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CORRECTION TO THE PAPER "BOUNDED LINEAR OPERATOR IN PROBABILISTIC NORMED SPACES"

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ABSTRACT. We show that Theorem 2.4 of a recent paper by I.H. Jebril and R.I.M. Ali is incorrect.

Key words and phrases: Probabilistic normed spaces; Bounded linear operator; Counterexample.

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The purpose of this note is to show, by means of an appropriate counterexample, that Theorem 2.4 of the recent paper [2] is incorrect.

In [2], a linear operator T from the PN space (V, ν, τ, τ^*) to the PN space $(V, \mu, \sigma, \sigma^*)$ is said to be strongly $B-$bounded if there exists a constant $h > 0$ such that, for every $p \in V$ and for every $x > 0$,

$$\mu_{Tp}(hx) \geq \nu_p(x)$$

and, similarly, T is said to be strongly $C-$bounded if there exists a constant $h \in (0, 1)$ such that, for every $p \in V$ and for every $x > 0$,

$$\nu_p(x) > 1 - x \Longrightarrow \mu_{Tp}(hx) > 1 - hx.$$

Theorem 2.4 of [2] asserts that if T is strongly $B-$bounded and μ_{Tp} is strictly increasing on $[0, 1]$, then T is strongly $C-$bounded. To show that this is not so, consider the simple PN space generated by the real line $\mathbb{R}$ with its usual norm and the distribution function G given by $G(x) = x/(1 + x)$, so that for any p in $\mathbb{R}$ and any $x \geq 0$, $\nu_p(x) = x/(x + |p|)$. This space is a Menger space under $\mathbf{M}$ and therefore a PN space in the sense of Šerstnev [1]. Now let

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$T : \mathbb{R} \to \mathbb{R}$ be the linear map defined by $Tp = 2p$ and note that ν_{2p} is strictly increasing on $[0, 1]$. Then if $h > 2$,

$$\nu_{Tp}(hx) = \frac{hx}{hx + 2|p|} \geq \frac{hx}{hx + h|p|} = \nu_p(x),$$

whence T is strongly $B-$bounded. (Note that this holds in any simple PN space.) But for $x = 1/2$ and $p = 1/4$, we have $\nu_p(x) = 2/3 > 1/2 = 1 - x$, whereas, for any h in $(0, 1)$, $\nu_{2p}(hx) = h/(1 + h) < 1 - h/2 = 1 - hx$, so that T is not strongly $C-$bounded.

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