

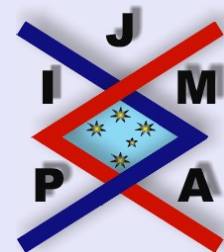
# Journal of Inequalities in Pure and Applied Mathematics

## GENERALIZATIONS OF A CLASS OF INEQUALITIES FOR PRODUCTS

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Abstract

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## Abstract

In this paper, a class of inequalities for products of positive numbers are generalized.

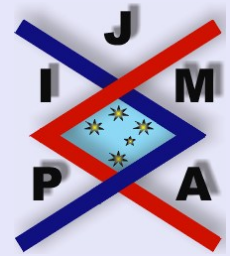
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*Key words:* Inequality, Product, Arithmetic-geometric mean inequality, Jensen's inequality.

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# 1. Introduction and Main Results

In 1987, H.-Sh. Huang [2] proved the following algebraic inequality for products:

$$(1.1) \quad \prod_{i=1}^n \left( \frac{1}{x_i} + x_i \right) \geq \left( n + \frac{1}{n} \right)^n,$$

where  $x_1, x_2, \dots, x_n$  are positive real numbers with  $\sum_{i=1}^n x_i = 1$ .

In 2002, X.-Y. Yang [4] considered an analogous form of inequality (1.1) and posed an interesting open problem as follows.

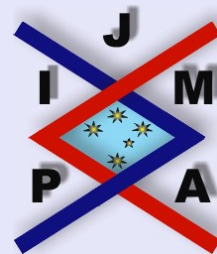
**Open Problem.** Assume  $x_1, x_2, \dots, x_n$  are positive real numbers with  $\sum_{i=1}^n x_i = 1$  for  $n \geq 3$ . Then

$$(1.2) \quad \prod_{i=1}^n \left( \frac{1}{x_i} - x_i \right) \geq \left( n - \frac{1}{n} \right)^n.$$

In [1], Ch.-H. Dai and B.-H. Liu gave an affirmative answer to the above open problem.

In this article, by using the arithmetic-geometric mean inequality, inequalities (1.1) and (1.2) are refined and generalized as follows.

**Theorem 1.1.** Let  $x_1, x_2, \dots, x_n$  be positive real numbers with  $\sum_{i=1}^n x_i = k$



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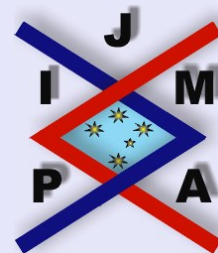
and  $k \leq n$ , where  $k$  and  $n$  are natural numbers. Then we have for  $m \in \mathbb{N}$

$$(1.3) \quad \prod_{i=1}^n \left( \frac{1}{x_i^m} + x_i^m \right) \geq \left( \frac{n^m}{k^m} + \frac{k^m}{n^m} \right)^n \left( \prod_{i=1}^n \frac{nx_i}{k} \right)^{\frac{m(k^{2m} - n^{2m})}{k^{2m} + n^{2m}}} \\ \geq \left( \frac{n^m}{k^m} + \frac{k^m}{n^m} \right)^n .$$

**Theorem 1.2.** Let  $x_1, x_2, \dots, x_n$  be positive real numbers with  $\sum_{i=1}^n x_i = k$  for  $k \leq 1$  and  $n \geq 3$ . Then for  $m \in \mathbb{N}$  we have

$$(1.4) \quad \prod_{i=1}^n \left( \frac{1}{x_i^m} - x_i^m \right) \geq \left( \frac{n^m}{k^m} - \frac{k^m}{n^m} \right)^n \left( \prod_{i=1}^n \frac{nx_i}{k} \right)^{\frac{m}{n} - \frac{m}{3}} \\ \geq \left( \frac{n^m}{k^m} - \frac{k^m}{n^m} \right)^n .$$

**Remark 1.1.** Choosing  $m = 1$  and  $k = 1$  in Theorem 1.1 and Theorem 1.2, we can obtain inequalities (1.1) and (1.2) respectively.



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## 2. Lemmas

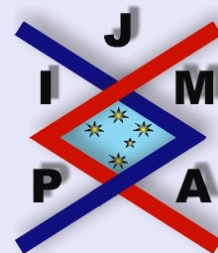
To prove Theorem 1.1 and Theorem 1.2, we will use following lemmas.

**Lemma 2.1.** Let  $x_1, x_2, \dots, x_n$  be positive real numbers with  $\sum_{i=1}^n x_i = 1$  and  $n \geq 3$ . Then

$$(2.1) \quad \prod_{i=1}^n \left( \frac{1}{x_i} - x_i \right) \geq \left( n - \frac{1}{n} \right)^n \left[ \prod_{i=1}^n (n x_i) \right]^{\frac{1}{n} - \frac{1}{3}}.$$

*Proof.* From the conditions of Lemma 2.1 and by using the arithmetic-geometric mean inequality, we have for  $1 \leq p, q \leq n$  and  $p \neq q$

$$\begin{aligned} (2.2) \quad (1 - x_p)(1 - x_q) &= 1 - x_p - x_q + x_p x_q \\ &= \sum_{k \neq p, q} x_k + x_p x_q \\ &= \sum_{k \neq p, q} \left( \underbrace{\frac{x_k}{n} + \dots + \frac{x_k}{n}}_n \right) + x_p x_q \\ &\geq [n(n-2) + 1] \left[ \prod_{k \neq p, q} \left( \frac{x_k}{n} \right)^n x_p x_q \right]^{\frac{1}{n(n-2)+1}} \\ &= (n-1)^2 \left[ \left( \frac{1}{n} \right)^{n(n-2)} \left( \prod_{k \neq p, q} x_k \right)^n x_p x_q \right]^{\frac{1}{(n-1)^2}} \end{aligned}$$



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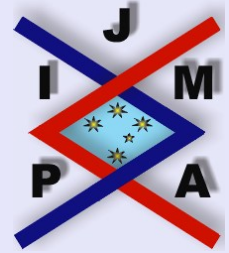


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$$= (n-1)^2 \left(\frac{1}{n}\right)^{\frac{n(n-2)}{(n-1)^2}} \left(\prod_{k=1}^n x_i\right)^{\frac{n}{(n-1)^2}} (x_p x_q)^{\frac{1}{1-n}}$$

then

$$(2.3) \quad \prod_{i=1}^n (1-x_i) \geq (n-1)^n \left(\frac{1}{n}\right)^{\frac{n^2(n-2)}{2(n-1)^2}} \left(\prod_{i=1}^n x_i\right)^{\frac{n^2-2n+2}{2(n-1)^2}}.$$

By the arithmetic-geometric mean inequality, we obtain

$$(2.4) \quad \begin{aligned} \prod_{i=1}^n (1+x_i) &= \prod_{i=1}^n \left( \underbrace{\frac{1}{n} + \dots + \frac{1}{n}}_n + x_i \right) \\ &\geq \prod_{i=1}^n \left\{ (n+1) \left[ \left(\frac{1}{n}\right)^n x_i \right]^{\frac{1}{n+1}} \right\} \\ &= (n+1)^n \left(\frac{1}{n}\right)^{\frac{n^2}{n+1}} \left(\prod_{i=1}^n x_i\right)^{\frac{1}{n+1}}. \end{aligned}$$

Utilizing (2.3) and (2.4) yields

$$(2.5) \quad \begin{aligned} \prod_{i=1}^n \left(\frac{1}{x_i} - x_i\right) &= \left[ \prod_{i=1}^n (1-x_i) \right] \left[ \prod_{i=1}^n (1+x_i) \right] \prod_{i=1}^n \frac{1}{x_i} \end{aligned}$$

$$\begin{aligned} &\geq (n-1)^n(n+1)^n \left(\frac{1}{n}\right)^{\frac{n^2(n-2)}{2(n-1)^2} + \frac{n^2}{n+1}} \left(\prod_{i=1}^n x_i\right)^{\frac{n^2-2n+2}{2(n-1)^2} + \frac{1}{n+1} - 1} \\ &= \left(n - \frac{1}{n}\right)^n \left[\prod_{i=1}^n (nx_i)\right]^{\frac{-n^3+3n^2-2n+2}{2(n+1)(n-1)^2}}. \end{aligned}$$

From the arithmetic-geometric mean inequality and  $\sum_{i=1}^n x_i = 1$  for  $n \geq 3$ , we have

$$(2.6) \quad 0 < \prod_{i=1}^n (nx_i) \leq \left(\sum_{i=1}^n x_i\right)^n = 1.$$

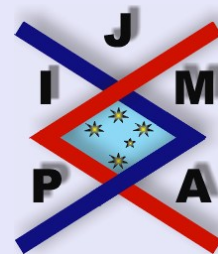
Since  $n \geq 3$ , it follows that

$$\begin{aligned} \frac{-n^3 + 3n^2 - 2n + 2}{2(n+1)(n-1)^2} &= \frac{1}{n} - \frac{1}{3} - \frac{n(n-3)(n^2+2n+8) + 10n + 6}{6n(n+1)(n-1)^2} \\ &< \frac{1}{n} - \frac{1}{3} \\ &\leq 0. \end{aligned}$$

Therefore, by the monotonicity of the exponential function, we obtain

$$(2.7) \quad \left[\prod_{i=1}^n (nx_i)\right]^{\frac{-n^3+3n^2-2n+2}{2(n+1)(n-1)^2}} \geq \left[\prod_{i=1}^n (nx_i)\right]^{\frac{1}{n} - \frac{1}{3}}.$$

Combining inequalities (2.5) and (2.7) leads to inequality (2.1). □




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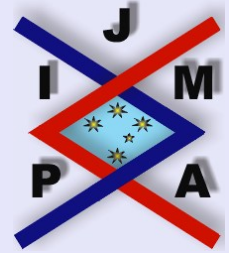


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**Lemma 2.2.** Let  $x_1, x_2, \dots, x_n$  be positive real numbers with  $\sum_{i=1}^n x_i = 1$  for  $n \geq 3$  and  $m$  a natural number. Then

$$(2.8) \quad \prod_{i=1}^n \left( \frac{1}{x_i^m} - x_i^m \right) \geq \left( n^m - \frac{1}{n^m} \right)^n \left[ \prod_{i=1}^n (nx_i) \right]^{\frac{m}{n} - \frac{m}{3}}.$$

*Proof.* Using the arithmetic-geometric mean inequality, we obtain

$$(2.9) \quad \begin{aligned} \sum_{j=0}^{m-1} x_i^{2j} &= \sum_{j=0}^{m-2} \left( \underbrace{\frac{x_i^{2j}}{n^{2(m-j-1)}} + \dots + \frac{x_i^{2j}}{n^{2(m-j-1)}}}_{n^{2(m-j-1)}} \right) + x_i^{2m-2} \\ &\geq \left[ \sum_{j=0}^{m-1} n^{2j} \right] \left[ x_i^{2(m-1)} \prod_{j=0}^{m-2} \left( \frac{x_i^{2j}}{n^{2(m-j-1)}} \right) n^{2(m-j-1)} \right] \sum_{j=0}^{m-1} n^{2j} \\ &= \frac{n^{2m} - 1}{n^{2(m-1)}(n^2 - 1)} (nx_i)^{\frac{\sum_{j=0}^{m-1} [2(m-j-1)]n^{2j}}{\sum_{j=0}^{m-1} n^{2j}}}. \end{aligned}$$

Hence

$$(2.10) \quad \begin{aligned} \frac{1}{x_i^m} - x_i^m &= \left( \frac{1}{x_i} - x_i \right) x_i^{1-m} \sum_{j=0}^{m-1} x_i^{2j} \\ &\geq \left( \frac{1}{x_i} - x_i \right) x_i^{1-m} \frac{n^{2m} - 1}{n^{2(m-1)}(n^2 - 1)} (nx_i)^{\frac{\sum_{j=0}^{m-1} [2(m-j-1)]n^{2j}}{\sum_{j=0}^{m-1} n^{2j}}} \\ &= \left( \frac{1}{x_i} - x_i \right) \frac{n^{2m} - 1}{n^{m-1}(n^2 - 1)} (nx_i)^{\frac{(m-1) \sum_{j=0}^{m-1} n^{2j} - \sum_{j=1}^{m-1} 2jn^{2j}}{\sum_{j=0}^{m-1} n^{2j}}}, \end{aligned}$$



and then

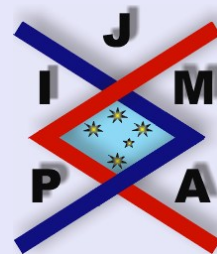
$$(2.11) \quad \prod_{i=1}^n \left( \frac{1}{x_i^m} - x_i^m \right) \geq n^{n(1-m)} \left[ \frac{n^{2m} - 1}{n^2 - 1} \right]^n \left[ \prod_{i=1}^n \left( \frac{1}{x_i} - x_i \right) \right] \left[ \prod_{i=1}^n (n x_i) \right]^{\frac{(m-1) \sum_{j=0}^{m-1} n^{2j} - \sum_{j=1}^{m-1} 2j n^{2j}}{\sum_{j=0}^{m-1} n^{2j}}}$$

In the following, we prove that for  $n \geq 3$

$$(2.12) \quad \frac{(m-1) \sum_{j=0}^{m-1} n^{2j} - \sum_{j=1}^{m-1} 2j n^{2j}}{\sum_{j=0}^{m-1} n^{2j}} \leq (m-1) \left( \frac{1}{n} - \frac{1}{3} \right).$$

For  $m = 1$ , the equality in (2.12) holds. For  $m \geq 2$ , we have

$$(2.13) \quad \frac{(m-1) \sum_{j=0}^{m-1} n^{2j} - \sum_{j=1}^{m-1} 2j n^{2j}}{\sum_{j=0}^{m-1} n^{2j}} - (m-1) \left( \frac{1}{n} - \frac{1}{3} \right) = \frac{(m-1) \left( \frac{4}{3} - \frac{1}{n} \right) \sum_{j=0}^{m-1} n^{2j} - \sum_{j=1}^{m-1} 2j n^{2j}}{\sum_{j=0}^{m-1} n^{2j}} = \frac{(m-1) \left( \frac{4}{3} - \frac{1}{n} \right) \sum_{j=0}^{m-2} n^{2j} - \sum_{j=1}^{m-2} 2j n^{2j}}{\sum_{j=0}^{m-2} n^{2j}} - \frac{(m-1) \left( \frac{1}{n} + \frac{2}{3} \right) n^{2(m-1)}}{\sum_{j=0}^{m-2} n^{2j}}$$



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$$\begin{aligned}
 &= \frac{(m-1) \left[ \frac{n^{2(m-1)}-1}{n^2-1} \left( \frac{4}{3} - \frac{1}{n} \right) - \left( \frac{1}{n} + \frac{2}{3} \right) n^{2(m-1)} \right] - \sum_{j=1}^{m-2} 2jn^{2j}}{\sum_{j=0}^{m-2} n^{2j}} \\
 &< \frac{(m-1) \left[ \frac{1}{8} \left( \frac{4}{3} - \frac{1}{n} \right) n^{2(m-1)} - \left( \frac{1}{n} + \frac{2}{3} \right) n^{2(m-1)} \right] - \sum_{j=1}^{m-2} 2jn^{2j}}{\sum_{j=0}^{m-2} n^{2j}} \\
 &= \frac{(m-1) \left( -\frac{9}{8n} - \frac{1}{2} \right) n^{2(m-1)} - \sum_{j=1}^{m-2} 2jn^{2j}}{\sum_{j=0}^{m-2} n^{2j}} \\
 &< 0.
 \end{aligned}$$

Hence inequality (2.12) holds.

Considering inequality (2.6) and the monotonicity of the exponential function and combining inequality (2.11) with (2.12) reveals

$$\begin{aligned}
 (2.14) \quad &\prod_{i=1}^n \left( \frac{1}{x_i^m} - x_i^m \right) \\
 &\geq n^{n(1-m)} \left( \frac{n^{2m}-1}{n^2-1} \right)^n \left[ \prod_{i=1}^n \left( \frac{1}{x_i} - x_i \right) \right] \left[ \prod_{i=1}^n (nx_i) \right]^{(m-1)\left(\frac{1}{n}-\frac{1}{3}\right)}.
 \end{aligned}$$

Substituting inequality (2.1) into (2.14) produces

$$\begin{aligned}
 (2.15) \quad &\prod_{i=1}^n \left( \frac{1}{x_i^m} - x_i^m \right) \geq n^{n(1-m)} \left( \frac{n^{2m}-1}{n^2-1} \right)^n \left( n - \frac{1}{n} \right)^n \\
 &\quad \times \left[ \prod_{i=1}^n (nx_i) \right]^{\frac{1}{n}-\frac{1}{3}} \left[ \prod_{i=1}^n (nx_i) \right]^{(m-1)\left(\frac{1}{n}-\frac{1}{3}\right)}
 \end{aligned}$$

$$= \left( n^m - \frac{1}{n^m} \right)^n \left[ \prod_{i=1}^n (nx_i) \right]^{m\left(\frac{1}{n} - \frac{1}{3}\right)}.$$

The proof is complete.  $\square$

**Lemma 2.3.** *Let  $x_1, x_2, \dots, x_n$  be positive real numbers with  $\sum_{i=1}^n x_i = k \leq 1$  for  $n \geq 3$ . Then for any natural number  $m$ , we have*

$$(2.16) \quad \prod_{i=1}^n \left( \frac{1}{x_i^m} - x_i^m \right) \geq \left( n^m - \frac{1}{n^m} \right)^{-n} \left( \frac{n^m}{k^m} - \frac{k^m}{n^m} \right)^n \prod_{i=1}^n \left( \frac{k^m}{x_i^m} - \frac{x_i^m}{k^m} \right).$$

*Proof.* It is easy to see that

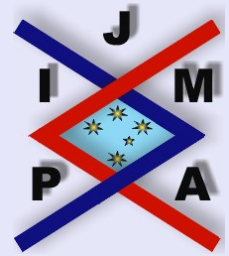
$$(2.17) \quad \prod_{i=1}^n \left( \frac{1}{x_i^m} - x_i^m \right) \prod_{i=1}^n \left( \frac{k^m}{x_i^m} - \frac{x_i^m}{k^m} \right)^{-1} = k^{nm} \prod_{i=1}^n \frac{1 - x_i^{2m}}{k^{2m} - x_i^{2m}}.$$

Define

$$(2.18) \quad f(x) = \ln \frac{1 - x^{2m}}{k^{2m} - x^{2m}}$$

for  $x \in (0, k)$ ,  $m \geq 1$  and  $k \leq 1$ . Direct calculation shows that

$$(2.19) \quad f'(x) = \frac{2m(1 - k^{2m})x^{2m-1}}{(1 - x^{2m})(k^{2m} - x^{2m})},$$



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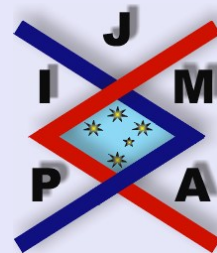


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$$(2.20) \quad f''(x) = \frac{2mx^{2(m-1)}(1-k^{2m})}{(1-x^{2m})^2(k^{2m}-x^{2m})^2} [(2m-1)(1-x^{2m})(k^{2m}-x^{2m}) + 2mx^{2m}(k^{2m}-x^{2m}+1-x^{2m})] \geq 0.$$

This means that  $f$  is convex in the interval  $(0, k)$ . Using Jensen's inequality [3], we obtain

$$(2.21) \quad \frac{1}{n} \sum_{i=1}^n \ln \frac{1-x_i^{2m}}{k^{2m}-x_i^{2m}} \geq \ln \frac{1-\left[\frac{1}{n} \sum_{i=1}^n x_i\right]^{2m}}{k^{2m}-\left[\frac{1}{n} \sum_{i=1}^n x_i\right]^{2m}}$$

for any  $0 < x_i < k \leq 1$  and  $i \in \mathbb{N}$ . Using  $\sum_{i=1}^n x_i = k$  in (2.21), it follows that

$$(2.22) \quad \prod_{i=1}^n \frac{1-x_i^{2m}}{k^{2m}-x_i^{2m}} \geq \left( \frac{1-\frac{k^{2m}}{n^{2m}}}{k^{2m}-\frac{k^{2m}}{n^{2m}}} \right)^n,$$

therefore

$$(2.23) \quad k^{nm} \prod_{i=1}^n \frac{1-x_i^{2m}}{k^{2m}-x_i^{2m}} \geq \left( n^m - \frac{1}{n^m} \right)^{-n} \left( \frac{n^m}{k^m} - \frac{k^m}{n^m} \right)^n.$$

Substituting (2.17) into (2.23) leads to (2.16). The proof is complete. □

### 3. Proofs of Theorems

*Proof of Theorem 1.1.* Using the arithmetic-geometric mean inequality, we obtain

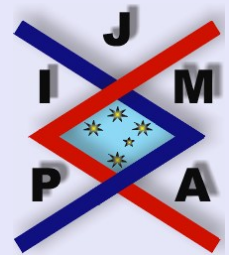
$$\begin{aligned}
 \frac{1}{x_i^m} + x_i^m &= \underbrace{\frac{1}{n^{2m}x_i^m} + \cdots + \frac{1}{n^{2m}x_i^m}}_{n^{2m}} + \underbrace{\frac{x_i^m}{k^{2m}} + \cdots + \frac{x_i^m}{k^{2m}}}_{k^{2m}} \\
 (3.1) \quad &\geq (n^{2m} + k^{2m}) \left[ \left( \frac{1}{n^{2m}x_i^m} \right)^{n^{2m}} \left( \frac{x_i^m}{k^{2m}} \right)^{k^{2m}} \right]^{\frac{1}{k^{2m} + n^{2m}}} \\
 &= (n^{2m} + k^{2m}) \left( k^{-2mk^{2m}} n^{-2mn^{2m}} x_i^{mk^{2m} - mn^{2m}} \right)^{\frac{1}{k^{2m} + n^{2m}}},
 \end{aligned}$$

therefore

$$\begin{aligned}
 (3.2) \quad &\prod_{i=1}^n \left( \frac{1}{x_i^m} + x_i^m \right) \\
 &\geq (n^{2m} + k^{2m})^n \left( k^{-2mk^{2m}} n^{-2mn^{2m}} \right)^{\frac{n}{k^{2m} + n^{2m}}} \left( \prod_{i=1}^n x_i \right)^{\frac{m(k^{2m} - n^{2m})}{k^{2m} + n^{2m}}},
 \end{aligned}$$

that is

$$(3.3) \quad \prod_{i=1}^n \left( \frac{1}{x_i^m} + x_i^m \right) \geq \left( \frac{n^m}{k^m} + \frac{k^m}{n^m} \right)^n \left( \prod_{i=1}^n \frac{nx_i}{k} \right)^{\frac{m(k^{2m} - n^{2m})}{k^{2m} + n^{2m}}}.$$



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From  $\sum_{i=1}^n x_i = k$  and the arithmetic-geometric mean inequality, it follows that

$$(3.4) \quad \prod_{i=1}^n \frac{nx_i}{k} \leq \left( \sum_{i=1}^n \frac{x_i}{k} \right)^n = 1,$$

and then, considering  $k \leq n$ , we have

$$(3.5) \quad \left( \frac{n^m}{k^m} + \frac{k^m}{n^m} \right)^n \left( \prod_{i=1}^n \frac{nx_i}{k} \right)^{\frac{m(k^{2m}-n^{2m})}{k^{2m}+n^{2m}}} \geq \left( \frac{n^m}{k^m} + \frac{k^m}{n^m} \right)^n.$$

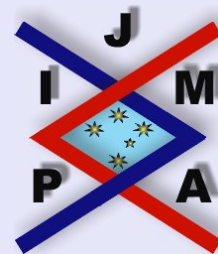
Inequality (1.3) is then deduced by combining (3.3) and (3.5). This completes the proof of Theorem 1.1.  $\square$

*Proof of Theorem 1.2.* Applying  $\sum_{i=1}^n \frac{x_i}{k} = 1$  to Lemma 2.2, we have

$$(3.6) \quad \prod_{i=1}^n \left( \frac{k^m}{x_i^m} - \frac{x_i^m}{k^m} \right) \geq \left( n^m - \frac{1}{n^m} \right)^n \left( \prod_{i=1}^n \frac{nx_i}{k} \right)^{\frac{m}{n} - \frac{m}{3}}.$$

Substituting inequality (3.6) into Lemma 2.3 gives

$$(3.7) \quad \begin{aligned} \prod_{i=1}^n \left( \frac{1}{x_i^m} - x_i^m \right) &\geq \left( n^m - \frac{1}{n^m} \right)^{-n} \left( \frac{n^m}{k^m} - \frac{k^m}{n^m} \right)^n \prod_{i=1}^n \left( \frac{k^m}{x_i^m} - \frac{x_i^m}{k^m} \right) \\ &\geq \left( \frac{n^m}{k^m} - \frac{k^m}{n^m} \right)^n \left( \prod_{i=1}^n \frac{nx_i}{k} \right)^{\frac{m}{n} - \frac{m}{3}}. \end{aligned}$$



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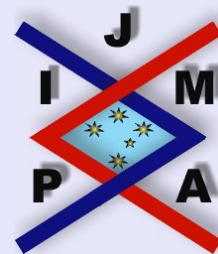
Since

$$(3.8) \quad 0 < \prod_{i=1}^n \frac{nx_i}{k} \leq \left( \sum_{i=1}^n \frac{x_i}{k} \right)^n = 1$$

and  $\frac{m}{n} - \frac{m}{3} \leq 0$ , we have

$$(3.9) \quad \left( \frac{n^m}{k^m} - \frac{k^m}{n^m} \right)^n \left( \prod_{i=1}^n \frac{nx_i}{k} \right)^{\frac{m}{n} - \frac{m}{3}} \geq \left( \frac{n^m}{k^m} - \frac{k^m}{n^m} \right)^n.$$

Combining (3.7) and (3.9), we immediately obtain inequality (1.4). This completes the proof of Theorem 1.2.  $\square$



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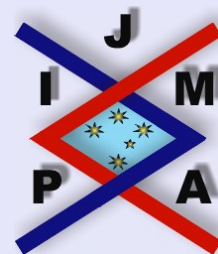
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