

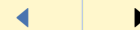
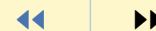
THE ALEXANDER TRANSFORMATION OF A SUBCLASS OF SPIRALLIKE FUNCTIONS OF TYPE β



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Qinghua Xu and Sanya Lu
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Abstract: In this paper, a subclass of spirallike function of type β denoted by \hat{S}_α^β is introduced in the unit disc of the complex plane. We show that the Alexander transformation of class of \hat{S}_α^β is univalent when $\cos \beta \leq \frac{1}{2(1-\alpha)}$, which generalizes the related results of some authors.

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1. Introduction

Let A denote the class of analytic functions f on the unit disk $D = \{z \in \mathbb{C} : |z| < 1\}$ normalized by $f(0) = 0$ and $f'(0) = 1$, S denote the subclass of A consisting of univalent functions, and S^* denote starlike functions on D . Obviously, $S^* \subset S \subset A$ holds.

In [1], Robertson introduced starlike functions of order α on D .

Definition 1.1. Let $\alpha \in [0, 1)$, $f \in S$ and

$$\Re \left[\frac{zf'(z)}{f(z)} \right] > \alpha, \quad z \in D.$$

We say that f is a starlike function of order α . Let $S^*(\alpha)$ denote the whole starlike functions of order α on D .

Spaček [2] extended the class of S^* , and obtained the class of spirallike functions of type β . In the same article, the author gave an analytical characterization of spirallikeness of type β on D .

Theorem 1.2. Let $f \in S$ and $\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Then $f(z)$ is a spirallike function of type β on D if and only if

$$\Re \left[e^{i\beta} \frac{zf'(z)}{f(z)} \right] > 0, \quad z \in D.$$

We denote the whole spirallike functions of type β on D by \hat{S}_β .

From Definition 1.1 and Theorem 1.2, it is easy to see that starlike functions of order α and spirallike functions of type β have some relationships on geometry. Spirallike functions of type β map D into the right half complex plane by the mapping



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$e^{i\beta} \frac{zf'(z)}{f(z)}$, while starlike functions of order α map D into the right half complex plane whose real part is greater than α by the mapping $\frac{zf'(z)}{f(z)}$. Since $\lim_{z \rightarrow 0} e^{i\beta} \frac{zf'(z)}{f(z)} = e^{i\beta}$, we can deduce that if we restrict the image of the mapping $e^{i\beta} \frac{zf'(z)}{f(z)}$ in the right complex plane whose real part is greater than a certain constant, then the constant must be smaller than $\cos \beta$. According to this, we introduce the functions class \hat{S}_α^β on D .

Definition 1.3. Let $\alpha \in [0, 1)$, $\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $f \in S$, then $f \in \hat{S}_\alpha^\beta$ if and only if

$$\Re \left[e^{i\beta} \frac{zf'(z)}{f(z)} \right] > \alpha \cos \beta, \quad z \in D.$$

Obviously, when $\beta = 0$, $f \in S^*(\alpha)$; while $\alpha = 0$, $f \in \hat{S}_\beta$.

Example 1.1. Let $f(z) = \frac{z}{(1-z)^{\frac{2(1-\alpha)}{1+i \tan \beta}}}$, $z \in D$. The branch of the power function is chosen such that

$$[(1-z)]^{\frac{2(1-\alpha)}{1+i \tan \beta}} \Big|_{z=0} = 1.$$

It is easily proved that $f \in \hat{S}_\alpha^\beta$. We omit the proof.

For our applications, we set $\hat{S} = \bigcup_\beta \hat{S}_\alpha^\beta$.

In this paper, we first establish the relationships among \hat{S}_α^β and some important subclasses of S , then investigate the Alexander transformation of \hat{S}_α^β preserving univalence. Furthermore, some other properties of the class of \hat{S}_α^β are obtained. These results generalize the related works of some authors.

2. Integral Transformations and Lemmas

Integral Transformation 1. *The integral transformation*

$$J[f](z) = \int_0^z \frac{f(\zeta)}{\zeta} d\zeta$$

is called the Alexander Transformation and it was introduced by Alexander in [4]. Alexander was the first to observe and prove that the Integral transformation J maps the class S^* of starlike functions onto the class K of convex functions in a one-to-one fashion.

In 1960, Biernacki conjectured that $J(S) \subset S$, but Krzyz and Lewandowski disproved it in 1963 by giving the example $f(z) = z(1 - iz)^{i-1}$, which is a spirallike function of type $\frac{\pi}{4}$ but is transformed into a non-univalent function by J [4]. In 1969, Robertson studied the Alexander Integral Transformation of spirallike functions of type β . The author showed that $J(\hat{S}_\beta) \subset S$ holds when β satisfies a certain condition, that is $\cos \beta \leq x_0$ (a constant). Robertson also noticed that x_0 cannot be replaced by any number greater than $\frac{1}{2}$ and asked about the best value for this [3]. In 2007, Y.C. Kim and T. Sugawa proved that $J(\hat{S}_\beta) \subset S$ holds precisely when $\cos \beta \leq \frac{1}{2}$ or $\beta = 0$ [4].

Integral Transformation 2. Let $\gamma \in \mathbb{C}$, $f(z) \in A$ be locally univalent, and the Integral transformation I_γ [5] be defined by

$$I_\gamma[f](z) = \int_0^z [f'(\zeta)]^\gamma d\zeta = z \int_0^1 [f'(tz)]^\gamma dt.$$

Based on the definition of I_γ , we may easily show that $I_\gamma \circ I_{\gamma'} = I_{\gamma\gamma'}$.

Let $A(F) = \{\gamma \in \mathbb{C} : I_\gamma(F) \subset S\}$, $F \subset A$ be locally univalent. According to the definition of the $A(F)$, $J(\hat{S}_\alpha^\beta) \subset S$ is equivalent to $1 \in A(J(\hat{S}_\alpha^\beta))$.



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For the proof of the theorems in this paper, we need the following lemma, which establishes the relationships among \hat{S}_α^β and some important subclasses of S .

Lemma 2.1. For $\alpha \in [0, 1)$, $\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $c = e^{-i\beta} \cos \beta$, the following assertions hold:

(i) ([6, 7]) $f \in S^*(\alpha)$ if and only if

$$\frac{f(z)}{z} = \left[\frac{u(z)}{z} \right]^{1-\alpha}, \quad z \in D,$$

where $u(z) \in S^*$. The branch of the power function is chosen such that

$$\left[\frac{u(z)}{z} \right]^{1-\alpha} \Big|_{z=0} = 1.$$

(ii) $f \in \hat{S}_\alpha^\beta$ if and only if

$$\frac{f(z)}{z} = \left[\frac{g(z)}{z} \right]^c, \quad z \in D,$$

where $g(z) \in S^*(\alpha)$. The branch of the power function is chosen such that

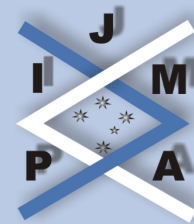
$$\left[\frac{g(z)}{z} \right]^c \Big|_{z=0} = 1.$$

(iii) $f \in \hat{S}_\alpha^\beta$ if and only if

$$\frac{f(z)}{z} = \left[\frac{s(z)}{z} \right]^{(1-\alpha)c}, \quad z \in D,$$

where $s(z) \in S^*$. The branch of the power function is chosen such that

$$\left[\frac{s(z)}{z} \right]^{(1-\alpha)c} \Big|_{z=0} = 1.$$



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Now we give the proof of (ii) and (iii).

Proof. (ii). First, assume that $f(z) \in \hat{S}_\alpha^\beta$. Setting $g(z) = z \left[\frac{f(z)}{z} \right]^{\frac{e^{i\beta}}{\cos \beta}}$, through simple calculations we may obtain the equality

$$\frac{zg'(z)}{g(z)} = (1 + i \tan \beta) \frac{zf'(z)}{f(z)} - i \tan \beta.$$

Therefore the following inequality holds,

$$\Re \left[\frac{zg'(z)}{g(z)} \right] = \frac{1}{\cos \beta} \Re \left[e^{i\beta} \frac{zf'(z)}{f(z)} \right] > \frac{\alpha \cos \beta}{\cos \beta} = \alpha,$$

namely $g(z) \in S^*(\alpha)$.

Conversely, suppose $g(z) \in S^*(\alpha)$, then according to the above calculation, we have the inequality

$$\frac{1}{\cos \beta} \Re \left[e^{i\beta} \frac{zf'(z)}{f(z)} \right] = \Re \left[\frac{zg'(z)}{g(z)} \right] > \alpha.$$

This implies

$$\Re \left[e^{i\beta} \frac{zf'(z)}{f(z)} \right] > \alpha \cos \beta,$$

i.e., $f(z) \in \hat{S}_\alpha^\beta$.

(iii). It is easy to see from (ii) that $f \in \hat{S}_\alpha^\beta$ if and only if $g \in S^*(\alpha)$ such that $\frac{f(z)}{z} = \left[\frac{g(z)}{z} \right]^c$, here $c = e^{-i\beta} \cos \beta$. Noting that $g(z) \in S^*(\alpha)$ if and only if $s(z) \in S^*$ such that $\frac{g(z)}{z} = \left[\frac{s(z)}{z} \right]^{1-\alpha}$ which holds in (i), we may obtain an important relationship



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between the class of \hat{S}_α^β and the class of $S^* : f \in \hat{S}_\alpha^\beta$ if and only if there exists $s(z) \in S^*$ such that $\frac{f(z)}{z} = \left[\frac{s(z)}{z} \right]^{(1-\alpha)c}$. Here, $c = e^{-i\beta} \cos \beta$ and the branch of the power function is chosen such that $\left[\frac{s(z)}{z} \right]^{(1-\alpha)c} \Big|_{z=0} = 1$. \square

Lemma 2.1 expresses the relations of the \hat{S}_α^β and S^* classes, which play a key role in this paper.

Lemma 2.2 ([5], [8]). $A(K) = \{|\gamma| \leq \frac{1}{2}\} \cup [\frac{1}{2}, \frac{3}{2}]$.

Lemma 2.3. For $\alpha \in [0, 1)$, $\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $J(\hat{S}_\alpha^\beta) = I_{(1-\alpha)e^{-i\beta} \cos \beta}(K)$.

Proof. Let $f \in J(\hat{S}_\alpha^\beta)$, then there exists $g(z) \in \hat{S}_\alpha^\beta$ such that $f(z) = \int_0^z \frac{g(\zeta)}{\zeta} d\zeta$. According to (iii) of Lemma 2.1 there is $s(z) \in S^*$ such that

$$g(z) = z \left[\frac{s(z)}{z} \right]^{(1-\alpha)e^{-i\beta} \cos \beta},$$

therefore

$$f(z) = \int_0^z \left[\frac{s(\zeta)}{\zeta} \right]^{(1-\alpha)e^{-i\beta} \cos \beta} d\zeta.$$

By the relationship of the S^* class and the K class, there exists $u(z) \in K$ such that $s(z) = zu'(z)$, thus

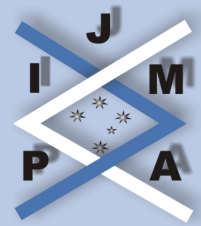
$$f(z) = \int_0^z [u'(\zeta)]^{(1-\alpha)e^{-i\beta} \cos \beta} d\zeta,$$

i.e., $f(z) \in I_{(1-\alpha)e^{-i\beta} \cos \beta}(K)$. As a result, $J(\hat{S}_\alpha^\beta) \subset I_{(1-\alpha)e^{-i\beta} \cos \beta}(K)$ holds.

Conversely, when $f(z) \in I_{(1-\alpha)e^{-i\beta} \cos \beta}(K)$, we can trace back the above procedure to get $f \in J(\hat{S}_\alpha^\beta)$, so $I_{(1-\alpha)e^{-i\beta} \cos \beta}(K) \subset J(\hat{S}_\alpha^\beta)$.

From the above proof, we obtain the assertion. □

Remark 1. If, in the hypothesis of Lemma 2.3, we set $\alpha = 0$, we arrive at Lemma 4 of [4].



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3. The Main Results and Their Proofs

In this section, we let $[z, w]$ denote the closed line segment with endpoints z and w for $z, w \in \mathbb{C}$.

Now we give the main results and their proofs.

Theorem 3.1. For $\alpha \in [0, 1)$, $\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$,

$$A(J(\hat{S}_\alpha^\beta)) = \left\{ |\gamma| \leq \frac{1}{2(1-\alpha)\cos\beta} \right\} \cup \left\{ \frac{e^{i\beta}}{2(1-\alpha)\cos\beta}, \frac{3e^{i\beta}}{2(1-\alpha)\cos\beta} \right\}.$$

Proof. By Lemma 2.3, we have

$$I_\gamma(J(\hat{S}_\alpha^\beta)) = I_\gamma(I_{(1-\alpha)e^{-i\beta}\cos\beta}(K)) = I_{\gamma(1-\alpha)e^{-i\beta}\cos\beta}(K).$$

Therefore, $\gamma \in A(J(\hat{S}_\alpha^\beta))$ if and only if $\gamma(1-\alpha)e^{-i\beta}\cos\beta \in A(K)$, and by Lemma 2.2 we may easily get the result. \square

Remark 2. In this theorem, if we set $\alpha = 0$, we obtain Theorem 3 of [4].

Theorem 3.2. For $\alpha \in [0, 1)$, $\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, the inclusion relation $J(\hat{S}_\alpha^\beta) \subset S$ holds precisely if either $\cos\beta \leq \frac{1}{2(1-\alpha)}$ or $\alpha = \beta = 0$.

Proof. As $\alpha = \beta = 0$, the result holds evidently by Integral transformation 1; while for $\alpha = 0$ and $\beta \neq 0$, the result is Theorem 1 of [4] and was proved by Y.C. Kim and T. Sugawa [4].

If $\alpha \neq 0$ and $\beta = 0$, then $f(z) \in S^*(\alpha)$. By Lemma 2.1(i), there exists $u(z) \in S^*$ such that $u(z) = z \left(\frac{f(z)}{z} \right)^{\frac{1}{1-\alpha}}$. The branch of the power function is chosen such that

$\left. \left(\frac{f(z)}{z} \right)^{\frac{1}{1-\alpha}} \right|_{z=0} = 1$. From Integral transformation 1, we can easily see that there

exists $g(z) \in J(\hat{S}_\alpha^\beta)$ such that

$$g(z) = \int_0^z \left(\frac{f(\zeta)}{\zeta} \right)^{\frac{1}{1-\alpha}} d\zeta.$$

For

$$\Re \left[1 + \frac{zg''(z)}{g'(z)} \right] = \Re \left[\frac{1}{1-\alpha} \frac{zf'(z)}{f(z)} \right]$$

and $\Re \left[\frac{zf'(z)}{f(z)} \right] > \alpha$, we can deduce that $\Re \left[1 + \frac{zg''(z)}{g'(z)} \right] > 0$. This implies $g(z) \in K$ and $J(S^*(\alpha)) \subset S$.

Now let $\alpha \neq 0$ and $\beta \neq 0$. Since $J(\hat{S}_\alpha^\beta) \subset S$ is equivalent to $1 \in A(J(\hat{S}_\alpha^\beta))$ and $1 \notin \left[\frac{e^{i\beta}}{2(1-\alpha)\cos\beta}, \frac{3e^{i\beta}}{2(1-\alpha)\cos\beta} \right]$, by Theorem 3.1, we deduce that $1 \leq \frac{1}{2(1-\alpha)\cos\beta}$, i.e., $\cos\beta \leq \frac{1}{2(1-\alpha)}$.

Summarizing the above procedure, for $\alpha \in [0, 1)$, $\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $J(\hat{S}_\alpha^\beta) \subset S$ holds when $\cos\beta \leq \frac{1}{2(1-\alpha)}$ or $\alpha = \beta = 0$. This completes the proof. \square

Remark 3. This theorem is an extension of Theorem 1 of [4]. Indeed, if we set $\alpha = 0$, we will obtain the result of [4].

Theorem 3.3. For $\alpha \in [0, 1)$, $\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$,

$$A(J(\hat{S})) = \left\{ |\gamma| \leq \frac{1}{2(1-\alpha)\cos\beta} \right\}.$$

Proof. In view of $\hat{S} = \bigcup_\beta \hat{S}_\alpha^\beta$ and $A(F) = \{\gamma \in \mathbb{C} : I_\gamma(F) \subset S\}$, we deduce $A(J(\hat{S})) = \bigcap_\beta (A(J(\hat{S}_\alpha^\beta)))$. With the aid of Theorem 3.1, a simple observation gives $A(J(\hat{S})) = \left\{ |\gamma| \leq \frac{1}{2(1-\alpha)\cos\beta} \right\}$. Thus the proof is now complete. \square



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Remark 4. For $\alpha = \beta = 0$, Theorem 3.3 implies the Theorem 2 of [4].

At the end of this paper, we mention the norm estimate of pre-Schwarzian derivatives. The hyperbolic norm of the pre-Schwarzian derivative $T_f = f''/f'$ of $f \in A$ is defined to be

$$\|f\| = \sup_{|z|<1} (1 - |z|^2) |T_f(z)|.$$

It is known that f is bounded if $\|f\| < 2$ and the bound depends only on the value of $\|f\|$ ([9]). Since

$$\begin{aligned} \|I_\gamma[f]\| &= \sup_{|z|<1} (1 - |z|^2) \left| \frac{(\int_0^z [f'(\zeta)]^\gamma d\zeta)''}{(\int_0^z [f'(\zeta)]^\gamma)' } \right| \\ &= \sup_{|z|<1} (1 - |z|^2) \left| \frac{([f'(z)]^\gamma)' }{f'(z)^\gamma} \right| \\ &= \sup_{|z|<1} (1 - |z|^2) \left| \frac{\gamma f''(z)}{f'(z)} \right| = |\gamma| \|f\|. \end{aligned}$$

We obtain the following assertion.

Proposition 3.4. *For each $\alpha \in [0, 1)$, $\beta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, the sharp inequality $\|f\| \leq 4(1 - \alpha) \cos \beta$ holds for $f \in J(\hat{S}_\alpha^\beta)$. Moreover, if $\cos \beta < \frac{1}{2(1-\alpha)}$, then a function in $J(\hat{S}_\alpha^\beta)$ is bounded by a constant depending on α and β .*

Proof. For each $f \in J(\hat{S}_\alpha^\beta)$, by Lemma 2.3, there is a function $k \in K$ such that $f = I_\gamma(k)$, where $\gamma = (1 - \alpha)e^{-i\beta} \cos \beta$. Noting that $\|k\| \leq 4$ [10], we obtain the following inequality

$$\|f\| = |\gamma| \|k\| \leq 4|\gamma| = 4(1 - \alpha) \cos \beta.$$



Since the inequality $\|k\| \leq 4$ is sharp, the above inequality is also sharp. If $\cos \beta < \frac{1}{2(1-\alpha)}$, the above inequality implies $\|f\| \leq 4(1-\alpha) \cos \beta < 2$, so f is bounded by a constant depending on α and β . \square

Remark 5. If, in the statement of Proposition 3.4, we set $\alpha = 0$, we arrive at the result of [4].

In the above proposition, the bound $\frac{1}{2}$ cannot be replaced by any number greater than $\frac{1}{\sqrt{2(1-\alpha)}}$. Indeed, by the Alexander transformation, if the function

$$g(z) = z(1-z)^{-2(1-\alpha)e^{-i\beta} \cos \beta} \in \hat{S}_\alpha^\beta,$$

then the function

$$f(z) = \frac{(1-z)^{1-2(1-\alpha)e^{-i\beta} \cos \beta} - 1}{2(1-\alpha)e^{-i\beta} \cos \beta - 1} \in J(\hat{S}_\alpha^\beta),$$

and we may verify that the latter is unbounded when $\cos \beta > \frac{1}{\sqrt{2(1-\alpha)}}$.

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