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GENERALIZED $(A,\eta)-$ RESOLVENT OPERATOR TECHNIQUE AND SENSITIVITY ANALYSIS FOR RELAXED COCOERCIVE VARIATIONAL INCLUSIONS

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ABSTRACT. Sensitivity analysis for relaxed cocoercive variational inclusions based on the generalized resolvent operator technique is discussed The obtained results are general in nature.

Key words and phrases: Sensitivity analysis, Quasivariational inclusions, Maximal relaxed monotone mapping, $(A,\eta)-monotone$ mapping, Generalized resolvent operator technique.

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1. INTRODUCTION

In [8] the author studied sensitivity analysis for quasivariational inclusions using the resolvent operator technique. Resolvent operator techniques have been frequently applied to a broad range of problems arising from several fields, including equilibria problems in economics, optimization and control theory, operations research, and mathematical programming. In this paper we intend to present the sensitivity analysis for $(A,\eta)-$monotone quasivariational inclusions involving relaxed cocoercive mappings. The notion of $(A,\eta)-$monotonicity [8] generalizes the notion of $A-$ monotonicity in [12]. The obtained results generalize a wide range of results on the sensitivity analysis for quasivariational inclusions, including [2] – [5] and others. For more details on nonlinear variational inclusions and related resolvent operator techniques, we recommend the reader [1] – [12].

2. (A,η)-MONOTONICITY

In this section we explore some basic properties derived from the notion of $(A,\eta)-$*monotonicity.* Let $\eta : X\times X \to X$ be $(\tau)-$Lipschitz continuous, that is, there exists a positive constant $\tau > 0$ such that

$$\|\eta(u,v)\| \leq \tau\|u-v\| \qquad \forall\, u,v\in X.$$

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Definition 2.1. Let $\eta : X \times X \to X$ be a single-valued mapping, and let $M : X \to 2^X$ be a multivalued mapping on X. The map M is said to be:

(i) (r, η)-*strongly* monotone if
$$\langle u^* - v^*, \eta(u, v)\rangle \geq r\|u - v\|^2 \qquad \forall (u, u^*), (v, v^*) \in Graph(M).$$

(ii) (r, η)-*strongly* pseudomonotone if
$$\langle v^*, \eta(u, v)\rangle \geq 0$$
implies
$$\langle u^*, \eta(u, v)\rangle \geq r\|u - v\|^2 \qquad \forall (u, u^*), (v, v^*) \in Graph(M).$$

(iii) (η)-*pseudomonotone* if
$$\langle v^*, \tau(u, v)\rangle \geq 0$$
implies
$$\langle u^*, \eta(u, v)\rangle \geq 0 \qquad \forall (u, u^*), (v, v^*) \in Graph(M).$$

(iii) (m, η)-*relaxed* monotone if there exists a positive constant m such that
$$\langle u^* - v^*, \eta(u, v)\rangle \geq (-m)\|u - v\|^2 \qquad \forall (u, u^*), (v, v^*) \in Graph(M).$$

Definition 2.2. A mapping $M : X \to 2^X$ is said to be maximal (m, η)-*relaxed* monotone if

(i) M is (m, η)-*relaxed* monotone,

(ii) For $(u, u^*) \in X \times X$, and
$$\langle u^* - v^*, \eta(u, v)\rangle \geq (-m)\|u - v\|^2 \qquad \forall (v, v^*) \in Graph(M),$$
we have $u^* \in M(u)$.

Definition 2.3. Let $A : X \to X$ and $\eta : X \times X \to X$ be two single-valued mappings. The map $M : X \to 2^X$ is said to be (A, η)-*monotone* if

(i) M is (m, η)-*relaxed* monotone

(ii) $R(A + \rho M) = X$ for $\rho > 0$.

Alternatively, we have

Definition 2.4. Let $A : X \to X$ and $\eta : X \times X \to X$ be two single-valued mappings. The map $M : X \to 2^X$ is said to be (A, η)-*monotone* if

(i) M is (m, η)-*relaxed* monotone

(ii) $A + \rho M$ is (η)-*pseudomonotone* for $\rho > 0$.

Proposition 2.1. *Let $A : X \to X$ be an (r, η)-strongly monotone single-valued mapping and let $M : X \to 2^X$ be an $(A), \eta)$-monotone mapping. Then M is maximal (m, η)-relaxed monotone for $0 < \rho < \frac{r}{m}$.*

Proposition 2.2. *Let $A : X \to X$ be an (r, η)-strongly monotone single-valued mapping and let $M : X \to 2^X$ be an (A, η)-monotone mapping. Then $(A + \rho M)$ is maximal (η)-monotone for $0 < \rho < \frac{r}{m}$.*

Proof. Since A is (r, η)-*strongly* monotone and M is (A, η)-*monotone,* it implies that $A + \rho M$ is $(r - \rho m, \eta)$-*strongly* monotone. This in turn implies that $A + \rho M$ is (η)-*pseudomonotone,* and hence $A + \rho M$ is maximal (η)-*monotone* under the given conditions. □

Proposition 2.3. *Let $A : X \to X$ be an (r, η)-strongly monotone mapping and let $M : X \to 2^X$ be an (A, η)-monotone mapping. Then the operator $(A + \rho M)^{-1}$ is single-valued.*

Definition 2.5. Let $A : X \to X$ be an (r,η)-*strongly* monotone mapping and let $M : X \to 2^X$ be an (A,η)-*monotone* mapping. Then the generalized resolvent operator $J_{\rho,A}^M : X \to X$ is defined by

$$J_{\rho,A}^M(u) = (A + \rho M)^{-1}(u).$$

Furthermore, we upgrade the notions of the monotonicity as well as strong monotonicity in the context of sensitivity analysis for nonlinear variational inclusion problems.

Definition 2.6. The map $T : X \times X \times L \to X$ is said to be:

(i) Monotone with respect to A in the first argument if

$$\langle T(x,u,\lambda) - T(y,u,\lambda), A(x) - A(y)\rangle \geq 0 \qquad \forall (x,y,u,\lambda) \in X \times X \times X \times L.$$

(ii) (r)-*strongly* monotone with respect to A in the first argument if there exists a positive constant r such that

$$\langle T(x,u,\lambda) - T(y,u,\lambda), A(x) - A(y)\rangle \geq (r)\|x-y\|^2 \qquad \forall (x,y,u,\lambda) \in X \times X \times X \times L.$$

(iii) (γ,α)-relaxed cocoercive with respect to A in the first argument if there exist positive constants γ and α such that

$$\langle T(x,u,\lambda) - T(y,u,\lambda), A(x) - A(y)\rangle \geq -\gamma\|T(x) - T(y)\|^2 + \alpha\|x-y\|^2$$

$\forall (x,y,u,\lambda) \in X \times X \times X \times L.$

(iv) (γ)-relaxed cocoercive with respect to A in the first argument if there exists a positive constant γ such that

$$\langle T(x,u,\lambda) - T(y,u,\lambda), A(x) - A(y)\rangle \geq -\gamma\|T(x) - T(y)\|^2$$

$\forall (x,y,u,\lambda) \in X \times X \times X \times L.$

3. RESULTS ON SENSITIVITY ANALYSIS

Let X denote a real Hilbert space with the norm $\|\cdot\|$ and inner product $\langle\cdot,\cdot\rangle$. Let $N : X \times X \times L \to X$ be a nonlinear mapping and $M : X \times X \times L \to 2^X$ be an A-*monotone* mapping with respect to the first variable, where L is a nonempty open subset of X. Furthermore, let $\eta : X \times X \to X$ be a nonlinear mapping. Then the problem of finding an element $u \in X$ for a given element $f \in X$ such that

$$f \in N(u,u,\lambda) + M(u,u,\lambda), \tag{3.1}$$

where $\lambda \in L$ is the perturbation parameter, is called a class of generalized strongly monotone mixed quasivariational inclusion (abbreviated SMMQVI) problems.

The solvability of the $SMMQVI$ problem (3.1) depends on the equivalence between (3.1) and the problem of finding the fixed point of the associated generalized resolvent operator.

Note that if M is (A,η)-*monotone,* then the corresponding generalized resolvent operator $J_{\rho,A}^M$ in first argument is defined by

$$J_{\rho,A}^{M(\cdot,y)}(u) = (A + \rho M(\cdot,y))^{-1}(u) \qquad \forall u \in X, \tag{3.2}$$

where $\rho > 0$ and A is an (r,η)-*strongly* monotone mapping.

Lemma 3.1. *Let X be a real Hilbert space, and let $\eta : X \times X \to X$ be a (τ)-Lipschitz continuous nonlinear mapping. Let $A : X \to X$ be (r,η)-strongly monotone, and let $M : X \times X \times L \to 2^X$ be (A,η)-monotone in the first variable. Then the generalized resolvent operator associated with $M(\cdot,y,\lambda)$ for a fixed $y \in X$ and defined by*

$$J_{\rho,A}^{M(\cdot,y,\lambda)}(u) = (A + \rho M(\cdot,y,\lambda))^{-1}(u) \qquad \forall\, u \in X,$$

is $(\frac{\tau}{r-\rho m})$-Lipschitz continuous.

Proof. By the definition of the generalized resolvent operator, we have

$$\frac{1}{\rho}\left(u - A\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u)\right)\right) \in M\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u)\right),$$

and

$$\frac{1}{\rho}\left(v - A\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right)\right) \in M\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right) \qquad \forall\, u, v \in X.$$

Given M is (m,η)-relaxed monotone, we find

$$\begin{aligned}\frac{1}{\rho}\left\langle u - v - \left(A\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u)\right) - A\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right)\right), \eta\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u), J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right)\right\rangle \\ \geq (-m)\left\|J_{\rho,A}^{M(\cdot,y,\lambda)}(u) - J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right\|^2.\end{aligned}$$

Therefore,

$$\begin{aligned}&\tau\|u-v\|\left\|J_{\rho,A}^{M(\cdot,y,\lambda)}(u) - J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right\| \\ &\geq \left\langle u - v, \eta\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u), J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right)\right\rangle \\ &\geq \left\langle A\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u)\right) - A\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right), \eta\left(J_{\rho,A}^{M(\cdot,y,\lambda)}(u), J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right)\right\rangle \\ &\qquad - (\rho m)\left\|J_{\rho,A}^{M(\cdot,y,\lambda)}(u) - J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right\|^2 \\ &\geq (r - \rho m)\left\|J_{\rho,A}^{M(\cdot,y,\lambda)}(u) - J_{\rho,A}^{M(\cdot,y,\lambda)}(v)\right\|^2.\end{aligned}$$

This completes the proof. □

Lemma 3.2. *Let X be a real Hilbert space, let $A : X \to X$ be $(r,\eta)-strongly$ monotone, and let $M : X \times X \times L \to 2^X$ be $(A),\eta-$monotone in the first variable. Let $\eta : X \times X \to X$ be a $(\tau) - Lipschitz$ continuous nonlinear mapping. Then the following statements are mutually equivalent:*

(i) *An element $u \in X$ is a solution to (3.1).*
(ii) *The map $G : X \times L \to X$ defined by*

$$G(u,\lambda) = J_{\rho,A}^{M(\cdot,u,\lambda)}(A(u) - \rho N(u,u,\lambda) + \rho f)$$

has a fixed point.

Theorem 3.3. *Let X be a real Hilbert space, and let $\eta : X \times X \to X$ be a (τ)-Lipschitz continuous nonlinear mapping. Let $A : X \to X$ be (r,η)-strongly monotone and (s)-Lipschitz continuous, and let $M : X \times X \times L \to 2^X$ be (A,η)-monotone in the first variable. Let $N : X \times X \times L \to X$ be (γ,α)-relaxed cocoercive (with respect to A) and (β)-Lipschitz continuous in the first variable, and let N be (μ)-Lipschitz continuous in the second variable. If, in addition,*

$$\left\|J_{\rho,A}^{M(\cdot,u,\lambda)}(w) - J_{\rho,A}^{M(\cdot,v,\lambda)}(w)\right\| \leq \delta\|u-v\| \qquad \forall (u,v,\lambda) \in X \times X \times L, \tag{3.3}$$

then

$$\|G(u,\lambda) - G(v,\lambda)\| \leq \theta\|u-v\| \qquad \forall (u,v,\lambda) \in X \times X \times L, \tag{3.4}$$

where

$$\theta = \frac{\tau}{r - \rho m}\left[\sqrt{s^2 - 2\rho\alpha + 2\rho\beta^2\gamma + \rho^2\beta^2} + \rho\mu\right] + \delta < 1,$$

$$\left|\rho - \frac{(\alpha-\gamma\beta^2)\tau^2 - r[\mu\tau + m(1-\delta)](1-\delta)}{\beta^2 - (\mu\tau + m(1-\delta))^2}\right| < \frac{\sqrt{[(\alpha-\gamma\beta^2)\tau^2 - r(\mu\tau + m(1-\delta))(1-\eta)]^2 - B}}{\beta^2 - (\mu\tau - m(1-\delta))^2},$$

$$B = [\beta^2 - (\mu\tau + m(1-\delta))^2](s^2\tau^2 - r^2(1-\delta)^2),$$

for

$$\alpha(\alpha-\gamma\beta^2)\tau^2 > r(\mu\tau + m(1-\delta))(1-\delta) + \sqrt{B},$$
$$\beta > \mu\tau + m(1-\delta),\ 0 < \delta < 1.$$

Consequently, for each $\lambda \in L$, *the mapping* $G(u,\lambda)$ *in light of* (3.4) *has a unique fixed point* $z(\lambda)$. *Hence, in light of Lemma 3.2,* $z(\lambda)$ *is a unique solution to* (3.1). *Thus, we have*

$$G(z(\lambda),\lambda) = z(\lambda).$$

Proof. For any element $(u,v,\lambda) \in X \times X \times L$, we have

$$G(u,\lambda) = J_{\rho,A}^{M(\cdot,u,\lambda)}(A(u) - \rho N(u,u,\lambda) + \rho f),$$
$$G(v,\lambda) = J_{\rho,A}^{M(\cdot,v,\lambda)}(A(v) - \rho N(v,v,\lambda) + \rho f).$$

It follows that

$$\begin{aligned}
&\|G(u,\lambda) - G(v,\lambda)\| \\
&= \left\|J_{\rho,A}^{M(\cdot,u,\lambda)}(A(u) - \rho N(u,u,\lambda) + \rho f) - J_{\rho,A}^{M(\cdot,v,\lambda)}(A(v) - \rho N(v,v,\lambda) + \rho f)\right\| \\
&\le \left\|J_{\rho,A}^{M(\cdot,u,\lambda)}(A(u) - \rho N(u,u,\lambda) + \rho f) - J_{\rho,A}^{M(\cdot,u,\lambda)}(A(v) - \rho N(v,v,\lambda) + \rho f)\right\| \\
&\quad + \left\|J_{\rho,A}^{M(\cdot,u,\lambda)}(A(v) - \rho N(v,v,\lambda) + \rho f) - J_{\rho,A}^{M(\cdot,v,\lambda)}(A(v) - \rho N(v,v,\lambda) + \rho f)\right\| \\
&\le \frac{\tau}{r-\rho m}\|A(u) - A(v) - \rho(N(u,u,\lambda) - N(v,v,\lambda))\| + \delta\|u-v\| \\
&\le \frac{\tau}{r-\rho m}[\|A(u) - A(v) - \rho(N(u,u,\lambda) - N(v,u,\lambda))\| \\
&\quad + \|\rho(N(v,u,\lambda) - N(v,v,\lambda))\|] + \delta\|u-v\|.
\end{aligned}$$

The (γ,α)-relaxed cocoercivity and (β)-*Lipschitz* continuity of N in the first argument imply that

$$\begin{aligned}
&\|A(u) - A(v) - \rho(N(u,u,\lambda) - N(v,u,\lambda))\|^2 \\
&= \|A(u) - A(v)\|^2 - 2\rho\langle N(u,u,\lambda) - N(v,u,\lambda), A(u) - A(v)\rangle \\
&\qquad + \rho^2\|N(u,u,\lambda) - N(v,u,\lambda)\|^2 \\
&\le (s^2 - 2\rho\alpha + 2\rho\beta^2\gamma + \rho^2\beta^2)\|u-v\|^2.
\end{aligned}$$

On the other hand, the (μ)-Lipschitz continuity of N in the second argument results

$$\|(N(v,u,\lambda)) - N(v,v,\lambda))\| \le \mu\|u-v\|.$$

In light of above arguments, we infer that

$$\|G(u,\lambda) - G(v,\lambda)\| \le \theta\|u-v\|, \tag{3.5}$$

where

$$\theta = \frac{\tau}{(r-\rho m)}\left[\sqrt{s^2 - 2\rho\alpha + 2\rho\beta^2\gamma + \rho^2\beta^2} + \rho\mu\right] + \delta < 1.$$

Since $\theta < 1$, it concludes the proof. □

Theorem 3.4. *Let X be a real Hilbert space, let $A : X \to X$ be (r,η)-strongly monotone and (s)-Lipschitz continuous, and let $M : X \times X \times L \to 2^X$ be (A,η)-monotone in the first variable. Let $N : X \times X \times L \to X$ be (γ,α)-relaxed cocoercive (with respect to A) and (β)-Lipschitz continuous in the first variable, and let N be (μ)-Lipschitz continuous in the second variable. Furthermore, let $\eta : X \times X \to X$ be (τ)-Lipschitz continuous. In addition, if*

$$\left\| J_{\rho,A}^{M(\cdot,u,\lambda)}(w) - J_{\rho,A}^{M(\cdot,v,\lambda)}(w) \right\| \le \delta\|u-v\| \qquad \forall (u,v,\lambda) \in X \times X \times L,$$

then

$$\|G(u,\lambda) - G(v,\lambda)\| \le \theta\|u-v\| \qquad \forall (u,v,\lambda) \in X \times X \times L, \tag{3.6}$$

where

$$\theta = \frac{\tau}{r-\rho m}\left[\sqrt{s^2 - 2\rho\alpha + 2\rho\beta^2\gamma + \rho^2\beta^2} + \rho\mu\right] + \delta < 1,$$

$$\left|\rho - \frac{(\alpha-\gamma\beta^2)\tau^2 - r[\mu\tau + m(1-\delta)](1-\delta)}{\beta^2 - (\mu\tau + m(1-\delta))^2}\right| < \frac{\sqrt{[(\alpha-\gamma\beta^2)\tau^2 - r(\mu\tau + m(1-\delta))(1-\eta)]^2 - B}}{\beta^2 - (\mu\tau - m(1-\delta))^2},$$

$$B = [\beta^2 - (\mu\tau + m(1-\delta))^2](s^2\tau^2 - r^2(1-\delta)^2),$$

for

$$(\alpha-\gamma\beta^2)\tau^2 > r(\mu\tau + m(1-\delta))(1-\delta) + \sqrt{B},$$
$$\beta > \mu\tau + m(1-\delta),\ 0 < \delta < 1.$$

If the mappings $\lambda \to N(u,v,\lambda)$ and $\lambda \to J_{\rho,A}^{M(\cdot,u,\lambda)}(w)$ both are continuous (or Lipschitz continuous) from L to X, then the solution $z(\lambda)$ of (3.1) is continuous (or Lipschitz continuous) from L to X.

Proof. From the hypotheses of the theorem, for any $\lambda, \lambda^* \in L$, we have

$$\begin{aligned}\|z(\lambda) - z(\lambda^*)\| &= \|G(z(\lambda),\lambda) - G(z(\lambda^*),\lambda^*)\| \\ &\le \|G(z(\lambda),\lambda) - G(z(\lambda^*),\lambda)\| + \|G(z(\lambda^*),\lambda) - G(z(\lambda^*),\lambda^*)\| \\ &\le \theta\|z(\lambda) - z(\lambda^*)\| + \|G(z(\lambda^*),\lambda) - G(z(\lambda^*),\lambda^*)\|.\end{aligned}$$

It follows that

$$\begin{aligned}&\|G(z(\lambda^*),\lambda) - G(z(\lambda^*),\lambda^*)\| \\ &\quad = \Big\| J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda)) \\ &\qquad\qquad - J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda^*)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \Big\| \\ &\quad \le \Big\| J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda)) \\ &\qquad\qquad - \Big\| J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \Big\| \\ &\qquad\qquad + \Big\| J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \Big\| \\ &\qquad\qquad - \ J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda^*)}(A(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \Big\|\end{aligned}$$

$$
\begin{aligned}
&\le \frac{\rho\tau}{r-\rho m}\left\|N(z(\lambda^*),z(\lambda^*),\lambda)-N(z(\lambda^*),z(\lambda^*),\lambda^*)\right\| \\
&\qquad + \left\| J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda)}(z(\lambda^*)-\rho N(z(\lambda^*),z(\lambda^*),\lambda^*)) \right. \\
&\qquad \left. - J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda^*)}(z(\lambda^*)-\rho N(z(\lambda^*),z(\lambda^*),\lambda^*)) \right\|.
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
\|z(\lambda)-z(\lambda^*)\| \le{}& \frac{\rho\tau}{(r-\rho m)(1-\theta)}\|N(z(\lambda^*),z(\lambda^*),\lambda)-N(z(\lambda^*),z(\lambda^*),\lambda^*)\| \\
&+\frac{1}{1-\theta}\left\| J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda)}(z(\lambda^*)-\rho N(z(\lambda^*),z(\lambda^*),\lambda^*)) \right. \\
&\qquad \left. - J_{\rho,A}^{M(\cdot,z(\lambda^*),\lambda^*)}(z(\lambda^*)-\rho N(z(\lambda^*),z(\lambda^*),\lambda^*)) \right\|.
\end{aligned}
$$

This completes the proof. □

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