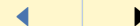
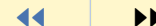




[Title Page](#)

[Contents](#)



Page 1 of 15

[Go Back](#)

[Full Screen](#)

[Close](#)

GENERALIZED OSTROWSKI'S INEQUALITY ON TIME SCALES

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Received: 31 July, 2008

Accepted: 08 October, 2008

Communicated by: [S.S. Dragomir](#)

2000 AMS Sub. Class.: 26D15.

Key words: Montgomery's identity, Ostrowski's inequality, time scales.

Abstract: In this paper, we generalize Ostrowski's inequality and Montgomery's identity on arbitrary time scales which were given in a recent paper [*J. Inequal. Pure. Appl. Math.*, **9**(1) (2008), Art. 6] by Bohner and Matthews. Some examples for the continuous, discrete and the quantum calculus cases are given as well.

Contents

1	Introduction	3
2	Time Scales Essentials	5
3	Generalization by Generalized Polynomials	7
4	Applications for Generalized Polynomials	11
5	Generalization by Arbitrary Functions	13



Ostrowski's Inequality on
Time Scales

B. Karpuz and U.M. Özkan

vol. 9, iss. 4, art. 112, 2008

Title Page

Contents



Page 2 of 15

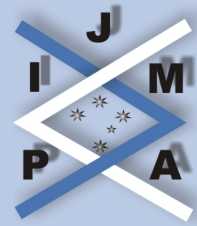
Go Back

Full Screen

Close

journal of **inequalities**
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mathematics

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[Title Page](#)

[Contents](#)

[◀◀](#) [▶▶](#)

[◀](#) [▶](#)

Page 3 of 15

[Go Back](#)

[Full Screen](#)

[Close](#)

journal of **inequalities**
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1. Introduction

In 1937, Ostrowski gave a very useful formula to estimate the absolute value of derivation of a differentiable function by its integral mean. In [9], the so-called Ostrowski's inequality

$$\left| f(t) - \frac{1}{b-a} \int_a^b f(\eta) d\eta \right| \leq \left\{ \sup_{\eta \in (a,b)} |f'(\eta)| \right\} \left(\frac{(t-a)^2 + (b-t)^2}{2(b-a)} \right)$$

is shown by the means of the Montgomery's identity (see [6, pp. 565]).

In a very recent paper [2], the Montgomery identity and the Ostrowski inequality were generalized respectively as follows:

Lemma A (Montgomery's identity). Let $a, b \in \mathbb{T}$ with $a < b$ and $f \in C_{\text{rd}}^1([a, b]_{\mathbb{T}}, \mathbb{R})$.

Then

$$f(t) = \frac{1}{b-a} \left(\int_a^b f^\sigma(\eta) \Delta\eta + \int_a^b \Psi(t, \eta) f^\Delta(\eta) \Delta\eta \right)$$

holds for all $t \in \mathbb{T}$, where $\Psi : [a, b]_{\mathbb{T}}^2 \rightarrow \mathbb{R}$ is defined as follows:

$$\Psi(t, s) := \begin{cases} s - a, & s \in [a, t]_{\mathbb{T}}; \\ s - b, & s \in [t, b]_{\mathbb{T}} \end{cases}$$

for $s, t \in [a, b]_{\mathbb{T}}$.

Theorem A (Ostrowski's inequality). Let $a, b \in \mathbb{T}$ with $a < b$ and $f \in C_{\text{rd}}^1([a, b]_{\mathbb{T}}, \mathbb{R})$.

Then

$$\left| f(t) - \frac{1}{b-a} \int_a^b f^\sigma(\eta) \Delta\eta \right| \leq \left\{ \sup_{\eta \in (a,b)} |f^\Delta(\eta)| \right\} \left(\frac{h_2(t, a) + h_2(t, b)}{b-a} \right)$$

holds for all $t \in \mathbb{T}$. Here, $h_2(t, s)$ is the second-order generalized polynomial on time scales.

In this paper, we shall apply a new method to generalize Lemma A, Theorem A, which is completely different to the method employed in [2], however following the routine steps in [2], our results may also be proved.

The paper is arranged as follows: in §2, we quote some preliminaries on time scales from [1]; §3 includes our main results which generalize Lemma A and Theorem A by the means of generalized polynomials on time scales; in §4, as applications, we consider particular time scales \mathbb{R} , \mathbb{Z} and $q^{\mathbb{N}_0}$; finally, in §5, we give extensions of the results stated in §3.



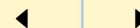
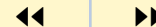
Ostrowski's Inequality on
Time Scales

B. Karpuz and U.M. Özkan

vol. 9, iss. 4, art. 112, 2008

Title Page

Contents



Page 4 of 15

Go Back

Full Screen

Close

journal of **inequalities**
in pure and applied
mathematics

issn: 1443-5756

2. Time Scales Essentials

Definition 2.1. A time scale is a nonempty closed subset of reals.

Definition 2.2. On an arbitrary time scale \mathbb{T} the following are defined: the forward jump operator $\sigma : \mathbb{T} \rightarrow \mathbb{T}$ is defined by $\sigma(t) := \inf(t, \infty)_{\mathbb{T}}$ for $t \in \mathbb{T}$, the backward jump operator $\rho : \mathbb{T} \rightarrow \mathbb{T}$ is defined by $\rho(t) := \sup(-\infty, t)_{\mathbb{T}}$ for $t \in \mathbb{T}$, and the graininess function $\mu : \mathbb{T} \rightarrow \mathbb{R}_0^+$ is defined by $\mu(t) := \sigma(t) - t$ for $t \in \mathbb{T}$. For convenience, we set $\inf \emptyset := \sup \mathbb{T}$ and $\sup \emptyset := \inf \mathbb{T}$.

Definition 2.3. Let t be a point in \mathbb{T} . If $\sigma(t) = t$ holds, then t is called right-dense, otherwise it is called right-scattered. Similarly, if $\rho(t) = t$ holds, then t is called left-dense, a point which is not left-dense is called left-scattered.

Definition 2.4. A function $f : \mathbb{T} \rightarrow \mathbb{R}$ is called rd-continuous provided that it is continuous at right-dense points of \mathbb{T} and its left-sided limits exist (finite) at left-dense points of \mathbb{T} . The set of rd-continuous functions is denoted by $C_{\text{rd}}(\mathbb{T}, \mathbb{R})$, and $C_{\text{rd}}^1(\mathbb{T}, \mathbb{R})$ denotes the set of functions for which the delta derivative belongs to $C_{\text{rd}}(\mathbb{T}, \mathbb{R})$.

Theorem 2.5 (Existence of antiderivatives). Let f be a rd-continuous function. Then f has an antiderivative F such that $F^\Delta = f$ holds.

Definition 2.6. If $f \in C_{\text{rd}}(\mathbb{T}, \mathbb{R})$ and $s \in \mathbb{T}$, then we define the integral

$$F(t) := \int_s^t f(\eta) \Delta \eta \quad \text{for } t \in \mathbb{T}.$$

Theorem 2.7. Let f, g be rd-continuous functions, $a, b, c \in \mathbb{T}$ and $\alpha, \beta \in \mathbb{R}$. Then, the following are true:

$$1. \int_a^b [\alpha f(\eta) + \beta g(\eta)] \Delta \eta = \alpha \int_a^b f(\eta) \Delta \eta + \beta \int_a^b g(\eta) \Delta \eta,$$



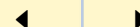
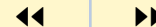
Ostrowski's Inequality on
Time Scales

B. Karpuz and U.M. Özkan

vol. 9, iss. 4, art. 112, 2008

Title Page

Contents



Page 5 of 15

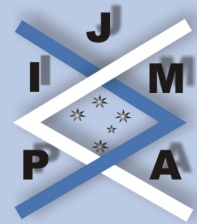
Go Back

Full Screen

Close

journal of **inequalities**
in pure and applied
mathematics

issn: 1443-5756



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 6 of 15

Go Back

Full Screen

Close

$$2. \int_a^b f(\eta)\Delta\eta = -\int_b^a f(\eta)\Delta\eta,$$

$$3. \int_a^c f(\eta)\Delta\eta = \int_a^b f(\eta)\Delta\eta + \int_b^c f(\eta)\Delta\eta,$$

$$4. \int_a^b f(\eta)g^\Delta(\eta)\Delta\eta = f(b)g(b) - f(a)g(a) - \int_a^b f^\Delta(\eta)g(\sigma(\eta))\Delta\eta.$$

Definition 2.8. Let $h_k : \mathbb{T}^2 \rightarrow \mathbb{R}$ be defined as follows:

$$(2.1) \quad h_k(t, s) := \begin{cases} 1, & k = 0 \\ \int_s^t h_{k-1}(\eta, s)\Delta\eta, & k \in \mathbb{N} \end{cases}$$

for all $s, t \in \mathbb{T}$ and $k \in \mathbb{N}_0$.

Note that the function h_k satisfies

$$(2.2) \quad h_k^{\Delta_t}(t, s) = \begin{cases} 0, & k = 0 \\ h_{k-1}(t, s), & k \in \mathbb{N} \end{cases}$$

for all $s, t \in \mathbb{T}$ and $k \in \mathbb{N}_0$.

Property 1. Using induction it is easy to see that $h_k(t, s) \geq 0$ holds for all $k \in \mathbb{N}$ and $s, t \in \mathbb{T}$ with $t \geq s$ and $(-1)^k h_k(t, s) \geq 0$ holds for all $k \in \mathbb{N}$ and $s, t \in \mathbb{T}$ with $t \leq s$.



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 7 of 15

Go Back

Full Screen

Close

3. Generalization by Generalized Polynomials

We start this section by quoting the following useful change of order formula for double(iterated) integrals which is employed in our proofs.

Lemma 3.1 ([8, Lemma 1]). Assume that $a, b \in \mathbb{T}$ and $f \in C_{\text{rd}}(\mathbb{T}^2, \mathbb{R})$. Then

$$\int_a^b \int_\xi^b f(\eta, \xi) \Delta\eta \Delta\xi = \int_a^b \int_a^{\sigma(\eta)} f(\eta, \xi) \Delta\xi \Delta\eta.$$

Now, we give a generalization for Montgomery's identity as follows:

Lemma 3.2. Assume that $a, b \in \mathbb{T}$ and $f \in C_{\text{rd}}^1([a, b]_{\mathbb{T}}, \mathbb{R})$. Define $\Psi, \Phi \in C_{\text{rd}}^1([a, b]_{\mathbb{T}}, \mathbb{R})$ by

$$\Psi(t, s) := \begin{cases} h_k(s, a), & s \in [a, t]_{\mathbb{T}} \\ h_k(s, b), & s \in [t, b]_{\mathbb{T}} \end{cases} \quad \text{and} \quad \Phi(t, s) := \begin{cases} h_{k-1}(s, a), & s \in [a, t]_{\mathbb{T}} \\ h_{k-1}(s, b), & s \in [t, b]_{\mathbb{T}} \end{cases}$$

for $s, t \in [a, b]_{\mathbb{T}}$ and $k \in \mathbb{N}$. Then

$$(3.1) \quad f(t) = \frac{1}{h_k(t, a) - h_k(t, b)} \left(\int_a^b \Phi(t, \eta) f^\sigma(\eta) \Delta\eta + \int_a^b \Psi(t, \eta) f^\Delta(\eta) \Delta\eta \right)$$

is true for all $t \in [a, b]_{\mathbb{T}}$ and all $k \in \mathbb{N}$.

Proof. Note that we have $\Psi^{\Delta s} = \Phi$. Clearly, for all $t \in [a, b]_{\mathbb{T}}$ and all $k \in \mathbb{N}$, from (3.1), (2.1) and (2.2) we have

$$\begin{aligned} & \int_a^t \Phi(t, \eta) f^\sigma(\eta) \Delta\eta + \int_a^t \Psi(t, \eta) f^\Delta(\eta) \Delta\eta \\ &= \int_a^t h_{k-1}(\eta, a) f^\sigma(\eta) \Delta\eta + \int_a^t h_k(\eta, a) f^\Delta(\eta) \Delta\eta \end{aligned}$$



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 8 of 15

Go Back

Full Screen

Close

$$(3.2) \quad = \int_a^t \int_a^{\sigma(\eta)} h_{k-1}(\eta, a) f^\Delta(\xi) \Delta\xi \Delta\eta + f(a) h_k(t, a) \\ + \int_a^t \int_a^\eta [h_k(\xi, a) f^\Delta(\eta)]^{\Delta\xi} \Delta\xi \Delta\eta.$$

Applying Lemma 3.1 and considering (2.1), the right-hand side of (3.2) takes the form

$$(3.3) \quad \int_a^t \int_\xi^t h_{k-1}(\eta, a) f^\Delta(\xi) \Delta\eta \Delta\xi + f(a) h_k(t, a) \\ + \int_a^t \int_a^\eta h_{k-1}(\xi, a) f^\Delta(\eta) \Delta\xi \Delta\eta \\ = \int_a^t \int_a^t h_{k-1}(\eta, a) f^\Delta(\xi) \Delta\eta \Delta\xi + f(a) h_k(t, a) \\ = f(t) h_k(t, a),$$

and very similarly, from Lemma 3.1, (3.1), (2.1) and (2.2), we obtain

$$\int_t^b \Phi(t, \eta) f^\sigma(\eta) \Delta\eta + \int_t^b \Psi(t, \eta) f^\Delta(\eta) \Delta\eta \\ = \int_t^b h_{k-1}(\eta, b) f^\sigma(\eta) \Delta\eta + \int_t^b h_k(\eta, b) f^\Delta(\eta) \Delta\eta \\ = \int_t^b \int_t^{\sigma(\eta)} h_{k-1}(\eta, b) f^\Delta(\xi) \Delta\xi \Delta\eta - f(t) h_k(t, b) \\ - \int_t^b \int_\eta^b [h_k(\xi, b) f^\Delta(\eta)]^{\Delta\xi} \Delta\xi \Delta\eta,$$



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 9 of 15

Go Back

Full Screen

Close

$$\begin{aligned}
 &= \int_t^b \int_\xi^b h_{k-1}(\eta, b) f^\Delta(\xi) \Delta\eta \Delta\xi - f(t) h_k(t, b) \\
 &\quad - \int_t^b \int_\eta^b h_{k-1}(\xi, b) f^\Delta(\eta) \Delta\xi \Delta\eta \\
 (3.4) \quad &= -f(t) h_k(t, b).
 \end{aligned}$$

By summing (3.3) and (3.4), we get the desired result. □

Now, we give the following generalization of Ostrowski's inequality.

Theorem 3.3. Assume that $a, b \in \mathbb{T}$ and $f \in C_{\text{rd}}^1([a, b]_{\mathbb{T}}, \mathbb{R})$. Then

$$\begin{aligned}
 &\left| f(t) - \frac{1}{h_k(t, a) - h_k(t, b)} \int_a^b \Phi(t, \eta) f^\sigma(\eta) \Delta\eta \right| \\
 &\leq M \left(\frac{h_{k+1}(t, a) + (-1)^{k+1} h_{k+1}(t, b)}{h_k(t, a) - h_k(t, b)} \right)
 \end{aligned}$$

is true for all $t \in [a, b]_{\mathbb{T}}$ and all $k \in \mathbb{N}$, where Φ is as introduced in (3.1) and $M := \sup_{\eta \in (a, b)} |f^\Delta(\eta)|$.

Proof. From Lemma 3.2 and (3.1), for all $k \in \mathbb{N}$ and $t \in [a, b]_{\mathbb{T}}$, we get

$$\begin{aligned}
 &\left| f(t) - \frac{1}{h_k(t, a) - h_k(t, b)} \int_a^b \Phi(t, \eta) f^\sigma(\eta) \Delta\eta \right| \\
 &= \left| \frac{1}{h_k(t, a) - h_k(t, b)} \int_a^b \Psi(t, \eta) f^\Delta(\eta) \Delta\eta \right| \\
 &= \left| \frac{1}{h_k(t, a) - h_k(t, b)} \left(\int_a^t h_k(\eta, a) f^\Delta(\eta) \Delta\eta + \int_t^b h_k(\eta, b) f^\Delta(\eta) \Delta\eta \right) \right|
 \end{aligned}$$



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 10 of 15

Go Back

Full Screen

Close

$$(3.5) \leq \frac{M}{h_k(t, a) - h_k(t, b)} \left(\left| \int_a^t h_k(\eta, a) \Delta\eta \right| + \left| \int_t^b h_k(\eta, b) \Delta\eta \right| \right),$$

and considering Property 1 and (2.1) on the right-hand side of (3.5), we have

$$\begin{aligned} & \frac{M}{h_k(t, a) - h_k(t, b)} \left(\int_a^t h_k(\eta, a) \Delta\eta + \int_t^b (-1)^k h_k(\eta, b) \Delta\eta \right) \\ &= \frac{M}{h_k(t, a) - h_k(t, b)} \left(\int_a^t h_k(\eta, a) \Delta\eta + (-1)^{k+1} \int_b^t h_k(\eta, b) \Delta\eta \right) \\ &= M \left(\frac{h_{k+1}(t, a) + (-1)^{k+1} h_{k+1}(t, b)}{h_k(t, a) - h_k(t, b)} \right), \end{aligned}$$

which completes the proof. \square

Remark 1. It is clear that Lemma 3.2 and Theorem 3.3 reduce to Lemma A and Theorem A respectively by letting $k = 1$.



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 11 of 15

Go Back

Full Screen

Close

4. Applications for Generalized Polynomials

In this section, we give examples on particular time scales for Theorem 3.3. First, we consider the continuous case.

Example 4.1. Let $\mathbb{T} = \mathbb{R}$. Then, we have $h_k(t, s) = (t - s)^k/k! = (-1)^k(s - t)^k/k!$ for all $s, t \in \mathbb{R}$ and $k \in \mathbb{N}$. In this case, Ostrowski's inequality reads as follows:

$$\left| f(t) - \frac{k!}{(t-a)^k + (-1)^{k+1}(b-t)^k} \int_a^b \Phi(t, \eta) f(\eta) d\eta \right| \leq \frac{M}{k+1} \left(\frac{(t-a)^{k+1} + (b-t)^{k+1}}{(t-a)^k + (-1)^{k+1}(b-t)^k} \right),$$

where M is the maximum value of the absolute value of the derivative f' over $[a, b]_{\mathbb{R}}$, and $\Phi(t, s) = (s - a)^k/k!$ for $s \in [a, t]_{\mathbb{R}}$ and $\Phi(t, s) = (s - b)^k/k!$ for $s \in [t, b]_{\mathbb{R}}$.

Next, we consider the discrete calculus case.

Example 4.2. Let $\mathbb{T} = \mathbb{Z}$. Then, we have $h_k(t, s) = (t - s)^{(k)}/k! = (-1)^k(s - t + k)^{(k)}/k!$ for all $s, t \in \mathbb{Z}$ and $k \in \mathbb{N}$, where the usual factorial function (k) is defined by $n^{(k)} := n!/k!$ for $k \in \mathbb{N}$ and $n^{(0)} := 1$ for $n \in \mathbb{Z}$. In this case, Ostrowski's inequality reduces to the following inequality:

$$\left| f(t) - \frac{k!}{(t-a)^{(k)} + (-1)^{k+1}(b-t+k)^{(k)}} \sum_{\eta=a}^{b-1} \Phi(t, \eta) f(\eta + 1) \right| \leq \frac{M}{k+1} \left(\frac{(t-a)^{(k+1)} + (b-t+k)^{(k+1)}}{(t-a)^{(k)} + (-1)^{k+1}(b-t+k)^{(k)}} \right),$$

where M is the maximum value of the absolute value of the difference Δf over $[a, b-1]_{\mathbb{Z}}$, and $\Phi(t, s) = (s - a)^{(k)}/k!$ for $s \in [a, t-1]_{\mathbb{Z}}$ and $\Phi(t, s) = (s - b)^{(k)}/k!$ for $s \in [t, b]_{\mathbb{Z}}$.



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 12 of 15

Go Back

Full Screen

Close

Before giving the quantum calculus case, we need to introduce the following notations from [7]:

$$[k]_q := \frac{q^k - 1}{q - 1} \quad \text{for } q \in \mathbb{R}/\{1\} \text{ and } k \in \mathbb{N}_0,$$

$$[k]! := \prod_{j=1}^k [j]_q \quad \text{for } k \in \mathbb{N}_0,$$

$$(t - s)_q^k := \prod_{j=0}^{k-1} (t - q^j s) \quad \text{for } s, t \in q^{\mathbb{N}_0} \text{ and } k \in \mathbb{N}_0.$$

It is shown in [1, Example 1.104] that the following holds:

$$h_k(t, s) := \frac{(t - s)_q^k}{[k]!} \quad \text{for } s, t \in q^{\mathbb{N}_0} \text{ and } k \in \mathbb{N}_0.$$

And finally, we consider the quantum calculus case.

Example 4.3. Let $\mathbb{T} = q^{\mathbb{N}_0}$ with $q > 1$. Therefore, for the quantum calculus case, Ostrowski's inequality takes the following form:

$$\left| f(t) - \frac{[k]!(q-1)a}{(t-a)_q^k - (t-b)_q^k} \sum_{\eta=0}^{\log_q(b/(qa))} q^\eta \Phi(t, q^\eta a) f(q^{\eta+1} a) \right| \leq \frac{M}{[k+1]_q} \left(\frac{(t-a)_q^{k+1} + (-1)^{k+1} (t-b)_q^{k+1}}{(t-a)_q^k - (t-b)_q^k} \right),$$

where M is the maximum value of the absolute value of the q -difference $D_q f$ over $[a, b/q]_{q^{\mathbb{N}_0}}$, and $\Phi(t, s) = (s-a)_q^k/[k]!$ for $s \in [a, t/q]_{q^{\mathbb{N}_0}}$ and $\Phi(t, s) = (s-b)_q^k/[k]!$ for $s \in [t, b]_{q^{\mathbb{N}_0}}$. Here, the q -difference operator D_q is defined by $D_q f(t) := [f(qt) - f(t)]/[(q-1)t]$.



Title Page

Contents



Page 13 of 15

Go Back

Full Screen

Close

journal of **inequalities**
in pure and applied
mathematics

issn: 1443-5756

5. Generalization by Arbitrary Functions

In this section, we replace the generalized polynomials $h_k(t, s)$ appearing in the definitions of $\Phi(t, s)$ and $\Psi(t, s)$ by arbitrary functions.

Since the proof of the following results can be done easily, we just give the statements of the results without proofs.

Lemma 5.1. Assume that $a, b \in \mathbb{T}$, $f \in C_{\text{rd}}^1([a, b]_{\mathbb{T}}, \mathbb{R})$, and that $\psi, \phi \in C_{\text{rd}}^1([a, b]_{\mathbb{T}}, \mathbb{R})$ with $\psi(b) = \phi(a) = 0$ and $\psi(t) - \phi(t) \neq 0$ for all $t \in [a, b]_{\mathbb{T}}$. Set $\Psi, \Phi \in C_{\text{rd}}([a, b]_{\mathbb{T}}, \mathbb{R})$ by

$$(5.1) \quad \Psi(t, s) := \begin{cases} \phi(s), & s \in [a, t]_{\mathbb{T}} \\ \psi(s), & s \in [t, b]_{\mathbb{T}} \end{cases} \quad \text{and} \quad \Phi(t, s) := \Psi^{\Delta s}(t, s)$$

for $s, t \in [a, b]_{\mathbb{T}}$. Then

$$\begin{aligned} f(t) &= \frac{1}{\psi(t) - \phi(t)} \int_a^b [\Psi(t, \eta) f(\eta)]^{\Delta \eta} \Delta \eta \\ &= \frac{1}{\psi(t) - \phi(t)} \left(\int_a^b \Phi(t, \eta) f^{\sigma}(\eta) \Delta \eta + \int_a^b \Psi(t, \eta) f^{\Delta}(\eta) \Delta \eta \right) \end{aligned}$$

is true for all $t \in [a, b]_{\mathbb{T}}$.

Theorem 5.2. Assume that $a, b \in \mathbb{T}$, $f \in C_{\text{rd}}^1([a, b]_{\mathbb{T}}, \mathbb{R})$, and that $\psi, \phi \in C_{\text{rd}}^1([a, b]_{\mathbb{T}}, \mathbb{R})$ with $\psi(b) = \phi(a) = 0$ and $\psi(t) - \phi(t) \neq 0$ for all $t \in [a, b]_{\mathbb{T}}$. Then

$$\left| f(t) - \frac{1}{\psi(t) - \phi(t)} \int_a^b \Phi(t, \eta) f^{\sigma}(\eta) \Delta \eta \right| \leq \frac{M}{|\psi(t) - \phi(t)|} \left(\int_a^b |\Psi(t, \eta)| \Delta \eta \right)$$

is true for all $t \in [a, b]_{\mathbb{T}}$, where Ψ, Φ are as introduced in (5.1) and $M := \sup_{\eta \in (a, b)} |f^{\Delta}(\eta)|$.



**Ostrowski's Inequality on
Time Scales**

B. Karpuz and U.M. Özkan

vol. 9, iss. 4, art. 112, 2008

Title Page

Contents



Page 14 of 15

Go Back

Full Screen

Close

journal of **inequalities**
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mathematics

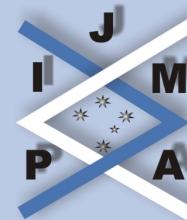
issn: 1443-5756

Remark 2. Letting $\phi(t) = h_k(t, a)$ and $\psi(t) = h_k(t, b)$ for some $k \in \mathbb{N}$, we obtain the results of §3, which reduce to the results in [2, § 3] by letting $k = 1$. This is for Ostrowski-polynomial type inequalities.

Remark 3. For instance, we may let $\phi(t) = e_\lambda(t, a) - 1$ and $\psi(t) = e_\lambda(t, b) - 1$ for some $\lambda \in \mathcal{R}^+([a, b]_{\mathbb{T}}, \mathbb{R}^+)$ to obtain new Ostrowski-exponential type inequalities.

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Ostrowski's Inequality on
Time Scales

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vol. 9, iss. 4, art. 112, 2008

Title Page

Contents



Page 15 of 15

Go Back

Full Screen

Close

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