ON THE COHEN p-NUCLEAR SUBLINEAR OPERATORS

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Abstract: Let $\mathcal{SB}(X,Y)$ be the set of all bounded sublinear operators from a Banach space X into a complete Banach lattice Y. In the present paper, we will introduce to this category the concept of Cohen p-nuclear operators. We give an analogue to "*Pietsch's domination theorem*" and we study some properties concerning this notion.

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1. Introduction and terminology

The notion of Cohen p-nuclear operators ($1 \leq p \leq \infty$) was initiated by Cohen in [7] and generalized to Cohen (p,q)-nuclear ($1 \leq q \leq \infty$) by Apiola in [4]. A linear operator u between two Banach spaces X, Y is Cohen p-nuclear for ($1 < p < \infty$) if there is a positive constant C such that for all $n \in \mathbb{N}$; $x_1, \dots, x_n \in X$ and $y_1^*, \dots, y_n^* \in Y^*$ we have

$$\left| \sum_{i=1}^{n} \left\langle u\left(x_i\right), y_i^* \right\rangle \right| \leq C \sup_{x^* \in B_{X^*}} \left\| \left(x^*\left(x_i\right)\right) \right\|_{l_p^n} \sup_{y \in B_Y} \left\| \left(y_i^*(y)\right) \right\|_{l_{p^*}^n} .$$

The smallest constant C which is noted by $n_p(u)$, such that the above inequality holds, is called the Cohen p-nuclear norm on the space $\mathcal{N}_p(X,Y)$ of all Cohen p-nuclear operators from X into Y which is a Banach space. For $p=1$ and $p=\infty$ we have $\mathcal{N}_1(X,Y) = \pi_1(X,Y)$ (the Banach space of all 1-summing operators) and $\mathcal{N}_\infty(X,Y) = \mathcal{D}_\infty(X,Y)$ (the Banach space of all strongly ∞-summing operators).

In [7, Theorem 2.3.2], Cohen proves that, if u verifies a domination theorem then u is p-nuclear and he asked if the statement of this theorem characterizes p-nuclear operators. The reciprocal of this statement is given in [8, Theorem 9.7, p.189], but these operators are called p-dominated operators. In this work, we generalize this notion to the sublinear maps and we give an analogue to "*Pietsch's domination theorem*" for this category of operators which is one of the main results of this paper. We study some properties concerning this class and treat some related results concerning the relations between linear and sublinear operators.

This paper is organized as follows. In the first section, we give some basic definitions and terminology concerning Banach lattices. We also recall some standard notations. In the second section, we present some definitions and properties concerning sublinear operators. We give the definition of positive p-summing operators

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introduced by Blasco [5, 6] and we present the notion of strongly p-summing sublinear operators initiated in [3].

In Section 3, we generalize the class of Cohen p-nuclear operators to the sublinear operators. This category verifies a domination theorem, which is the principal result. We use Ky Fan's lemma to prove it.

We end in Section 4, by studying some relations between the different classes of sublinear operators (p-nuclear, strongly p-summing and p-summing). We study also the relation between T and ∇T concerning the notion of Cohen p-nuclear sublinear operators, where $\nabla T = \{u \in \mathcal{L}(X, Y) : u \leq T\}$ ($\mathcal{L}(X, Y)$ is the space of all linear operators from X into Y). We prove that, if T is a Cohen positive p-nuclear sublinear operator, then u is Cohen positive p-nuclear and consequently u^* is positive p^*-summing. For the converse, we add one condition concerning T.

We start by recalling the abstract definition of Banach lattices. Let X be a Banach space. If X is a vector lattice and $\|x\| \leq \|y\|$ whenever $|x| \leq |y|$ ($|x| = \sup\{x, -x\}$) we say that X is a Banach lattice. If the lattice is complete, we say that X is a complete Banach lattice. Note that this implies obviously that for any $x \in X$ the elements x and $|x|$ have the same norm. We denote by $X_+ = \{x \in X : x \geq 0\}$. An element x of X is positive if $x \in X_+$.

The dual X^* of a Banach lattice X is a complete Banach lattice endowed with the natural order

$$x_1^* \leq x_2^* \Longleftrightarrow \langle x_1^*, x \rangle \leq \langle x_2^*, x \rangle\,, \quad \forall x \in X_+ \tag{1.1}$$

where $\langle \cdot, \cdot \rangle$ denotes the bracket of duality.

By a sublattice of a Banach lattice X we mean a linear subspace E of X so that $\sup\{x, y\}$ belongs to E whenever $x, y \in E$. The canonical embedding $i : X \longrightarrow X^{**}$ such that $\langle i(x), x^* \rangle = \langle x^*, x \rangle$ of X into its second dual X^{**} is an order isometry from X onto a sublattice of X^{**}, see [9, Proposition 1.a.2]. If we consider X as a

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sublattice of X^{**} we have for $x_1, x_2 \in X$

$$x_1 \leq x_2 \Longleftrightarrow \langle x_1, x^* \rangle \leq \langle x_2, x^* \rangle, \quad \forall x^* \in X^*_+. \tag{1.2}$$

For more details on this, the interested reader can consult the references [9, 11].

We continue by giving some standard notations. Let X be a Banach space and $1 \leq p \leq \infty$. We denote by $l_p(X)$ (resp. $l_p^n(X)$) the space of all sequences (x_i) in X with the norm

$$\|(x_i)\|_{l_p(X)} = \left(\sum_1^\infty \|x_i\|^p\right)^{\frac{1}{p}} < \infty$$

$$\left[\text{resp. } \left\|(x_i)_{1\leq i\leq n}\right\|_{l_p^n(X)} = \left(\sum_1^n \|x_i\|^p\right)^{\frac{1}{p}}\right]$$

and by $l_p^\omega(X)$ (resp. $l_p^{n\ \omega}(X)$) the space of all sequences (x_i) in X with the norm

$$\|(x_n)\|_{l_p^\omega(X)} = \sup_{\|\xi\|_{X^*}=1} \left(\sum_1^\infty |\langle x_i, \xi\rangle|^p\right)^{\frac{1}{p}} < \infty$$

$$\left[\text{resp. } \|(x_n)\|_{l_p^{n\ \omega}(X)} = \sup_{\|\xi\|_{X^*}=1} \left(\sum_1^n |\langle x_i, \xi\rangle|^p\right)^{\frac{1}{p}}\right]$$

where X^* denotes the dual (topological) of X and B_X denotes the closed unit ball of X. We know (see [8]) that $l_p(X) = l_p^\omega(X)$ for some $1 \leq p < \infty$ iff $\dim(X)$ is finite. If $p = \infty$, we have $l_\infty(X) = l_\infty^\omega(X)$. We have also if $1 < p \leq \infty$, $l_p^\omega(X) \equiv B(l_{p^*}, X)$ isometrically (where p^* is the conjugate of p, i.e., $\frac{1}{p} + \frac{1}{p^*} = 1$).

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In other words, let $v : l_{p^*} \longrightarrow X$ be a linear operator such that $v(e_i) = x_i$ (namely, $v = \sum_1^\infty e_j \otimes x_j$, e_j denotes the unit vector basis of l_p) then

$$\|v\| = \|(x_n)\|_{l_p^\omega(X)} . \tag{1.3}$$

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2. Sublinear Operators

For our convenience, we give in this section some elementary definitions and fundamental properties relative to sublinear operators. For more information see [1, 2, 3]. We also recall some notions concerning the summability of operators.

Definition 2.1. *A mapping T from a Banach space X into a Banach lattice Y is said to be sublinear if for all x, y in X and λ in $\mathbb{R}_+$, we have*

(i) $T(\lambda x) = \lambda T(x)$ *(i.e., positively homogeneous),*

(ii) $T(x+y) \leq T(x) + T(y)$ *(i.e., subadditive).*

Note that the sum of two sublinear operators is a sublinear operator and the multiplication by a positive number is also a sublinear operator.

Let us denote by

$$\mathcal{SL}(X,Y) = \{\text{sublinear mappings } T : X \longrightarrow Y\}$$

and we equip it with the natural order induced by Y

$$T_1 \leq T_2 \Longleftrightarrow T_1(x) \leq T_2(x), \quad \forall x \in X \tag{2.1}$$

and

$$\nabla T = \{u \in L(X,Y) : u \leq T \quad (\text{i.e., } \forall x \in X,\ u(x) \leq T(x))\}\,.$$

A very general case when the set ∇T is not empty is provided by Proposition 2.3 below.

Consequently,

$$u \leq T \Longleftrightarrow -T(-x) \leq u(x) \leq T(x), \quad \forall x \in X. \tag{2.2}$$

Let T be sublinear from a Banach space X into a Banach lattice Y. Then we have,

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- T is continuous if and only if there is $C > 0$ such that for all $x \in X$, $\|T(x)\| \leq C\|x\|$.

In this case we say that T is bounded and we put

$$\|T\| = \sup\left\{\|T(x)\| : \|x\|_{B_X} = 1\right\}.$$

We will denote by $\mathcal{SB}(X,Y)$ the set of all bounded sublinear operators from X into Y.

We say that a sublinear operator T is positive if for all x in X, $T(x) \geq 0$; is increasing if for all x, y in X, $T(x) \leq T(y)$ when $x \leq y$.

Also, there is no relation between positive and increasing like the linear case (a linear operator $u \in \mathcal{L}(X,Y)$ is positive if $u(x) \geq 0$ for $x \geq 0$).

We will need the following obvious properties.

Proposition 2.2. *Let X be an arbitrary Banach space. Let Y, Z be Banach lattices.*

(i) Consider T in $\mathcal{SL}(X,Y)$ and u in $\mathcal{L}(Y,Z)$. Assume that u is positive. Then, $u \circ T \in \mathcal{SL}(X,Z)$.

(ii) Consider u in $\mathcal{L}(X,Y)$ and T in $\mathcal{SL}(Y,Z)$. Then, $T \circ u \in \mathcal{SL}(X,Z)$.

(iii) Consider S in $\mathcal{SL}(X,Y)$ and T in $\mathcal{SL}(Y,Z)$. Assume that S is increasing. Then, $S \circ T \in \mathcal{SL}(X,Z)$.

The following proposition will be used implicitly in the sequel. For its proof, see [1, Proposition 2.3].

Proposition 2.3. *Let X be a Banach space and let Y be a complete Banach lattice. Let $T \in \mathcal{SL}(X,Y)$. Then, for all x in X there is $u_x \in \nabla T$ such that $T(x) = u_x(x)$ (i.e., the supremum is attained, $T(x) = \sup\{u(x) : u \in \nabla T\}$).*

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We have thus that ∇T is not empty if Y is a complete Banach lattice. If Y is simply a Banach lattice then ∇T is empty in general (see [10]).

As an immediate consequence of Proposition 2.3, we have:

- the operator T is bounded if and only if for all $u \in \nabla T$, $u \in \mathcal{B}(X,Y)$ (the space of all bounded linear operators).

We briefly continue by defining the notion of strongly p-summing introduced by Cohen [7] and generalized to sublinear operators in [3].

Definition 2.4. *Let X be a Banach space and Y be a Banach lattice. A sublinear operator $T : X \longrightarrow Y$ is strongly p-summing ($1 < p < \infty$), if there is a positive constant C such that for any $n \in \mathbb{N}$; $x_1, \ldots, x_n \in X$ and $y_1^*, \ldots, y_n^* \in Y^*$ we have*

$$\sum_{i=1}^{n} \left| \left\langle T(x_i), y_i^* \right\rangle \right| \le C \left\| (x_i) \right\|_{l_p^n(X)} \sup_{y \in B_Y} \left\| (y_i^*(y)) \right\|_{l_{p^*}^{n}{}^{\omega}} . \tag{2.3}$$

We denote by $\mathcal{D}_p(X,Y)$ the class of all strongly p-summing sublinear operators from X into Y and by $d_p(T)$ the smallest constant C such that the inequality (2.3) holds. For $p = 1$, we have $\mathcal{D}_1(X,Y) = \mathcal{SB}(X,Y)$.

Theorem 2.5 ([3]). *Let X be a Banach space and Y be a Banach lattice. An operator $T \in \mathcal{SB}(X,Y)$ is strongly p-summing ($1 < p < \infty$), if and only if, there exists a positive constant $C > 0$ and a Radon probability measure μ on $B_{Y^{**}}$ such that for all $x \in X$, we have*

$$\left| \left\langle T(x), y^* \right\rangle \right| \le C \left\| x \right\| \left(\int_{B_{Y^{**}}} \left| y^*(y^{**}) \right|^{p^*} d\mu(y^{**}) \right)^{\frac{1}{p^*}} . \tag{2.4}$$

Moreover, in this case

$$d_p(T) = \inf \left\{ C > 0 : \text{ for all } C \text{ verifying the inequality (2.4)} \right\} .$$

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For the definition of positive strongly p-summing, we replace Y^* by Y_+^* and $d_p(T)$ by $d_p^+(T)$.

To conclude this section, we recall the definition of positive p-summing sublinear operators, which was first stated in the linear case by Blasco in [5]. For the definition of p-summing and related properties, the reader can see [1].

Definition 2.6. *Let X, Y be Banach lattices. Let $T : X \longrightarrow Y$ be a sublinear operator. We will say that T is "positive p-summing" ($0 \leq p \leq \infty$) (we write $T \in \pi_p^+(X, Y)$), if there exists a positive constant C such that for all $n \in \mathbb{N}$ and all $\{x_1, ..., x_n\} \subset X_+$, we have*

$$\|(T(x_i))\|_{l_p^n(Y)} \leq C \|(x_i)\|_{l_p^{n\,\omega}(X)} . \tag{2.5}$$

We put

$$\pi_p^+(T) = \inf \{C \text{ verifying the inequality (2.5)}\} .$$

Theorem 2.7. *A sublinear operator between Banach lattices X, Y is positive p-summing ($1 \leq p < \infty$), if and only if, there exists a positive constant $C > 0$ and a Borel probability μ on $B_{X^*}^+$ such that*

$$\|T(x)\| \leq C \left(\int_{B_{X^*}^+} \langle x, x^* \rangle^p \, d\mu(x^*) \right)^{\frac{1}{p}} \tag{2.6}$$

for every $x \in X_+$. Moreover, in this case

$$\pi_p^+(T) = \inf \{C > 0 : \text{ for all } C \text{ verifying the inequality (2.6)}\} .$$

Proof. It is similar to the linear case (see [5, 12]). □

If T is positive p-summing then u is positive p-summing for all $u \in \nabla T$ and by [1, Corollary 2.4], we have $\pi_p^+(u) \leq 2\pi_p^+(T)$. We do not know if the converse is true.

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3. Cohen $p-$Nuclear Sublinear Operators

We introduce the following generalization of the class of Cohen p-nuclear operators. We give the domination theorem for such a category by using Ky Fan's Lemma.

Definition 3.1. *Let X be a Banach space and Y be a Banach lattice. A sublinear operator $T : X \longrightarrow Y$ is Cohen p-nuclear ($1 < p < \infty$), if there is a positive constant C such that for any $n \in \mathbb{N}$, $x_1, ..., x_n \in X$ and $y_1^*, ..., y_n^* \in Y^*$, we have*

$$\left| \sum_{i=1}^{n} \langle T(x_i), y_i^* \rangle \right| \leq C \sup_{x^* \in B_{X^*}} \|(x^*(x_i))\|_{l_p^n} \sup_{y \in B_Y} \|(y_i^*(y))\|_{l_{p^*}^n} . \tag{3.1}$$

We denote by $\mathcal{N}_p(X, Y)$ the class of all Cohen p-nuclear sublinear operators from X into Y and by $n_p(T)$ the smallest constant C such that the inequality (3.1) holds. For the definition of positive Cohen p-nuclear, we replace Y^* by Y_+^* and $n_p(T)$ by $n_p^+(T)$.

Let $T \in \mathcal{SB}(X, Y)$ and $v : l_p^n \longrightarrow Y^*$ be a bounded linear operator. By (1.3), the sublinear operator T is Cohen p-nuclear, if and only if,

$$\left| \sum_{i=1}^{n} \langle T(x_i), v(e_i) \rangle \right| \leq C \sup_{x^* \in B_{X^*}} \|(x^*(x_i))\|_{l_p^n} \|v\| . \tag{3.2}$$

Similar to the linear case, for $p = 1$ and $p = \infty$, we have $\mathcal{N}_1(X, Y) = \pi_1(X, Y)$ and $\mathcal{N}_\infty(X, Y) = \mathcal{D}_\infty(X, Y)$.

Proposition 3.2. *Let X be a Banach space and Y, Z be two Banach lattices. Consider T in $\mathcal{SB}(X, Y)$, u a positive operator in $\mathcal{B}(Y, Z)$ and S in $\mathcal{B}(E, X)$.*

(i) If T is a Cohen p-nuclear sublinear operator, then $u \circ T$ is a Cohen p-nuclear sublinear operator and $n_p(u \circ T) \leq \|u\| n_p(T)$.

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(ii) If T is a Cohen p-nuclear sublinear operator, then $T \circ S$ is a Cohen p-nuclear sublinear operator and $n_p(T \circ S) \leq \|S\| \, n_p(T)$.

Proof. (i) Let $n \in \mathbb{N}$; $x_1, \dots, x_n \in X$ and $z_1^*, \dots, z_n^* \in Z^*$. It suffices by (3.2) to prove that

$$\left| \sum_{i=1}^{n} \langle uT(x_i), z_i^* \rangle \right| \leq C \sup_{x^* \in B_{X^*}} \|(x^*(x_i))\|_{l_p^n} \|v\|$$

where $v : Z \longrightarrow l_{p^*}^n$ such that $v(z) = \sum_{i=1}^n z_i^*(z) e_i$. We have

$$\begin{aligned}\left| \sum_{i=1}^{n} \langle uT(x_i), z_i^* \rangle \right| &= \left| \sum_{i=1}^{n} \langle T(x_i), u^*(z_i^*) \rangle \right| \\ &\leq n_p(T) \sup_{x^* \in B_{X^*}} \|(x^*(x_i))\|_{l_p^n} \|w\|\end{aligned}$$

where

$$\begin{aligned} w(y) &= \sum_{i=1}^{n} \langle u^*(z_i^*), y \rangle e_i, \\ &= \sum_{i=1}^{n} \langle z_i^*, u(y) \rangle e_i, \\ &= \|u(y)\| \sum_{i=1}^{n} \left\langle z_i^*, \frac{u(y)}{\|u(y)\|} \right\rangle e_i. \end{aligned}$$

This implies that

$$\begin{aligned} \|w\| &\leq \|u\| \sup_{y \in B_Y} \left\| (z_i^*(z))_{1 \leq i \leq n} \right\| \\ &\leq \|u\| \, \|v\| \, . \end{aligned}$$

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(ii) Let $n \in \mathbb{N}$; $e_1, ..., e_n \in E$ and $y_1^*, ..., y_n^* \in Y^*$. We have

$$\left| \sum_{i=1}^{n} \langle T \circ S(e_i), y_i^* \rangle \right| \leq n_p(T) \sup_{x^* \in B_{X^*}} \left(\sum_{i=1}^{n} |\langle S(e_i), x^* \rangle|^p \right)^{\frac{1}{p}} \|v\|$$
$$\leq n_p(T) \sup_{x^* \in B_{X^*}} \|S^*(x^*)\| \left(\sum_{i=1}^{n} \left| \left\langle e_i, \frac{S^*(x^*)}{\|S^*(x^*)\|} \right\rangle \right|^p \right)^{\frac{1}{p}} \|v\|$$
$$\leq n_p(T) \|S\| \sup_{e^* \in B_{E^*}} \left(\sum_{i=1}^{n} |\langle e_i, e^* \rangle|^p \right)^{\frac{1}{p}} \|v\|.$$

This implies that T is Cohen p-nuclear and $n_p(T \circ S) \leq \|S\| \, n_p(T)$. □

The main result of this section is the next extension of "*Pietsch's domination theorem*" for the class of sublinear operators. For the proof we will use the following lemma due to Ky Fan, see [8].

Lemma 3.3. *Let E be a Hausdorff topological vector space, and let $\mathcal{C}$ be a compact convex subset of E. Let M be a set of functions on $\mathcal{C}$ with values in $(-\infty, \infty]$ having the following properties:*

(a) each $f \in M$ is convex and lower semicontinuous;

(b) if $g \in conv(M)$, there is an $f \in M$ with $g(x) \leq f(x)$, for every $x \in \mathcal{C}$;

(c) there is an $r \in \mathbb{R}$ such that each $f \in M$ has a value not greater than r.

Then there is an $x_0 \in \mathcal{C}$ such that $f(x_0) \leq r$ for all $f \in M$.

We now give the domination theorem by using the above lemma.

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Theorem 3.4. *Let X be a Banach space and Y be a Banach lattice. Consider $T \in \mathcal{SB}(X,Y)$ and C a positive constant.*

1. *The operator T is Cohen p-nuclear and $n_p(T) \leq C$.*
2. *For any n in $\mathbb{N}$, $x_1, ..., x_n$ in X and $y_1^*, ..., y_n^*$ in Y^* we have*

$$\sum_{i=1}^{n} |\langle T(x_i), y_i^* \rangle| \leq C \sup_{x^* \in B_{X^*}} \|(x^*(x_i))\|_{l_p^n} \sup_{y \in B_Y} \|(y_i^*(y))\|_{l_{p^*}^n}.$$

3. *There exist Radon probability measures μ_1 on B_{X^*} and μ_2 on $B_{Y^{**}}$, such that for all $x \in X$ and $y^* \in Y^*$, we have*

$$|\langle T(x), y^* \rangle| \leq C \left(\int_{B_{X^*}} |x(x^*)|^p \, d\mu_1(x^*) \right)^{\frac{1}{p}} \times \left(\int_{B_{Y^{**}}} |y^*(y^{**})|^{p^*} \, d\mu_2(y^{**}) \right)^{\frac{1}{p^*}}. \tag{3.3}$$

Moreover, in this case

$$n_p(T) = \inf \{ C > 0 : \text{ for all } C \text{ verifying the inequality (3.3)} \}.$$

Proof. (1)⇒(2). Let T be in $\mathcal{N}_p(X,Y)$ and (λ_i) be a scalar sequence. We have

$$\left| \sum_{i=1}^{n} \lambda_i \langle T(x_i), y_i^* \rangle \right| \leq n_p(T) \sup \|(\lambda_i)\|_{l_\infty} \|(x_i)\|_{l_p^{n\,\omega}(X)} \sup_{y \in B_Y} \|(y_i^*(y))\|_{l_{p^*}^n}.$$

Taking the supremum over all sequences (λ_i) with $\|(\lambda_i)\|_{l_\infty} \leq 1$, we obtain

$$\sum_{i=1}^{n} |\langle T(x_i), y_i^* \rangle| \leq n_p(T) \|(x_i)\|_{l_p^{n\,\omega}(X)} \sup_{y \in B_Y} \|(y_i^*(y))\|_{l_{p^*}^n}.$$

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To prove that (2) implies (3). We consider the sets $P(B_{X^*})$ and $P(B_{Y^{**}})$ of probability measures in $C(B_{X^*})^*$ and $C(B_{Y^{**}})^*$, respectively. These are convex sets which are compact when we endow $C(B_{X^*})^*$ and $C(B_{Y^{**}})^*$ with their weak$*$ topologies. We are going to apply Ky Fan's Lemma with $E = C(B_{X^*})^* \times C(B_{Y^{**}})^*$ and $\mathcal{C} = P(B_{X^*}) \times P(B_{Y^{**}})$.

Consider the set M of all functions $f : \mathcal{C} \to \mathbb{R}$ of the form

$$f_{((x_i),(y_i^*))}(\mu_1,\mu_2) := \sum_{i=1}^{n} |\langle T(x_i), y_i^*\rangle| - C\left(\frac{1}{p}\sum_{i=1}^{n}\int_{B_{X^*}} |x_i(x^*)|^p \, d\mu_1(x^*) + \frac{1}{p^*}\sum_{i=1}^{n}\int_{B_{Y^{**}}} |y_i^*(y^{**})|^{p^*} \, d\mu_2(y^{**})\right), \tag{3.4}$$

where $x_1, ..., x_n \in X$ and $y_1^*, ..., y_n^* \in Y^*$.

These functions are convex and continuous. We now apply Ky Fan's Lemma (the conditions (a) and (b) of Ky Fan's Lemma are satisfied). Let f, g be in M and $\alpha \in [0,1]$ such that

$$f_{((x_i'),(y_i'^*))}(\mu_1,\mu_2) = \sum_{i=1}^{k} |\langle T(x_i'), y_i'^*\rangle| - C\left[\frac{1}{p}\sum_{i=1}^{k}\int_{B_{X^*}} |\langle x_i', x^*\rangle|^p \, d\mu_1(x^*) + \frac{1}{p^*}\sum_{i=1}^{k}\int_{B_{Y^{**}}} |\langle y_i'^*, y^{**}\rangle|^{p^*} \, d\mu_2(y^{**})\right],$$

and

$$g_{((x_i''),(y_i''^*))}(\mu_1,\mu_2) = \sum_{i=k+1}^{l} |\langle T(x_i''), y_i''^*\rangle| - C\left[\frac{1}{p}\sum_{i=k+1}^{l}\int_{B_{X^*}} |\langle x_i'', x^*\rangle|^p \, d\mu_1(x^*)\right.$$

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$$+\frac{1}{p^*}\sum_{i=k+1}^{l}\left|\langle y_i''^*, y^{**}\rangle\right|^{p^*} d\mu_2(y^{**})\Bigg].$$

It follows that

$$\alpha f = \alpha\left[\sum_{i=1}^{k}\left|\langle T(x_i'), y_i'^*\rangle\right| - C\left(\frac{1}{p}\sum_{i=1}^{k}\int_{B_{X^*}}\left|\langle x_i', x^*\rangle\right|^p d\mu_1(x^*)\right.\right.$$
$$\left.\left.+\frac{1}{p^*}\sum_{i=1}^{k}\int_{B_{Y^{**}}}\left|\langle y_i'^*, y^{**}\rangle\right|^{p^*} d\mu_2(y^{**})\right)\right]$$
$$=\sum_{i=1}^{k}\left|\left\langle T\left(\alpha^{\frac{1}{p}}x_i'\right), \alpha^{\frac{1}{p^*}}y_i'^*\right\rangle\right| - C\left(\frac{1}{p}\sum_{i=1}^{k}\int_{B_{X^*}}\left|\left\langle \alpha^{\frac{1}{p}}x_i', x^*\right\rangle\right|^p d\mu_1(x^*)\right.$$
$$\left.+\frac{1}{p^*}\sum_{i=1}^{k}\int_{B_{Y^{**}}}\left|\left\langle \alpha^{\frac{1}{p^*}}y_i'^*, y^{**}\right\rangle\right|^{p^*} d\mu_2(y^{**})\right)$$
$$= f_{\left(\left(\alpha^{\frac{1}{p}}x_i'\right),\left(\alpha^{\frac{1}{p^*}}y_i'^*\right)\right)}(\mu_1,\mu_2),$$

and

$$f+g = \sum_{i=1}^{k}\left|\langle T(x_i'), y_i'^*\rangle\right| - C\left(\frac{1}{p}\sum_{i=1}^{k}\int_{B_{X^*}}\left|\langle x_i', x^*\rangle\right|^p d\mu_1(x^*)\right.$$
$$\left.+\frac{1}{p^*}\sum_{i=1}^{k}\int_{B_{Y^{**}}}\left|\langle y_i'^*, y^{**}\rangle\right|^{p^*} d\mu_2(y^{**})\right) + \sum_{i=k+1}^{l}\left|\langle T(x_i''), y_i''^*\rangle\right|$$
$$-C\left(\frac{1}{p}\sum_{i=k+1}^{l}\int_{B_{X^*}}\left|\langle x_i'', x^*\rangle\right|^p d\mu_1(x^*) + \frac{1}{p^*}\sum_{i=k+1}^{l}\left|\langle y_i''^*, y^{**}\rangle\right|^{p^*} d\mu_2(y^{**})\right)$$

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$$= \sum_{i=1}^{k} |\langle T(x_i'), y_i'^* \rangle| + \sum_{i=k+1}^{l} |\langle T(x_i''), y_i''^* \rangle| - C \left(\frac{1}{p} \sum_{i=1}^{n} \int_{B_{X^*}} |\langle x_i, x^* \rangle|^p \, d\mu_1(x^*) + \frac{1}{p^*} \sum_{i=1}^{n} \int_{B_{Y^{**}}} |\langle y_i^*, y^{**} \rangle|^{p^*} \, d\mu_2(y^{**}) \right)$$

$$= \sum_{i=1}^{n} |\langle T(x_i), y_i^* \rangle| - C \left(\frac{1}{p} \sum_{i=1}^{n} \int_{B_{X^*}} |\langle x_i, x^* \rangle|^p \, d\mu_1(x^*) + \frac{1}{p^*} \sum_{i=1}^{n} \int_{B_{Y^{**}}} |\langle y_i^*, y^{**} \rangle|^{p^*} \, d\mu_2(y^{**}) \right)$$

with $n = k + l$,

$$x_i = \begin{cases} x_i' & \text{if} \quad 1 \leq i \leq k, \\ x_i'' & \text{if} \quad k+1 \leq i \leq l \end{cases}$$

and

$$y_i^* = \begin{cases} y_i'^* & \text{if} \quad 1 \leq i \leq k, \\ y_i''^* & \text{if} \quad k+1 \leq i \leq l. \end{cases}$$

For the condition (c), since B_{X^*} and $B_{Y^{**}}$ are weak$*$ compact and norming sets, there exist for $f \in M$ two elements, $x_0^* \in B_{X^*}$ and $y_0 \in B_{Y^{**}}$ such that

$$\sup_{x^* \in B_{X^*}} \sum_{i=1}^{n} |\langle x_i, x^* \rangle|^p = \sum_{i=1}^{n} |\langle x_i, x_0^* \rangle|^p$$

and

$$\sup_{y \in B_Y} \|(y_i^*(y))\|_{l_{p^*}^n}^{p^*} = \sum_{i=1}^{n} |\langle y_i^*, y_0 \rangle|^{p^*}.$$

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Using the elementary identity

$$\forall \alpha, \beta \in \mathbb{R}_+^* \quad \alpha\beta = \inf_{\epsilon>0} \left\{ \frac{1}{p} \left(\frac{\alpha}{\epsilon} \right)^p + \frac{1}{p^*} (\epsilon\beta)^{p^*} \right\}, \tag{3.5}$$

taking

$$\alpha = \sup_{x^* \in B_{X^*}} \left(\sum_{i=1}^n |\langle x_i, x^* \rangle|^p \right)^{\frac{1}{p}}, \qquad \beta = \sup_{y \in B_Y} \|(y_i^*(y))\|_{l_{p^*}^n}$$

and $\epsilon = 1$, then

$$\begin{aligned} f\left(\delta_{x_0^*}, \delta_{y_0}\right) &= \sum_{i=1}^n |\langle T(x_i), y_i^* \rangle| - \frac{C}{p} \left(\sup_{x^* \in B_{X^*}} \sum_{i=1}^n |\langle x_i, x^* \rangle|^p \right) - \frac{C}{p^*} \sup_{y \in B_Y} \|(y_i^*(y))\|_{l_{p^*}^n}^{p^*} \\ &\leq \sum_{i=1}^n |\langle T(x_i), y_i^* \rangle| - C \left(\sup_{x^* \in B_{X^*}} \sum_{i=1}^n |\langle x_i, x^* \rangle|^p \right)^{\frac{1}{p}} \sup_{y \in B_Y} \|(y_i^*(y))\|_{l_{p^*}^n}. \end{aligned}$$

The last quantity is less than or equal to zero (by hypothesis (2)) and hence condition (c) is verified by taking $r = 0$. By Ky Fan's Lemma, there is $(\mu_1, \mu_2) \in \mathcal{C}$ with $f(\mu_1, \mu_2) \leq 0$ for all $f \in M$. Then, if f is generated by the single elements $x \in X$ and $y^* \in Y^*$,

$$|\langle T(x), y^* \rangle| \leq \frac{C}{p} \int_{B_{X^*}} |\langle x, x^* \rangle|^p \, d\mu_1(x^*) + \frac{C}{p^*} \int_{B_{Y^{**}}} |\langle y^*, y^{**} \rangle|^{p^*} \, d\mu_2(y^{**}).$$

Fix $\epsilon > 0$. Replacing x by $\frac{1}{\epsilon}x$, and y^* by ϵy^* and taking the infimum over all $\epsilon > 0$

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(using the elementary identity (3.5)), we find

$$|\langle T(x), y^*\rangle| \leq C\left\{\frac{1}{p}\left[\frac{1}{\epsilon}\left(\int_{B_{X^*}}|\langle x, x^*\rangle|^p \, d\mu_1(x^*)\right)^{\frac{1}{p}}\right]^p + \frac{1}{p^*}\left[\epsilon\left(\int_{B_{Y^{**}}}|\langle y^*, y^{**}\rangle|^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}}\right]^{p^*}\right\}$$
$$\leq C\left(\int_{B_{X^*}}|\langle x, x^*\rangle|^p \, d\mu_1(x^*)\right)^{\frac{1}{p}}\left(\int_{B_{Y^{**}}}|\langle y^*, y^{**}\rangle|^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}}.$$

To prove that (3)$\Longrightarrow$(1), let $x_1, \ldots, x_n \in X$ and $y_1^*, \ldots, y_n^* \in Y^*$. We have by (3.3)

$$|\langle T(x_i), y_i^*\rangle| \leq C\left(\int_{B_{X^*}}|x_i(x^*)|^p \, d\mu_1(x^*)\right)^{\frac{1}{p}}\left(\int_{B_{Y^{**}}}|y_i^*(y^{**})|^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}}$$

for all $1 \leq i \leq n$. Thus we obtain by using Hölder's inequality

$$\left|\sum_{i=1}^n \langle T(x_i), y_i^*\rangle\right| \leq \sum_{i=1}^n |\langle T(x_i), y_i^*\rangle|$$
$$\leq C\sum_{i=1}^n\left(\int_{B_{X^*}}|x_i(x^*)|^p \, d\mu_1(x^*)\right)^{\frac{1}{p}}\left(\int_{B_{Y^{**}}}|y_i^*(y^{**})|^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}}$$
$$\leq C\left(\int_{B_{X^*}}\sum_{i=1}^n|x_i(x^*)|^p \, d\mu_1(x^*)\right)^{\frac{1}{p}}\left(\sum_{i=1}^n\int_{B_{Y^{**}}}|y_i^*(y^{**})|^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}}$$

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$$\leq C \sup_{x^* \in B_{X^*}} \left(\sum_{i=1}^{n} |x_i(x^*)|^p \right)^{\frac{1}{p}} \sup_{y \in B_Y} \left\| (y_i^*(y))_{1 \leq i \leq n} \right\|_{l_{p^*}^n} .$$

This implies that $T \in \mathcal{N}_p(X, Y)$ and $n_p(T) \leq C$ and this concludes the proof. $\square$

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4. Relationships Between $\pi_p(X,Y)$, $D_p(X,Y)$ and $N_p(X,Y)$

In this section we investigate the relationships between the various classes of sublinear operators discussed in Section 2 and 4. We also give a relation between T and ∇T concerning the notion of Cohen p-nuclear.

Theorem 4.1. *Let X be a Banach space and Y be a Banach lattice. We have:*

1. $\mathcal{N}_p(X,Y) \subseteq \mathcal{D}_p(X,Y)$ *and* $d_p(T) \leq n_p(T)$.
2. $\mathcal{N}_p(X,Y) \subseteq \pi_p(X,Y)$ *and* $\pi_p(T) \leq n_p(T)$.

Proof. (1) Let $T \in \mathcal{N}_p(X,Y)$. Let $x \in X$ and $y^* \in Y^*$. We have by (3.3)

$$\begin{aligned}
|\langle T(x), y^* \rangle| &\leq n_p(T) \left(\int_{B_{X^*}} |x^*(x)|^p \, d\mu_1(x^*) \right)^{\frac{1}{p}} \left(\int_{B_{Y^{**}}} |y^*(y^{**})|^{p^*} \, d\mu_2(y^{**}) \right)^{\frac{1}{p^*}} \\
&\leq n_p(T) \sup_{x^* \in B_{X^*}} |x^*(x)| \left(\int_{B_{Y^{**}}} |y^*(y^{**})|^{p^*} \, d\mu_2(y^{**}) \right)^{\frac{1}{p^*}} \\
&\leq n_p(T) \|x\| \left(\int_{B_{Y^{**}}} |y^*(y^{**})|^{p^*} \, d\mu_2(y^{**}) \right)^{\frac{1}{p^*}}
\end{aligned}$$

so

$$|\langle T(x), y^* \rangle| \leq n_p(T) \|x\| \left(\int_{B_{Y^{**}}} |y^*(y^{**})|^{p^*} \, d\mu_2(y^{**}) \right)^{\frac{1}{p^*}}.$$

Then, by Theorem 2.5, T is strongly p-summing and $d_p(T) \leq n_p(T)$.

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(2) Let T be an operator in $\mathcal{N}_p(X,Y)$

$$\begin{aligned}\|T(x)\| &= \sup_{y^*\in B_{Y^*}} |\langle T(x), y^*\rangle| \\ &\le \sup_{y^*\in B_{Y^*}} n_p(T)\left(\int_{B_{X^*}} |x^*(x)|^p \, d\mu_1(x^*)\right)^{\frac{1}{p}} \left(\int_{B_{Y^{**}}} |y^*(y^{**})|^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}} \\ &\le n_p(T)\left(\int_{B_{X^*}} |x^*(x)|^p \, d\mu_1(x^*)\right)^{\frac{1}{p}} \sup_{y^*\in B_{Y^*}} \|y^*\| .\end{aligned}$$

Then

$$\|T(x)\| \le n_p(T)\left(\int_{B_{X^*}} |x^*(x)|^p \, d\mu_1(x^*)\right)^{\frac{1}{p}}$$

and by Theorem 2.7, T is p-summing and $\pi_p(T) \le n_p(T)$. The proof is complete. □

Theorem 4.2. *Let X be Banach space and Y, Z be two Banach lattices. Let $1 < p < \infty$.*

1. *Let $T \in \mathcal{SB}(X,Y)$ and $L \in \mathcal{SB}(Y,Z)$. Assume that L is increasing. If L is a strongly p-summing sublinear operator, and T is a p-summing sublinear operator, then $L \circ T$ is a Cohen p-nuclear sublinear operator and $n_p(L\circ T) \le d_p(L)\pi_p(T)$.*
2. *Consider u in $\mathcal{B}(Z,X)$ a p-summing operator and T in $\mathcal{SB}(X,Y)$ a strongly p-summing one. Then, $T \circ u$ is a Cohen p-nuclear sublinear operator and $n_p(T\circ u) \le d_p(T)\pi_p(u)$.*

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3. *Consider T in $\mathcal{SB}(X,Y)$ a p-summing operator and v in $\mathcal{B}(Y,Z)$ a strongly p-summing one. Assume that v is positive. Then, $v \circ T$ is a Cohen p-nuclear sublinear operator and $n_p(v \circ T) \leq d_p(v)\pi_p(T)$.*

Proof. (1) The operator $L \circ T$ is sublinear by Proposition 2.2(iii). Let $x \in X$ and $z^* \in Z^*$. By Theorem 2.5, we have

$$\begin{aligned} \left|\left\langle L \circ T\left(x\right), z^*\right\rangle\right| &= \left|\left\langle L\left(T\left(x\right)\right), z^*\right\rangle\right| \\ &\leq d_p(L)\left\|T\left(x\right)\right\| \left(\int_{B_{Z^{**}}} \left|z^*(z^{**})\right|^{p^*} d\lambda(z^{**})\right)^{\frac{1}{p^*}} \end{aligned}$$

and by Theorem 2.7

$$\leq d_p(L)\pi_p(T)\left(\int_{B_{X^*}} |x(x^*)|^p \, d\mu(x^*)\right)^{\frac{1}{p}} \left(\int_{B_{Z^{**}}} \left|z^*(z^{**})\right|^{p^*} d\lambda(z^{**})\right)^{\frac{1}{p^*}},$$

so

$$\begin{aligned} &\left|\left\langle L \circ T\left(x\right), z^*\right\rangle\right| \\ &\leq d_p(L)\pi_p(T)\left(\int_{B_{X^*}} |x(x^*)|^p \, d\mu(x^*)\right)^{\frac{1}{p}} \left(\int_{B_{Z^{**}}} \left|z^*(z^{**})\right|^{p^*} d\lambda(z^{**})\right)^{\frac{1}{p^*}}. \end{aligned}$$

This implies that $L \circ T \in \mathcal{N}_p(X,Y)$ and $n_p(L \circ T) \leq d_p(L)\pi_p(T)$.
(2) Follows immediately by using Proposition 2.2(ii), Theorem 2.5 and Theorem 2.7.
(3) The operator $v \circ T$ is sublinear by Proposition 2.2(i). Letting $x \in X$ and $z^* \in Z^*$, we have

$$\begin{aligned} \left|\left\langle v(T\left(x\right)), z^*\right\rangle\right| &= \left|\left\langle T\left(x\right), v^*(z^*)\right\rangle\right| \\ &\leq \left\|T\left(x\right)\right\| \left\|v^*(z^*)\right\| \end{aligned}$$

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because, v is strongly $p-$summing iff v^* is p^*-summing and $d_p(v) = \pi_{p^*}(v^*)$ (see [7, Theorem 2.2.1 part(ii)]), so

$$\begin{aligned}
&\|T(x)\| \, \|v^*(z^*)\| \\
&\leq d_p(v) \, \|T(x)\| \left(\int_{B_{Z^{**}}} |z^{**}(z^*)|^{p^*} \, d\mu_2(z^{**}) \right)^{\frac{1}{p^*}} \\
&\leq \pi_p(T) d_p(v) \left(\int_{B_{X^*}} |x^*(x)|^p \, d\mu_1(x^*) \right)^{\frac{1}{p}} \left(\int_{B_{Z^{**}}} |z^{**}(z^*)|^{p^*} \, d\mu_2(z^{**}) \right)^{\frac{1}{p^*}} .
\end{aligned}$$

This implies that $v \circ T \in \mathcal{N}_p(X, Z)$ and $n_p(v \circ T) \leq d_p(v)\pi_p(T)$. □

We now present an example of Cohen p-nuclear sublinear operators.

Example 4.1. Let $1 \leq p < \infty$ and $n, N \in \mathbb{N}$. Let u be a linear operator from l_2^n into l_p^N such that $S(x) = |u(x)|$. Let v be a linear operator from $L_q(\mu)$ $(1 \leq q < \infty)$ into l_2^n. Then $T = S \circ v$ is a Cohen 2-nuclear sublinear operator.

Proof. Indeed, $S(x) = |u(x)|$ is a strongly 2-summing sublinear operator by [3], and by [7, Lemma 3.2.2], v is 2-summing. Then by Theorem 4.2 part (2), $T = S \circ v$ is a Cohen 2-nuclear sublinear operator. □

Proposition 4.3. *Let X be a Banach lattice and Y be a complete Banach lattice. Let T be a bounded sublinear operator from X into Y. Suppose that T is positive Cohen p-nuclear $(1 < p < \infty)$. Then for all $S \in \mathcal{SB}(X, Y)$ such that $S \leq T$, S is positive Cohen p-nuclear.*

Proof. Letting $x_i \in X_1$ and $y_i^* \in Y_+^*$, by (1.2), we have

$$\langle S(x_i), y_i^* \rangle \leq \langle T(x_i), y_i^* \rangle$$

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and consequently, by (2.2),

$$-\langle S(x_i), y_i^*\rangle \leq \langle T(-x_i), y_i^*\rangle$$

for all $1 \leq i \leq n$. This implies that

$$\begin{aligned}\sum_{i=1}^{n} |\langle S(x_i), y_i^*\rangle| &\leq \sum_{i=1}^{n} \sup\{\langle T(x), y_i^*\rangle, \langle T(-x), y_i^*\rangle\}\\ &\leq \sum_{i=1}^{n} \sup\{|\langle T(x), y_i^*\rangle|, |\langle T(-x), y_i^*\rangle|\}\\ &\leq \sum_{i=1}^{n} |\langle T(x), y_i^*\rangle| + \sum_{i=1}^{n} |\langle T(-x), y_i^*\rangle|\end{aligned}$$

and hence

$$\sum_{i=1}^{n} |\langle S(x_i), y_i^*\rangle| \leq 2n_p^+(T) \sup_{x^*\in B_{X^*}} \|(x^*(x_i))\|_{l_p^n} \sup_{y\in B_Y^+} \|(y_i^*(y))\|_{l_{p^*}^n}.$$

Thus the operator S is positive Cohen p-nuclear and $n_p^+(S) \leq 2n_p^+(T)$. □

Remark 1. If S, T are any sublinear operators, we have no answer.

Corollary 4.4. *If T is positive Cohen p-nuclear ($1 < p < \infty$), then for all $u \in \nabla T$, u is positive Cohen p-nuclear and consequently u^* is positive p^*-summing.*

Proof. Let T be a positive Cohen p-nuclear sublinear operator. Then for all $u \in \nabla T$, u is positive Cohen p-nuclear (replacing S by u in Proposition 4.3). If u is positive Cohen p-nuclear (by Theorem 4.1, u is positive strongly p-summing), then u^* is p^*-summing (see [7, Theorem 2.2.1 part(ii)]). □

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We now study the converse of the preceding corollary with some conditions.

Theorem 4.5. *Let X be Banach space and Y be a complete Banach lattice. Let $T : X \to Y$ be a sublinear operator. Suppose that there is a constant $C > 0$, a set I, an ultrafilter $\mathcal{U}$ on I and $\{u_i\}_{i\in I} \subset \nabla T$ such that for all x in X and y^* in Y^*,*

$$|\langle u_i(x), y^*\rangle| \underset{\mathcal{U}}{\longrightarrow} |\langle T(x), y^*\rangle|$$

and $n_p(u_i) \leq C$ uniformly. Then, $T \in \mathcal{N}_p(X,Y)$ and $n_p(T) \leq C$.

Proof. Since u_i is Cohen p-nuclear, by Theorem 3.4 there is a Radon probability measure (μ_i, ν_i) on $K = B_{X^*} \times B_{Y^{**}}$ such that for all $x \in X$ and y^* in Y^*, we have

$$|\langle u_i(x), y^*\rangle| \leq n_p(u_i) \left(\int_{B_{X^*}} |x(x^*)|^p \, d\mu_i\right)^{\frac{1}{p}} \left(\int_{B_{Y^{**}}} |y^*(y^{**})|^{p^*} \, d\nu_i\right)^{\frac{1}{p^*}}.$$

As we have for all x in X and $y^* \in Y^*$,

$$|\langle u_i(x), y^*\rangle| \underset{\mathcal{U}}{\longrightarrow} |\langle T(x), y^*\rangle|$$

thus we obtain that for all x in X and $y^* \in Y^*$,

$$|\langle T(x), y^*\rangle| \leq \lim_{\mathcal{U}} n_p(u_i) \left(\int_{B_{X^*}} |x(x^*)|^p \, d\mu_i\right)^{\frac{1}{p}} \left(\int_{B_{Y^{**}}} |y^*(y^{**})|^{p^*} \, d\nu_i\right)^{\frac{1}{p^*}}.$$

The set $K = B_{X^*} \times B_{Y^{**}}$ is weak* compact, hence (μ_i, ν_i) converge weak* to a probability (μ, ν) on $K = B_{X^*} \times B_{Y^{**}}$ and consequently, for all x in X and $y^* \in Y^*$

$$|\langle T(x), y^*\rangle| \leq C \left(\int_{B_{X^*}} |x(x^*)|^p \, d\mu\right)^{\frac{1}{p}} \left(\int_{B_{Y^{**}}} |y^*(y^{**})|^{p^*} \, d\nu\right)^{\frac{1}{p^*}}.$$

This implies that $n_p(T) \leq C$. □

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