.3. Let A be an n -by- n complex positive definite matrix, and B be an n -by- n positive definite Hermitian matrix ($n \geq 2$). Then for $t \geq \frac{2}{n}$

$$(2.3) \quad d[A + B]^t \geq d[A]^t + [\det(B)]^t.$$

Proof. Observing $C = B^{-1}A$ is similar to $B^{-\frac{1}{2}}AB^{-\frac{1}{2}}$ and $\operatorname{Re} \lambda(B^{-\frac{1}{2}}AB^{-\frac{1}{2}}) > 0$, where $\lambda(B^{-\frac{1}{2}}AB^{-\frac{1}{2}})$ is an arbitrary eigenvalue of $B^{-\frac{1}{2}}AB^{-\frac{1}{2}}$. Therefore, $\operatorname{Re} \lambda_k \geq 0$ and $\sigma(C) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$. Hence, Theorem 2.1 yields Corollary 2.3. \square

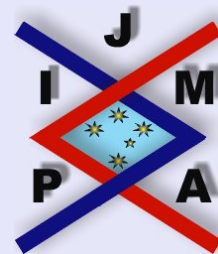
When $t = \frac{2}{n}$, inequality (2.3) gives Theorem 4 of [3]. When $t = 1$, inequality (2.3) gives Theorem 1 of [3]. To merit attention, Theorem 2 in [8] proves that if A is real positive definite and B is real positive definite symmetric, then (2.3) holds for $t = \frac{1}{n}$. It is untenable for example: $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Corollary 2.7 and Corollary 2.8 in this paper have been given correction.

Theorem 2.4. Let $A, B \in M_n(C)$. If B is nonsingular, and $\operatorname{Re} \lambda_k \geq 0$ ($k = 1, 2, \dots, n$), where $\sigma(C) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$, then n eigenvalues of C are pure imaginary complex numbers with the same modulus if and only if

$$(2.4) \quad d[A + B]^{\frac{2}{n}} = d[A]^{\frac{2}{n}} + d[B]^{\frac{2}{n}},$$

Proof. If n eigenvalues of C are $\pm id$ ($i = \sqrt{-1}, d > 0, d \in R$), then

$$\prod_{i=1}^n |\lambda_i + 1|^{\frac{2}{n}} = \prod_{i=1}^n (1 + d^2)^{\frac{1}{n}} = 1 + d^2 = \prod_{i=1}^n |\lambda_i|^{\frac{2}{n}} + 1.$$



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Hence equality (2.4) holds by Lemma 1.1.

Conversely, suppose (2.4) holds, then

$$\prod_{i=1}^n |\lambda_i + 1|^{\frac{2}{n}} = \prod_{i=1}^n |\lambda_i|^{\frac{2}{n}} + 1.$$

So

$$\prod_{i=1}^n (1 + 2 \operatorname{Re} \lambda_i + |\lambda_i|^2)^{\frac{1}{n}} = \prod_{i=1}^n (|\lambda_i|^2)^{\frac{1}{n}} + 1.$$

Obviously, $\operatorname{Re} \lambda_k = 0$ ($k = 1, 2, \dots, n$), otherwise

$$\prod_{i=1}^n (1 + 2 \operatorname{Re} \lambda_i + |\lambda_i|^2)^{\frac{1}{n}} > \prod_{i=1}^n (1 + |\lambda_i|^2)^{\frac{1}{n}} \geq \prod_{i=1}^n (|\lambda_i|^2)^{\frac{1}{n}} + 1,$$

with illogicality. Therefore

$$\prod_{i=1}^n [1 + (\operatorname{Im} \lambda_i)^2]^{\frac{1}{n}} = \prod_{i=1}^n [(\operatorname{Im} \lambda_i)^2]^{\frac{1}{n}} + 1.$$

By Lemma 1.2 we obtain $(\operatorname{Im} \lambda_k)^2 = d^2$ and $\lambda_k = \pm id$ ($k = 1, 2, \dots, n$). This completes the proof. \square

Corollary 2.5. *If $A, B \in M_n(C)$ with B is nonsingular and $C = B^{-1}A$ is skew-Hermitian, then formula (2.4) holds if and only if $A = idBUU^*$, where $i^2 = -1$, $d > 0$, U is a unitary matrix, $E = \operatorname{diag}(e_1, e_2, \dots, e_n)$ with $e_i = \pm 1$, $i = 1, 2, \dots, n$.*



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Proof. Since C is skew-Hermitian and its real parts of n eigenvalues are zero, then Theorem 2.4 implies that (2.4) holds if and only if

$$C = B^{-1}A = U \operatorname{diag}(\pm id, \pm id, \dots, \pm id)U^*,$$

where $\sigma(C) = \{\pm id, \pm id, \dots, \pm id\}$, $d > 0$ and U is unitary. Hence $A = idBEU^*$, where $i^2 = -1$, $d > 0$, U is a unitary matrix, $E = \operatorname{diag}(e_1, e_2, \dots, e_n)$ and $e_i = \pm 1$, $i = 1, 2, \dots, n$. \square

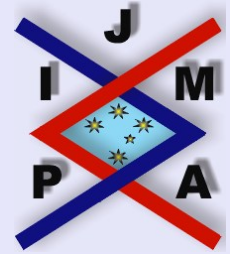
Theorem 2.6. Suppose $A, B \in M_n(C)$ with B nonsingular and $\operatorname{Re}\lambda_k \geq 0$ ($k = 1, 2, \dots, n$), where $\sigma(C) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$. If the number of the real eigenvalues of C is r , and the non-real eigenvalues of C are pair wise conjugate, then inequality (1.2) holds for $t \geq \frac{2}{n+r}$.

Proof. By Lemma 1.1, we need to prove (1.3) for $t \geq \frac{2}{n+r}$. Without loss of generality, suppose $\lambda_j \geq 0$ ($j = 1, 2, \dots, r$) are the real eigenvalues of C and $\lambda_k, \bar{\lambda}_k$ ($k = r+1, r+2, \dots, r+s$) are s pairs of non-real eigenvalues of C , where $n = r + 2s$. Then the right-hand side of (1.3) becomes

$$(2.5) \quad \prod_{i=1}^r \lambda_i^t \prod_{j=r+1}^{r+s} (|\lambda_j|^2)^t + 1,$$

and the left-hand side of (1.3) is

$$(2.6) \quad \prod_{i=1}^r (\lambda_i + 1)^t \prod_{j=r+1}^{r+s} (|1 + \lambda_j|^2)^t.$$



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Given $\operatorname{Re} \lambda_k \geq 0$ ($k = 1, 2, \dots, r + s$), so $|1 + \lambda_j|^2 \geq 1 + |\lambda_j|^2$, then

$$(2.7) \quad \prod_{i=1}^r (1 + \lambda_i)^t \prod_{j=r+1}^{r+s} (|1 + \lambda_j|^2)^t \geq \prod_{i=1}^r (1 + \lambda_i)^t \prod_{j=r+1}^{r+s} (1 + |\lambda_j|^2)^t.$$

By Lemma 1.2 and (2.7), we obtain that

$$\prod_{i=1}^r (\lambda_i + 1)^t \prod_{j=r+1}^{r+s} (|1 + \lambda_j|^2)^t \geq \prod_{i=1}^r \lambda_i^t \prod_{j=r+1}^{r+s} (|\lambda_j|^2)^t + 1,$$

for $\frac{1}{r + s} = \frac{2}{n + r}.$

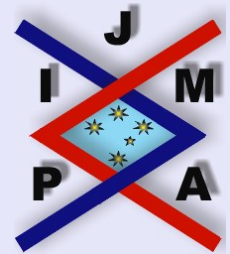
This completes the proof. □

In the following, we present some generalizations of the Minkowski inequality. By Theorem 2.6, it is easy to show:

Corollary 2.7. *Let $A, B \in M_n(C)$. If B is nonsingular and n eigenvalues of C are positive numbers, then for $t \geq \frac{1}{n}$*

$$(2.8) \quad d[A + B]^{\frac{1}{n}} \geq d[A]^{\frac{1}{n}} + d[B]^{\frac{1}{n}}.$$

If A is an n -by- n complex positive definite matrix and B is an n -by- n positive definite Hermitian matrix, with n eigenvalues of C being real numbers, then $\sigma(C) = \sigma(B^{\frac{1}{2}}CB^{-\frac{1}{2}})$, and $B^{\frac{1}{2}}CB^{-\frac{1}{2}} = B^{-\frac{1}{2}}AB^{-\frac{1}{2}}$ is positive definite, so any eigenvalue of C has a positive real part. Thus n eigenvalues of C are positive numbers. By Corollary 2.7 we have



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Corollary 2.8. Suppose $A, B \in M_n(C)$, where A is a complex positive definite matrix and B is a positive definite Hermitian matrix. If n eigenvalues of C are real numbers, then inequality (2.8) holds for $t \geq \frac{1}{n}$.

Corollary 2.9 (Minkowski inequality). Suppose $A, B \in M_n(C)$ are positive definite Hermitian matrices, then inequality (1.1) holds.

Proof. Note that $C = B^{-1}A$ is similar to a real diagonal matrix, and its eigenvalues are real numbers, using Corollary 2.8 and letting $t = 1$, the proof is completed. \square

Corollary 2.10. Suppose $A, B \in M_n(C)$, where A is a complex positive definite matrix and B is a positive definite Hermitian matrix. If the non-real eigenvalues of C are m pairs conjugate complex numbers, then inequality (1.2) holds for $t \geq \frac{1}{n-m}$.

Proof. Obviously $\operatorname{Re} \lambda_k \geq 0$ ($k = 1, 2, \dots, n$), where $\sigma(C) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$. Applying Theorem 2.6 completes the proof. \square

Let $A = H + K \in M_n(C)$, where $H = \frac{1}{2}(A + A^*)$, and $K = \frac{1}{2}(A - A^*)$, then we have

Theorem 2.11. Let $A = H + K$ be an n -by- n complex positive definite matrix, then for $t \geq \frac{2}{n}$

$$(2.9) \quad d[A]^t \geq d[H]^t + d[K]^t,$$

with equality if and only if $K = idHQ^*EQ$ as $t = \frac{2}{n}$, where $i^2 = -1$, $d > 0$, Q is a unitary matrix, $E = \operatorname{diag}(e_1, e_2, \dots, e_n)$ with $e_i = \pm 1$, $i = 1, 2, \dots, n$.



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Proof. Since $H^{-\frac{1}{2}}KH^{-\frac{1}{2}}$ is a skew-Hermitian matrix and is similar to $H^{-1}K$, $\operatorname{Re} \lambda(H^{-1}K) = \operatorname{Re} \lambda(H^{-\frac{1}{2}}KH^{-\frac{1}{2}}) = 0$. By Theorem 2.1 and Corollary 2.5, we get the desired result. \square

Let $t = 1$, we have the following interesting result.

Corollary 2.12. *If $A = H + K$ is an n -by- n complex positive definite matrix ($n \geq 2$), then*

$$(2.10) \quad d[A] \geq d[H] + d[K].$$

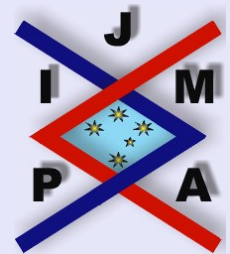
Corollary 2.13 (Ostrowski-Taussky Inequality). *If $A = H + K$ is an n -by- n positive definite matrix ($n \geq 2$), then $\det H \leq d[A]$ with equality if and only if A is Hermitian.*

Theorem 2.14. *Let A, B be two n -by- n complex positive definite matrices, and n eigenvalues of B be real numbers. Suppose A, B are simultaneously upper triangularizable, namely, there exists a nonsingular matrix P , such that $P^{-1}AP$ and $P^{-1}BP$ are upper triangular matrices, then inequality (1.2) holds for any $t \geq \frac{2}{n}$.*

Proof. If $P^{-1}AP$ and $P^{-1}BP$ are upper triangular matrices, then

$$P^{-1}B^{-1}AP = (P^{-1}BP)^{-1}(P^{-1}AP)$$

is an upper triangular matrix, with the product of the eigenvalues of B^{-1} and A on its diagonal. We denote the eigenvalue of X by $\lambda(X)$. Notice that positive definiteness of A and B^{-1} , $\operatorname{Re} \lambda(A)$ and $\lambda(B^{-1})$ are positive numbers by hypothesis, it is easy to see that $\operatorname{Re} \lambda(B^{-1}A) \geq 0$. By Theorem 2.1, we get the desired result. \square



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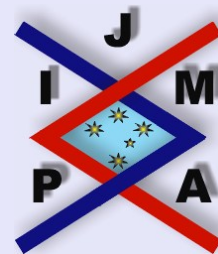
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Corollary 2.15. Let A, B be two n -by- n complex positive definite matrices, and all the eigenvalues of B be real numbers. If $r([A, B]) \leq 1$, then inequality (1.2) holds for $t \geq \frac{2}{n}$, where $[A, B] = AB - BA$, $r([A, B])$ is the rank of $[A, B]$.

Proof. It is easy to see that B^{-1} is a complex positive definite matrix and n eigenvalues of B^{-1} are real numbers. By the hypothesis and $r[B^{-1}, A] = r[A, B]$, we have $r([B^{-1}, A]) \leq 1$. By the Laffey-Choi Theorem (see [5], [1]), there exists a non-singular matrix P , such that $P^{-1}AP$ and $P^{-1}BP$ are upper triangular matrices. The result holds by Theorem 2.14. \square

Corollary 2.16. Let A, B be two n -by- n complex positive definite matrices ($n \geq 2$). Suppose $AB = BA$ and n eigenvalues of B are real numbers, then inequality (1.2) holds for $t \geq \frac{2}{n}$.

Proof. Follows from Corollary 2.15 and the fact that $r([A, B]) = 0$. \square



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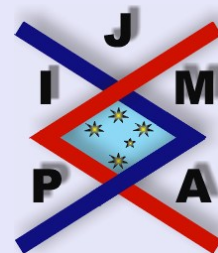
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