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GENERALIZED (A, η) - RESOLVENT OPERATOR TECHNIQUE AND SENSITIVITY ANALYSIS FOR RELAXED COCOERCIVE VARIATIONAL INCLUSIONS

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Abstract

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Abstract

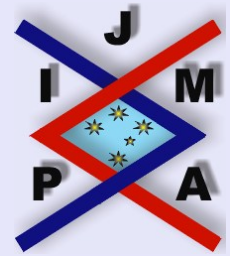
Sensitivity analysis for relaxed cocoercive variational inclusions based on the generalized resolvent operator technique is discussed. The obtained results are general in nature.

2000 Mathematics Subject Classification: 49J40, 47H10.

Key words: Sensitivity analysis, Quasivariational inclusions, Maximal relaxed monotone mapping, (A, η) – *monotone* mapping, Generalized resolvent operator technique.

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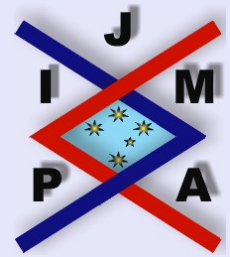
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1. Introduction

In [8] the author studied sensitivity analysis for quasivariational inclusions using the resolvent operator technique. Resolvent operator techniques have been frequently applied to a broad range of problems arising from several fields, including equilibria problems in economics, optimization and control theory, operations research, and mathematical programming. In this paper we intend to present the sensitivity analysis for (A, η) –monotone quasivariational inclusions involving relaxed cocoercive mappings. The notion of (A, η) –monotonicity [8] generalizes the notion of A –monotonicity in [12]. The obtained results generalize a wide range of results on the sensitivity analysis for quasivariational inclusions, including [2] – [5] and others. For more details on nonlinear variational inclusions and related resolvent operator techniques, we recommend the reader [1] – [12].



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2. (A, η) -Monotonicity

In this section we explore some basic properties derived from the notion of (A, η) -monotonicity. Let $\eta : X \times X \rightarrow X$ be (τ) -Lipschitz continuous, that is, there exists a positive constant $\tau > 0$ such that

$$\|\eta(u, v)\| \leq \tau \|u - v\| \quad \forall u, v \in X.$$

Definition 2.1. Let $\eta : X \times X \rightarrow X$ be a single-valued mapping, and let $M : X \rightarrow 2^X$ be a multivalued mapping on X . The map M is said to be:

(i) (r, η) -strongly monotone if

$$\langle u^* - v^*, \eta(u, v) \rangle \geq r \|u - v\|^2 \quad \forall (u, u^*), (v, v^*) \in \text{Graph}(M).$$

(ii) (r, η) -strongly pseudomonotone if

$$\langle v^*, \eta(u, v) \rangle \geq 0$$

implies

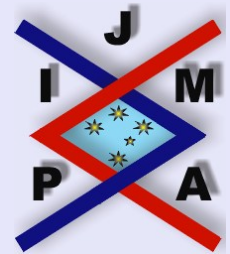
$$\langle u^*, \eta(u, v) \rangle \geq r \|u - v\|^2 \quad \forall (u, u^*), (v, v^*) \in \text{Graph}(M).$$

(iii) (η) -pseudomonotone if

$$\langle v^*, \tau(u, v) \rangle \geq 0$$

implies

$$\langle u^*, \eta(u, v) \rangle \geq 0 \quad \forall (u, u^*), (v, v^*) \in \text{Graph}(M).$$



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(iii) (m, η) -relaxed monotone if there exists a positive constant m such that

$$\langle u^* - v^*, \eta(u, v) \rangle \geq (-m)\|u - v\|^2 \quad \forall (u, u^*), (v, v^*) \in \text{Graph}(M).$$

Definition 2.2. A mapping $M : X \rightarrow 2^X$ is said to be maximal (m, η) -relaxed monotone if

(i) M is (m, η) -relaxed monotone,

(ii) For $(u, u^*) \in X \times X$, and

$$\langle u^* - v^*, \eta(u, v) \rangle \geq (-m)\|u - v\|^2 \quad \forall (v, v^*) \in \text{Graph}(M),$$

we have $u^* \in M(u)$.

Definition 2.3. Let $A : X \rightarrow X$ and $\eta : X \times X \rightarrow X$ be two single-valued mappings. The map $M : X \rightarrow 2^X$ is said to be (A, η) -monotone if

(i) M is (m, η) -relaxed monotone

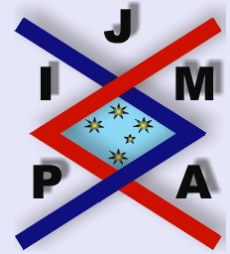
(ii) $R(A + \rho M) = X$ for $\rho > 0$.

Alternatively, we have

Definition 2.4. Let $A : X \rightarrow X$ and $\eta : X \times X \rightarrow X$ be two single-valued mappings. The map $M : X \rightarrow 2^X$ is said to be (A, η) -monotone if

(i) M is (m, η) -relaxed monotone

(ii) $A + \rho M$ is (η) -pseudomonotone for $\rho > 0$.



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Proposition 2.1. Let $A : X \rightarrow X$ be an (r, η) -strongly monotone single-valued mapping and let $M : X \rightarrow 2^X$ be an (A, η) -monotone mapping. Then M is maximal (m, η) -relaxed monotone for $0 < \rho < \frac{r}{m}$.

Proposition 2.2. Let $A : X \rightarrow X$ be an (r, η) -strongly monotone single-valued mapping and let $M : X \rightarrow 2^X$ be an (A, η) -monotone mapping. Then $(A + \rho M)$ is maximal (η) -monotone for $0 < \rho < \frac{r}{m}$.

Proof. Since A is (r, η) -strongly monotone and M is (A, η) -monotone, it implies that $A + \rho M$ is $(r - \rho m, \eta)$ -strongly monotone. This in turn implies that $A + \rho M$ is (η) -pseudomonotone, and hence $A + \rho M$ is maximal (η) -monotone under the given conditions. \square

Proposition 2.3. Let $A : X \rightarrow X$ be an (r, η) -strongly monotone mapping and let $M : X \rightarrow 2^X$ be an (A, η) -monotone mapping. Then the operator $(A + \rho M)^{-1}$ is single-valued.

Definition 2.5. Let $A : X \rightarrow X$ be an (r, η) -strongly monotone mapping and let $M : X \rightarrow 2^X$ be an (A, η) -monotone mapping. Then the generalized resolvent operator $J_{\rho, A}^M : X \rightarrow X$ is defined by

$$J_{\rho, A}^M(u) = (A + \rho M)^{-1}(u).$$

Furthermore, we upgrade the notions of the monotonicity as well as strong monotonicity in the context of sensitivity analysis for nonlinear variational inclusion problems.

Definition 2.6. The map $T : X \times X \times L \rightarrow X$ is said to be:



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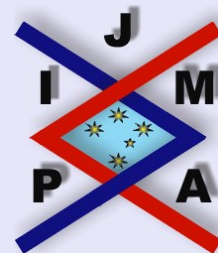


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(i) *Monotone with respect to A in the first argument if*

$$\langle T(x, u, \lambda) - T(y, u, \lambda), A(x) - A(y) \rangle \geq 0$$

$$\forall (x, y, u, \lambda) \in X \times X \times X \times L.$$

(ii) *(r)-strongly monotone with respect to A in the first argument if there exists a positive constant r such that*

$$\langle T(x, u, \lambda) - T(y, u, \lambda), A(x) - A(y) \rangle \geq (r)\|x - y\|^2$$

$$\forall (x, y, u, \lambda) \in X \times X \times X \times L.$$

(iii) *(γ, α)-relaxed cocoercive with respect to A in the first argument if there exist positive constants γ and α such that*

$$\langle T(x, u, \lambda) - T(y, u, \lambda), A(x) - A(y) \rangle \geq -\gamma\|T(x) - T(y)\|^2 + \alpha\|x - y\|^2$$

$$\forall (x, y, u, \lambda) \in X \times X \times X \times L.$$

(iv) *(γ)-relaxed cocoercive with respect to A in the first argument if there exists a positive constant γ such that*

$$\langle T(x, u, \lambda) - T(y, u, \lambda), A(x) - A(y) \rangle \geq -\gamma\|T(x) - T(y)\|^2$$

$$\forall (x, y, u, \lambda) \in X \times X \times X \times L.$$

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3. Results On Sensitivity Analysis

Let X denote a real Hilbert space with the norm $\|\cdot\|$ and inner product $\langle \cdot, \cdot \rangle$. Let $N : X \times X \times L \rightarrow X$ be a nonlinear mapping and $M : X \times X \times L \rightarrow 2^X$ be an A -monotone mapping with respect to the first variable, where L is a nonempty open subset of X . Furthermore, let $\eta : X \times X \rightarrow X$ be a nonlinear mapping. Then the problem of finding an element $u \in X$ for a given element $f \in X$ such that

$$(3.1) \quad f \in N(u, u, \lambda) + M(u, u, \lambda),$$

where $\lambda \in L$ is the perturbation parameter, is called a class of generalized strongly monotone mixed quasivariational inclusion (abbreviated SMMQVI) problems.

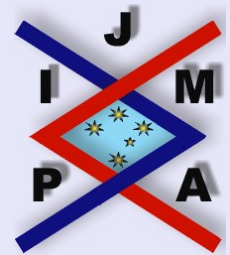
The solvability of the *SMMQVI* problem (3.1) depends on the equivalence between (3.1) and the problem of finding the fixed point of the associated generalized resolvent operator.

Note that if M is (A, η) -monotone, then the corresponding generalized resolvent operator $J_{\rho, A}^M$ in first argument is defined by

$$(3.2) \quad J_{\rho, A}^{M(\cdot, y)}(u) = (A + \rho M(\cdot, y))^{-1}(u) \quad \forall u \in X,$$

where $\rho > 0$ and A is an (r, η) -strongly monotone mapping.

Lemma 3.1. *Let X be a real Hilbert space, and let $\eta : X \times X \rightarrow X$ be a (τ) -Lipschitz continuous nonlinear mapping. Let $A : X \rightarrow X$ be (r, η) -strongly monotone, and let $M : X \times X \times L \rightarrow 2^X$ be (A, η) -monotone in the first*



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variable. Then the generalized resolvent operator associated with $M(\cdot, y, \lambda)$ for a fixed $y \in X$ and defined by

$$J_{\rho, A}^{M(\cdot, y, \lambda)}(u) = (A + \rho M(\cdot, y, \lambda))^{-1}(u) \quad \forall u \in X,$$

is $(\frac{\tau}{r-\rho m})$ -Lipschitz continuous.

Proof. By the definition of the generalized resolvent operator, we have

$$\frac{1}{\rho} \left(u - A \left(J_{\rho, A}^{M(\cdot, y, \lambda)}(u) \right) \right) \in M \left(J_{\rho, A}^{M(\cdot, y, \lambda)}(u) \right),$$

and

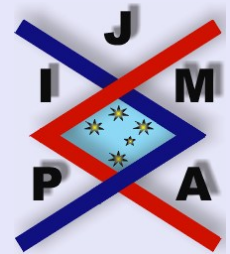
$$\frac{1}{\rho} \left(v - A \left(J_{\rho, A}^{M(\cdot, y, \lambda)}(v) \right) \right) \in M \left(J_{\rho, A}^{M(\cdot, y, \lambda)}(v) \right) \quad \forall u, v \in X.$$

Given M is (m, η) -relaxed monotone, we find

$$\begin{aligned} \frac{1}{\rho} \left\langle u - v - \left(A \left(J_{\rho, A}^{M(\cdot, y, \lambda)}(u) \right) - A \left(J_{\rho, A}^{M(\cdot, y, \lambda)}(v) \right) \right), \right. \\ \left. \eta \left(J_{\rho, A}^{M(\cdot, y, \lambda)}(u), J_{\rho, A}^{M(\cdot, y, \lambda)}(v) \right) \right\rangle \\ \geq (-m) \left\| J_{\rho, A}^{M(\cdot, y, \lambda)}(u) - J_{\rho, A}^{M(\cdot, y, \lambda)}(v) \right\|^2. \end{aligned}$$

Therefore,

$$\tau \|u - v\| \left\| J_{\rho, A}^{M(\cdot, y, \lambda)}(u) - J_{\rho, A}^{M(\cdot, y, \lambda)}(v) \right\|$$



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$$\begin{aligned}
&\geq \left\langle u - v, \eta \left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u), J_{\rho,A}^{M(\cdot,y,\lambda)}(v) \right) \right\rangle \\
&\geq \left\langle A \left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u) \right) - A \left(J_{\rho,A}^{M(\cdot,y,\lambda)}(v) \right), \eta \left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u), J_{\rho,A}^{M(\cdot,y,\lambda)}(v) \right) \right\rangle \\
&\quad - (\rho m) \left\| J_{\rho,A}^{M(\cdot,y,\lambda)}(u) - J_{\rho,A}^{M(\cdot,y,\lambda)}(v) \right\|^2 \\
&\geq (r - \rho m) \left\| J_{\rho,A}^{M(\cdot,y,\lambda)}(u) - J_{\rho,A}^{M(\cdot,y,\lambda)}(v) \right\|^2.
\end{aligned}$$

This completes the proof. \square

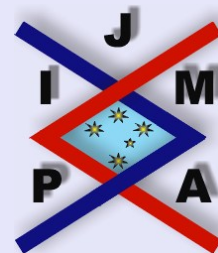
Lemma 3.2. *Let X be a real Hilbert space, let $A : X \rightarrow X$ be (r, η) -strongly monotone, and let $M : X \times X \times L \rightarrow 2^X$ be (A, η) -monotone in the first variable. Let $\eta : X \times X \rightarrow X$ be a (τ) -Lipschitz continuous nonlinear mapping. Then the following statements are mutually equivalent:*

- (i) An element $u \in X$ is a solution to (3.1).
- (ii) The map $G : X \times L \rightarrow X$ defined by

$$G(u, \lambda) = J_{\rho,A}^{M(\cdot,u,\lambda)}(A(u) - \rho N(u, u, \lambda) + \rho f)$$

has a fixed point.

Theorem 3.3. *Let X be a real Hilbert space, and let $\eta : X \times X \rightarrow X$ be a (τ) -Lipschitz continuous nonlinear mapping. Let $A : X \rightarrow X$ be (r, η) -strongly monotone and (s) -Lipschitz continuous, and let $M : X \times X \times L \rightarrow 2^X$ be (A, η) -monotone in the first variable. Let $N : X \times X \times L \rightarrow X$ be (γ, α) -relaxed cocoercive (with respect to A) and (β) -Lipschitz continuous in the first*



variable, and let N be (μ) -Lipschitz continuous in the second variable. If, in addition,

$$(3.3) \quad \left\| J_{\rho,A}^{M(\cdot,u,\lambda)}(w) - J_{\rho,A}^{M(\cdot,v,\lambda)}(w) \right\| \leq \delta \|u - v\| \quad \forall (u, v, \lambda) \in X \times X \times L,$$

then

$$(3.4) \quad \|G(u, \lambda) - G(v, \lambda)\| \leq \theta \|u - v\| \quad \forall (u, v, \lambda) \in X \times X \times L,$$

where

$$\theta = \frac{\tau}{r - \rho m} \left[\sqrt{s^2 - 2\rho\alpha + 2\rho\beta^2\gamma + \rho^2\beta^2} + \rho\mu \right] + \delta < 1,$$

$$\left| \rho - \frac{(\alpha - \gamma\beta^2)\tau^2 - r[\mu\tau + m(1 - \delta)](1 - \delta)}{\beta^2 - (\mu\tau + m(1 - \delta))^2} \right| < \frac{\sqrt{[(\alpha - \gamma\beta^2)\tau^2 - r(\mu\tau + m(1 - \delta))(1 - \eta)]^2 - B}}{\beta^2 - (\mu\tau - m(1 - \delta))^2},$$

$$B = [\beta^2 - (\mu\tau + m(1 - \delta))^2](s^2\tau^2 - r^2(1 - \delta)^2),$$

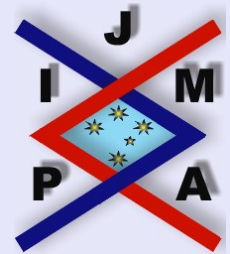
for

$$\alpha(\alpha - \gamma\beta^2)\tau^2 > r(\mu\tau + m(1 - \delta))(1 - \delta) + \sqrt{B},$$

$$\beta > \mu\tau + m(1 - \delta), \quad 0 < \delta < 1.$$

Consequently, for each $\lambda \in L$, the mapping $G(u, \lambda)$ in light of (3.4) has a unique fixed point $z(\lambda)$. Hence, in light of Lemma 3.2, $z(\lambda)$ is a unique solution to (3.1). Thus, we have

$$G(z(\lambda), \lambda) = z(\lambda).$$



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Proof. For any element $(u, v, \lambda) \in X \times X \times L$, we have

$$G(u, \lambda) = J_{\rho, A}^{M(\cdot, u, \lambda)}(A(u) - \rho N(u, u, \lambda) + \rho f),$$

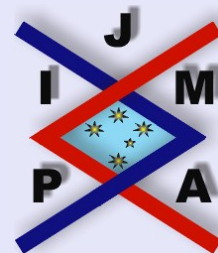
$$G(v, \lambda) = J_{\rho, A}^{M(\cdot, v, \lambda)}(A(v) - \rho N(v, v, \lambda) + \rho f).$$

It follows that

$$\begin{aligned} & \|G(u, \lambda) - G(v, \lambda)\| \\ &= \left\| J_{\rho, A}^{M(\cdot, u, \lambda)}(A(u) - \rho N(u, u, \lambda) + \rho f) - J_{\rho, A}^{M(\cdot, v, \lambda)}(A(v) - \rho N(v, v, \lambda) + \rho f) \right\| \\ &\leq \left\| J_{\rho, A}^{M(\cdot, u, \lambda)}(A(u) - \rho N(u, u, \lambda) + \rho f) - J_{\rho, A}^{M(\cdot, u, \lambda)}(A(v) - \rho N(v, v, \lambda) + \rho f) \right\| \\ &\quad + \left\| J_{\rho, A}^{M(\cdot, u, \lambda)}(A(v) - \rho N(v, v, \lambda) + \rho f) - J_{\rho, A}^{M(\cdot, v, \lambda)}(A(v) - \rho N(v, v, \lambda) + \rho f) \right\| \\ &\leq \frac{\tau}{r - \rho m} \|A(u) - A(v) - \rho(N(u, u, \lambda) - N(v, v, \lambda))\| + \delta \|u - v\| \\ &\leq \frac{\tau}{r - \rho m} [\|A(u) - A(v) - \rho(N(u, u, \lambda) - N(v, u, \lambda))\| \\ &\quad + \|\rho(N(v, u, \lambda) - N(v, v, \lambda))\|] + \delta \|u - v\|. \end{aligned}$$

The (γ, α) -relaxed cocoercivity and (β) -Lipschitz continuity of N in the first argument imply that

$$\begin{aligned} & \|A(u) - A(v) - \rho(N(u, u, \lambda) - N(v, u, \lambda))\|^2 \\ &= \|A(u) - A(v)\|^2 - 2\rho \langle N(u, u, \lambda) - N(v, u, \lambda), A(u) - A(v) \rangle \\ &\quad + \rho^2 \|N(u, u, \lambda) - N(v, u, \lambda)\|^2 \\ &\leq (s^2 - 2\rho\alpha + 2\rho\beta^2\gamma + \rho^2\beta^2) \|u - v\|^2. \end{aligned}$$



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On the other hand, the (μ) -Lipschitz continuity of N in the second argument results

$$\|(N(v, u, \lambda)) - N(v, v, \lambda)\| \leq \mu \|u - v\|.$$

In light of above arguments, we infer that

$$(3.5) \quad \|G(u, \lambda) - G(v, \lambda)\| \leq \theta \|u - v\|,$$

where

$$\theta = \frac{\tau}{(r - \rho m)} \left[\sqrt{s^2 - 2\rho\alpha + 2\rho\beta^2\gamma + \rho^2\beta^2} + \rho\mu \right] + \delta < 1.$$

Since $\theta < 1$, it concludes the proof. \square

Theorem 3.4. *Let X be a real Hilbert space, let $A : X \rightarrow X$ be (r, η) -strongly monotone and (s) -Lipschitz continuous, and let $M : X \times X \times L \rightarrow 2^X$ be (A, η) -monotone in the first variable. Let $N : X \times X \times L \rightarrow X$ be (γ, α) -relaxed cocoercive (with respect to A) and (β) -Lipschitz continuous in the first variable, and let N be (μ) -Lipschitz continuous in the second variable. Furthermore, let $\eta : X \times X \rightarrow X$ be (τ) -Lipschitz continuous. In addition, if*

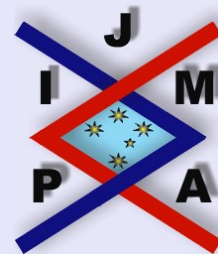
$$\left\| J_{\rho, A}^{M(\cdot, u, \lambda)}(w) - J_{\rho, A}^{M(\cdot, v, \lambda)}(w) \right\| \leq \delta \|u - v\| \quad \forall (u, v, \lambda) \in X \times X \times L,$$

then

$$(3.6) \quad \|G(u, \lambda) - G(v, \lambda)\| \leq \theta \|u - v\| \quad \forall (u, v, \lambda) \in X \times X \times L,$$

where

$$\theta = \frac{\tau}{r - \rho m} \left[\sqrt{s^2 - 2\rho\alpha + 2\rho\beta^2\gamma + \rho^2\beta^2} + \rho\mu \right] + \delta < 1,$$



$$\left| \rho - \frac{(\alpha - \gamma\beta^2)\tau^2 - r[\mu\tau + m(1 - \delta)](1 - \delta)}{\beta^2 - (\mu\tau + m(1 - \delta))^2} \right| < \frac{\sqrt{[(\alpha - \gamma\beta^2)\tau^2 - r(\mu\tau + m(1 - \delta))(1 - \eta)]^2 - B}}{\beta^2 - (\mu\tau - m(1 - \delta))^2},$$

$$B = [\beta^2 - (\mu\tau + m(1 - \delta))^2](s^2\tau^2 - r^2(1 - \delta)^2),$$

for

$$\begin{aligned} (\alpha - \gamma\beta^2)\tau^2 &> r(\mu\tau + m(1 - \delta))(1 - \delta) + \sqrt{B}, \\ \beta &> \mu\tau + m(1 - \delta), \quad 0 < \delta < 1. \end{aligned}$$

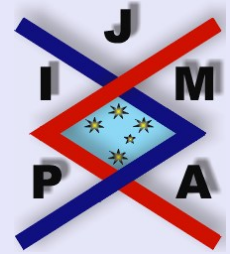
If the mappings $\lambda \rightarrow N(u, v, \lambda)$ and $\lambda \rightarrow J_{\rho, A}^{M(\cdot, u, \lambda)}(w)$ both are continuous (or Lipschitz continuous) from L to X , then the solution $z(\lambda)$ of (3.1) is continuous (or Lipschitz continuous) from L to X .

Proof. From the hypotheses of the theorem, for any $\lambda, \lambda^* \in L$, we have

$$\begin{aligned} \|z(\lambda) - z(\lambda^*)\| &= \|G(z(\lambda), \lambda) - G(z(\lambda^*), \lambda^*)\| \\ &\leq \|G(z(\lambda), \lambda) - G(z(\lambda^*), \lambda)\| + \|G(z(\lambda^*), \lambda) - G(z(\lambda^*), \lambda^*)\| \\ &\leq \theta\|z(\lambda) - z(\lambda^*)\| + \|G(z(\lambda^*), \lambda) - G(z(\lambda^*), \lambda^*)\|. \end{aligned}$$

It follows that

$$\|G(z(\lambda^*), \lambda) - G(z(\lambda^*), \lambda^*)\|$$



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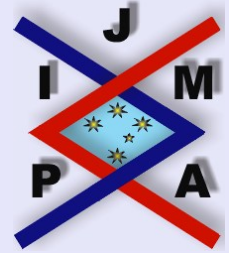


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$$\begin{aligned}
 &= \left\| J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda)) \right. \\
 &\quad \left. - J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda^*)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \right\| \\
 &\leq \left\| J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda)) \right. \\
 &\quad \left. - \left\| J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \right\| \right. \\
 &\quad \left. + \left\| J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \right\| \right. \\
 &\quad \left. - J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda^*)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \right\| \\
 &\leq \frac{\rho\tau}{r - \rho m} \|N(z(\lambda^*), z(\lambda^*), \lambda) - N(z(\lambda^*), z(\lambda^*), \lambda^*)\| \\
 &\quad + \left\| J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda)}(z(\lambda^*) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \right. \\
 &\quad \left. - J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda^*)}(z(\lambda^*) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \right\|.
 \end{aligned}$$

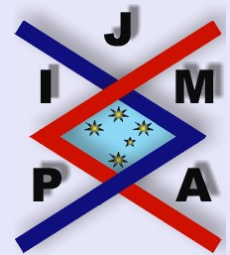
Hence, we have

$$\begin{aligned}
 \|z(\lambda) - z(\lambda^*)\| &\leq \frac{\rho\tau}{(r - \rho m)(1 - \theta)} \|N(z(\lambda^*), z(\lambda^*), \lambda) - N(z(\lambda^*), z(\lambda^*), \lambda^*)\| \\
 &\quad + \frac{1}{1 - \theta} \left\| J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda)}(z(\lambda^*) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \right. \\
 &\quad \left. - J_{\rho, A}^{M(\cdot, z(\lambda^*), \lambda^*)}(z(\lambda^*) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \right\|.
 \end{aligned}$$

This completes the proof. \square

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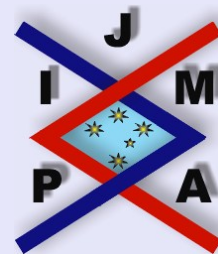
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